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ON LIFTING OF AUTOMORPHIC FORMS

by Hiroshi SAITO

Q. - Let  $F$  be a totally real algebraic number field with the degree  $[F:\underline{Q}] = \ell$ , and  $\mathcal{O}$  its maximal order. For the sake of simplicity, we assume that the class number of  $F$  is one, and  $\mathcal{O}$  has a unit with arbitrary signature distribution. For an even positive integer  $k$  and for the subgroup  $\Gamma = GL_2(\mathcal{O})^+$  of  $GL_2(\mathcal{O})$  consisting of all elements with totally positive determinants, we denote by  $S_k(\Gamma)$  the space of Hilbert cusp forms of weight  $k$  with respect to  $\Gamma$ , namely the set of all holomorphic functions  $f$  on the  $\ell$ -fold product of the complex upper half plane  $H$ , which satisfy

1°  $f(\gamma^{(1)} z_1, \gamma^{(2)} z_2, \dots, \gamma^{(\ell)} z_\ell) = \prod_i (c^{(i)} z_i + d^{(i)})^k f(z_1, \dots, z_\ell)$   
 for  $\gamma \in \Gamma$ ,

2°  $f$  vanishes at every cusp,

where  $\gamma^{(i)} = \begin{pmatrix} a^{(i)} & b^{(i)} \\ c^{(i)} & d^{(i)} \end{pmatrix}$  are all distinct conjugates of  $\gamma$  over  $\underline{Q}$ . It is

known that an element  $f$  of  $S_k(\Gamma)$  has a Fourier expansion of the form

$$f(z_1, \dots, z_\ell) = \sum_{\mathfrak{u}} C(\mathfrak{u}) \sum_{(\nu) = \mathfrak{u}/\mathfrak{D}, \nu > 0} \exp 2\pi i (\nu^{(1)} z_1 + \dots + \nu^{(\ell)} z_\ell),$$

where  $\mathfrak{u}$  runs through all integral ideals of  $F$ , and  $\mathfrak{D}$  is the different of the extension  $F/\underline{Q}$ . We denote by  $\Phi_f$  the associated Dirichlet series of  $f$ , that is,

$$\Phi_f(s) = \sum_{\mathfrak{u}} C(\mathfrak{u}) N\mathfrak{u}^{-s}.$$

For a place (archimedean or non-archimedean) of  $F$ , we denote by  $F_v$  the completion of  $F$  at  $v$ , and for a non-archimedean prime  $v = \mathfrak{p}$ , we denote by  $\mathcal{O}_{\mathfrak{p}}$  the ring of all  $\mathfrak{p}$ -adic integers of  $F_{\mathfrak{p}}$ . Let  $F_A$  be the adèle ring of  $F$ , and  $\mathfrak{u}_F$  be the open subgroup of  $GL_2(F_A)$  given by

$$\prod_{\mathfrak{p}: \text{non-archimedean}} GL_2(\mathcal{O}_{\mathfrak{p}}) \times \prod_{v: \text{archimedean}} GL_2(F_v).$$

Then we can consider the Hecke ring  $R(\mathfrak{u}_F, GL_2(F_A))$  with respect to  $GL_2(F_A)$  and  $\mathfrak{u}_F$ , and its action  $T$  on  $S_k(\Gamma)$  as in G. SHIMURA [9]. It is known that  $S_k(\Gamma)$  has a basis consisting of common eigen functions for all Hecke operators and that if  $f$  is a common eigen function for all Hecke operators with  $C(\mathcal{O}) = 1$ , then the associated Dirichlet series  $\Phi_f$  has an Euler product of the form

$$\Phi_f(s) = \prod_{\mathfrak{p}} (1 - C(\mathfrak{p})N\mathfrak{p}^{-s} + N\mathfrak{p}^{k-1-s})^{-1},$$

where  $\mathfrak{p}$  runs through all prime ideals of  $F$ .

1. - On the following, we assume that  $F$  is a totally real algebraic number field which satisfies

1°  $F$  is a cyclic extension of  $\underline{Q}$  with a prime degree  $\ell$ ,

2°  $F$  is a tamely ramified extension of  $\underline{Q}$ ,

3° The class number of  $F$  is one,

4° The index  $[E:E_+]$  is  $2^\ell$ ,

where  $E$  is the group of all units of  $\mathcal{O}$  and  $E_+$  is its subgroup consisting of all totally positive units. It follows from these conditions that the conductor of the extension  $F/\underline{Q}$  is a prime  $q$  with  $q \equiv 1 \pmod{\ell}$ .

We fix an embedding of  $F$  into the real number field  $\underline{R}$ , and consider  $F$  as a subfield of  $\underline{R}$ . We fix a generator  $\sigma$  of the Galois group  $\text{Gal}(F/\underline{Q})$ . With this  $\sigma$ , we consider  $\text{GL}_2(F)$  as a subgroup of  $\text{GL}_2(\underline{R})^\ell$  by

$$\gamma \longmapsto (\gamma, \sigma\gamma, \dots, \sigma^{\ell-1}\gamma) \text{ for } \gamma \in \text{GL}_2(F).$$

For this fixed generator  $\sigma$ , we define a linear operator  $T_\sigma$  on  $S_k(\Gamma)$  by

$$T_\sigma f(z_1, z_2, \dots, z_\ell) = f(z_2, \dots, z_\ell, z_1).$$

Using this  $T_\sigma$  and Hecke operators, we define a subspace  $SS_k(\Gamma)$  of  $S_k(\Gamma)$  as follows

$$SS_k(\Gamma) = \{f \in S_k(\Gamma) ; T_\sigma T(e) f = T(e) T_\sigma f \text{ for any } e \in R(\underline{u}_F, \text{GL}_2(\underline{F}_A))\}.$$

It is easy to see that this subspace is stable under the action of Hecke operators, and that if  $f$  is a common eigen function for all Hecke operators, then

$$f \in SS_k(\Gamma) \iff C(\mathfrak{u}) = C(\sigma\mathfrak{u}) = \dots = C(\sigma^{\ell-1}\mathfrak{u}) \text{ for any integral ideal } \mathfrak{u}.$$

Our purpose is to show that this subspace  $SS_k(\Gamma)$  is closely related with spaces of cusp forms of one variable, in fact, this subspace can be lifted from spaces of cusp forms of one variable.

Let  $S_k(\text{SL}_2(\underline{Z}))$  be the space of cusp forms of weight  $k$  with respect to  $\text{SL}_2(\underline{Z})$ . Let us introduce other spaces of cusp forms of one variable. From the condition on  $F$ , it follows that there exist  $\ell - 1$  characters mod  $q$  of order  $\ell$  corresponding to the extension  $F/\underline{Q}$  in the sense of class field theory. We denote them by  $\chi_i$ ,  $1 \leq i \leq \ell - 1$ . For each character  $\chi_i$ , we denote by  $S_k(\Gamma_0(q), \chi_i)$  the space of cusp forms  $g$  which satisfy

$$g(\gamma Z) = (cZ + d)^k \chi_i(d) g(Z) \text{ for } \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(q),$$

where

$$\Gamma_0(q) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{SL}_2(\underline{Z}), c \equiv 0 \pmod{q} \right\}.$$

The Hecke ring  $R(\underline{u}_Q, \text{GL}_2(\underline{Q}_A))$  acts on these spaces of cusp forms. On  $S_k(\text{SL}_2(\underline{Z}))$ , it acts in the usual manner. On the other spaces, we make it act in the following way. For a prime  $p$ , let  $T(p)$  and  $T(p, p)$  be the elements of  $R(\underline{u}_Q, \text{GL}_2(\underline{Q}_A))$

given in the next section. For  $p \neq q$ ,  $T(p)$  and  $T(p, p)$  acts in the usual manner. For  $p = q$ , we define the action of  $T(q)$  and  $T(q, q)$  on  $S_k(\Gamma_0(q), \chi_i)$  by

$$T(q) g = g \left[ \Gamma_0(q) \begin{pmatrix} 1 & 0 \\ 0 & q \end{pmatrix} \Gamma_0(q) \right]_{k, \chi_i} + g \left[ \Gamma_0(q) \begin{pmatrix} 1 & 0 \\ 0 & q \end{pmatrix} \Gamma_0(q) \right]_{k, \chi_i}^* ,$$

$$T(q, q) g = q^{k-2} g ,$$

where  $\left[ \Gamma_0(q) \begin{pmatrix} 1 & 0 \\ 0 & q \end{pmatrix} \Gamma_0(q) \right]_{k, \chi_i}$  is the action of the double coset  $\Gamma_0(q) \begin{pmatrix} 1 & 0 \\ 0 & q \end{pmatrix} \Gamma_0(q)$  defined in G. SHIMURA [10], and  $[\ ]_{k, \chi_i}^*$  means the adjoint operator of  $[\ ]_{k, \chi_i}$  with respect to the Petersson inner product. To compare the above two kinds of representations of Hecke rings, namely the representation of  $R(\mathfrak{u}_F, GL_2(\mathbb{F}_A))$  on  $SS_k(\Gamma)$  and those of  $R(\mathfrak{u}_Q, GL_2(\mathbb{Q}_A))$  on  $S_k(SL_2(\mathbb{Z}))$  and  $S_k(\Gamma_0(q), \chi_i)$ , we define a natural homomorphism  $\lambda$  from  $R(\mathfrak{u}_F, GL_2(\mathbb{F}_A))$  to  $R(\mathfrak{u}_Q, GL_2(\mathbb{Q}_A))$  in the next section. First assuming this  $\lambda$ , we will state our theorem. By means of  $\lambda$ , the spaces  $S_k(SL_2(\mathbb{Z}))$  and  $S_k(\Gamma_0(q), \chi_i)$ ,  $1 \leq i \leq \ell - 1$ , can be regarded as  $R(\mathfrak{u}_F, GL_2(\mathbb{F}_A))$ -modules. On these notations, we can prove [7], the following theorem.

THEOREM. - If  $k \geq 4$ , there exists a subspace  $S$  of  $\bigoplus_{i=1}^{\ell-1} S_k(\Gamma_0(q), \chi_i)$  such that

$$SS_k(\Gamma) \simeq S_k(SL_2(\mathbb{Z})) \oplus S ,$$

(and  $\bigoplus_{i=1}^{\ell-1} S_k(\Gamma_0(q), \chi_i) \simeq S \oplus S$ ) as  $R(\mathfrak{u}_F, GL_2(\mathbb{F}_A))$ -modules.

Let  $g \in S_k(SL_2(\mathbb{Z}))$  be a common eigen function for all Hecke operators and let  $f \in SS_k(\Gamma)$  be a common eigen function for all Hecke operators which corresponds to  $g$  in the above isomorphism, then it holds the following relation between the associated Dirichlet series  $\varphi_g$  of  $g$  and  $\Phi_f$  of  $f$ , namely,

$$\Phi_f(s) = \varphi_g(s) \prod_{i=1}^{\ell-1} \varphi_{\chi_i}(s, \chi_i) ,$$

where

$$\varphi_g(s, \chi_i) = \sum_{n=1}^{\infty} a_n \chi_i(n) n^{-s} \quad \text{for} \quad \varphi_g(s) = \sum_{n=1}^{\infty} a_n n^{-s} .$$

This theorem can be considered an analogue for automorphic forms of the decomposition theorem of Dedekind zeta-functions.

The above theorem can be derived easily from the following theorem on trace of Hecke operators.

THEOREM. - If  $k \geq 4$ ,

$$\text{tr } T(e)/SS_k(\Gamma) = \text{tr } T(\lambda(e))/S_k(SL_2(\mathbb{Z})) + \frac{1}{2} \sum_{i=1}^{\ell-1} \text{tr } T(\lambda(e))/S_k(\Gamma_0(q), \chi_i)$$

for any  $e \in R(\mathfrak{u}_F, GL_2(\mathbb{F}_A))$ , where  $\text{tr } T(e^*)$  is the trace of  $T(e^*)$  on the space  $*$ .

Remark. - The above theorem is a generalization and a refinement of the result of

K. DOI and H. NAGANUMA ([2], [6]), which treated the lifting for quadratic extensions. H. JACQUET [3] studied the lifting for quadratic extensions from the view point of representation theory. Alternative proofs for K. DOI and H. NAGANUMA's result are given by D. ZAGIER [12] and S. KUDLA [4]. T. ASAI [1] treated the lifting in the case of imaginary quadratic extensions over  $\underline{Q}$ .

2. - In this section, we give the definition of  $\lambda$ . Since it is known that

$$R(\mathfrak{u}_F, GL_2(\mathbb{F}_A)) = \bigotimes_{\mathfrak{p}} R(GL_2(\mathfrak{o}_{\mathfrak{p}}), GL_2(\mathbb{F}_{\mathfrak{p}})),$$

$$R(\mathfrak{u}_{\underline{Q}}, GL_2(\mathbb{Q}_A)) = \bigotimes_{\mathfrak{p}} R(GL_2(\mathbb{Z}_{\mathfrak{p}}), GL_2(\mathbb{Q}_{\mathfrak{p}})),$$

it is enough to define a homomorphism  $\lambda_{\mathfrak{p}}$  from  $R_{\mathfrak{p}} = R(GL_2(\mathfrak{o}_{\mathfrak{p}}), GL_2(\mathbb{F}_{\mathfrak{p}}))$  to  $R_{\mathfrak{p}} = R(GL_2(\mathbb{Z}_{\mathfrak{p}}), GL_2(\mathbb{Q}_{\mathfrak{p}}))$  for each prime ideal  $\mathfrak{p}$  of  $F$ , where  $\mathfrak{p}$  is a prime such as  $\mathfrak{p}|p$ . Let  $T(\mathfrak{p})$  and  $T(\mathfrak{p}, \mathfrak{p})$  (resp.  $T(p)$  and  $T(p, p)$ ) be the elements of  $R_{\mathfrak{p}}$  (resp.  $R_p$ ) given by the double cosets

$$GL_2(\mathfrak{o}_{\mathfrak{p}}) \begin{pmatrix} 1 & 0 \\ 0 & \pi \end{pmatrix} GL_2(\mathfrak{o}_{\mathfrak{p}}) \quad \text{and} \quad GL_2(\mathfrak{o}_{\mathfrak{p}}) \begin{pmatrix} \pi & 0 \\ 0 & \pi \end{pmatrix} GL_2(\mathfrak{o}_{\mathfrak{p}})$$

(resp.  $GL_2(\mathbb{Z}_{\mathfrak{p}}) \begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix} GL_2(\mathbb{Z}_{\mathfrak{p}})$  and  $GL_2(\mathbb{Z}_{\mathfrak{p}}) \begin{pmatrix} p & 0 \\ 0 & p \end{pmatrix} GL_2(\mathbb{Z}_{\mathfrak{p}})$ ) respectively, where  $\pi$  is a prime element of  $\mathfrak{o}_{\mathfrak{p}}$ . We denote by  $R_{\mathfrak{p}}^I$  (resp.  $R_p^I$ ) the subring of  $R_{\mathfrak{p}}$  (resp.  $R_p$ ) generated by  $T(\mathfrak{p})$  and  $T(\mathfrak{p}, \mathfrak{p})$  (resp.  $T(p)$  and  $T(p, p)$ ). If we put

$$\begin{aligned} T(\mathfrak{p}) &= X + Y & T(p) &= x + y \\ T(\mathfrak{p}, \mathfrak{p}) &= NpXY & T(p, p) &= pxy \end{aligned} \quad (\text{resp.} \quad ),$$

we can embed  $R_{\mathfrak{p}}$  (resp.  $R_p$ ) into the polynomial ring  $\underline{Q}[X, Y]$  (resp.  $\underline{Q}[x, y]$ ) of two variables over  $\underline{Q}$ . Now, consider the mapping from  $\underline{Q}[X, Y]$  to  $\underline{Q}[x, y]$  given by

$$\begin{aligned} X &\longmapsto x^f, \\ Y &\longmapsto y^f, \end{aligned}$$

where  $f$  is an integer such that  $Np = p^f$ . Then we see easily that this mapping can be extended to a homomorphism from  $R_{\mathfrak{p}}$  to  $R_p$ .

3. - On this section, we give a numerical example of our theorem. We take as  $F$  the maximal real subfield of 7-th root of unity, then  $[F:\underline{Q}] = 3$ , and  $F$  satisfies the condition in § 1. Let  $\chi$  be the character mod 7 of order 3 given by  $\chi(3) = \omega$ ,  $\omega = (-1 + \sqrt{-3})/2$ . For  $k = 4$ , we have  $\dim S_4(\Gamma) = 1$  and  $\dim S_4(\Gamma_0(7), \chi) = 1$ . In this case, the subspace  $SS_4(\Gamma)$  coincides with  $S_4(\Gamma)$ , hence  $S_4(\Gamma)$  is isomorphic to  $S_4(\Gamma_0(7), \chi)$  as  $R(\mathfrak{u}_F, GL_2(\mathbb{F}_A))$ -modules. Let  $f$  (resp.  $g$ ) be an element of  $S_4(\Gamma)$  (resp.  $S_4(\Gamma_0(7), \chi)$ ) with the associated Dirichlet series

$$\Phi_f(s) = \sum C(\mathfrak{u}) N\mathfrak{u}^{-s} \quad (\text{resp.} \quad \varphi_g(s) = \sum_{n=1}^{\infty} a_n n^{-s}).$$

We may assume  $C(\mathfrak{o}) = 1$  and  $a_1 = 1$ . Then our theorem asserts that it holds the

following relation between  $C(p)$  and  $a_p$ , namely

$$C(p) = \begin{cases} a_p & (p) = pp' p'' \\ a_p^3 - 3\chi(p) p^3 a_p & (p) = p \\ a_p + \bar{a}_p & (p) = p^3, \end{cases}$$

where  $p, p', p''$  are the distinct prime divisors of  $(p)$ . This relation can be checked for several  $p$  and  $p$ . The coefficients  $a_p$  can be calculated by Eichler-Selberg's trace formula using the class numbers of imaginary quadratic fields. On the other hand,  $C(p)$  can be obtained by Shimizu's trace formula [8] using the class numbers of totally imaginary quadratic extensions of  $F$ . For example, to calculate  $C((2))$ , we need the following class numbers.

$$h(\mathbb{F}(\sqrt{-8})) = 1, \quad h(\mathbb{F}(\sqrt{-7})) = 1, \quad h(\mathbb{F}(\sqrt{\alpha^2 - 8})) = 1, \quad h(\mathbb{F}(\sqrt{\alpha^2 + 2\alpha - 7})) = 1.$$

Here  $h(K)$  is the class number of  $K$  and  $\alpha$  is a root of the equation

$$X^3 + X^2 - 2X - 1 = 0.$$

On this way, we have the following table.

$p$	$\chi(p)$	$a_p$	$p$	$C(p)$
2	$\omega^2$	$2\omega$	(2)	- 40
3	$\omega$	$7\omega^2$	(3)	- 224
7	0	$7 - 14\omega$	$(2 - \alpha)$	28
13	1	- 14	$(\alpha^2 + 1)$	- 14
29	1	58	$(3 - \alpha)$	58

4. 1. - Let  $F$  be as in § 1, and  $\mathfrak{A}$  an integral ideal of  $F$  such as  $\sigma\mathfrak{A} = \mathfrak{A}$ , then we can define a subspace  $SS_k(\Gamma_0(\mathfrak{A}))$  of  $S_k(\Gamma_0(\mathfrak{A}))$  in the same way as in § 1, where  $\Gamma_0(\mathfrak{A}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma, c \equiv 0 \pmod{\mathfrak{A}} \right\}$ , and we can prove a similar but more complicated result on this  $SS_k(\Gamma_0(\mathfrak{A}))$ . Also, in the case of definite quaternion algebras, we can consider a similar problem, and the case where  $\ell \geq 3$  has been treated by H. HIJIKATA. In the case of quadratic extensions, we can prove the following. Let  $F$  be  $\mathbb{Q}(\sqrt{q})$  with a prime  $q$ ,  $q \equiv 1 \pmod{4}$ , and  $B$  a definite quaternion algebra over  $\mathbb{Q}$  which ramifies at  $q$  and at the archimedean prime. Let  $R$  be a maximal order of  $B \otimes F$  which satisfies  $\sigma R = R$ , where  $\sigma$  is the generator of  $\text{Gal}(F/\mathbb{Q})$ . For a non-negative even integer  $k$ , let  $M(\text{id}, \{k, k\})$  be the space of continuous functions on  $(B \otimes F)_A^\times$  defined in H. SHIMIZU [8] with respect to the open subgroup  $\prod_p R_p^\times \times \prod_v (B \otimes F)_v^\times$  of  $(B \otimes F)_A^\times$ . We can define the action of  $T_\sigma$  on  $M(\text{id}, \{k, k\})$  by means of the action of  $\sigma$  on  $(B \otimes F)_A$ , and in these notations we can prove the following theorem.

THEOREM. - For any  $e \in \bigotimes_{p \neq q} R(R_p^\times, (B \otimes F)_A^\times)$ , it holds

$$\text{tr } T(e) T_{\sigma}/M(\text{id}, \{k, k\})$$

$$= - \text{tr } T(\lambda(e))/S_{k+2}(\text{SL}_2(\mathbb{Z})) + \frac{1}{2} \text{tr } T(\lambda(e))/S_{k+2}(\Gamma_0(q), (\bar{q})),$$

where  $q$  is the prime ideal such as  $q^2 = (q)$  and  $(\bar{q})$  is the quadratic residue symbol mod  $q$ .

4. 2. - The theorem in § 1 has been generalized from the view point of representation theory by T. SHINTANI [11] and R. P. LANGLANDS [5] by a similar method as ours. Especially, R. P. LANGLANDS found an important application of his generalization of our theorem to the conjecture of Artin on the poles of Artin's L-functions.

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