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Automorphisms of p-Groups of Maximal Class (*).

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Dedicated to Guido Zappa on his 90th birthday

ABSTRACT - Juhász has proved that the automorphism group of a group G of maximal class of order p^n, with $p \geq 5$ and $n > p + 1$, has order divisible by $p^{\lceil (3n-2p+5)/2 \rceil}$.

We show that by translating the problem in terms of derivations, the result can be deduced from the case where G is metabelian. Here one can use a general result of Caranti and Scoppola concerning automorphisms of two-generator, nilpotent metabelian groups.

1. Introduction.

Baartmans and Woeppel have proved [BW76, Theorem 3.1] the following

THEOREM 1.1. *Let p be a prime, and let G be a p-group of maximal class of order p^n, which has an abelian maximal subgroup. Suppose G has exponent p. Then* $\operatorname{Aut}(G)$ *contains a subgroup of order* p^{2n-3}.

The exponent restriction limits the size of G to p^p (see 3.2 below). However, the main point of this result holds true more generally. Caranti and Scoppola have proved [CS91] (see also [CM96]) that any finite, metabelian p-group has a subgroup of its group of automorphisms of order $\gamma_2(G)^2$, where $\gamma_2(G)$ is the derived subgroup of G. We thus have in particular

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THEOREM 1.2. *Let p be a prime, and let G be a p-group of maximal class of order p^n, which is metabelian. Then* $\mathrm{Aut}(G)$ *contains a subgroup of order p^{2n-4}.*

Juhász has proved in [Juh82] among others the following result.

THEOREM 1.3. *Let $p \geq 5$ be a prime, and let G be a p-group of maximal class of order p^n, with $n > p+1$. Then* $\mathrm{Aut}(G)$ *contains a subgroup of order $p^{\lceil (3n-2p+5)/2 \rceil}$.*

The aim of this paper is to show that if one reformulates the problem in terms of derivations, as we do in Section 2, then the general case of an arbitrary p-group of maximal class of Theorem 1.3 can be shown to follow from the special case of a metabelian p-group of maximal class of Theorem 1.2.

A particular case of Theorem 1.3 has been used by Malinowska in [Mal01].

A stronger estimate holds for 3-groups of maximal class, because these are all metabelian (see [LGM02, Theorem 3.4.3]). According to Theorem 1.2, such a group G of order 3^n has at least 3^{2n-4} automorphisms.

After discussing relations between automorphisms and derivations according to our needs in Section 2, we give our proof of Theorem 1.3 in Section 3. The same approach allows us prove also the following result.

THEOREM 1.4. *Let $p \geq 5$ be a prime, and let G be a p-group of maximal class of order p^n, with $n > p+1$. Then* $\mathrm{Aut}(G)$ *has an abelian normal subgroup of order p^{n-2p+7}.*

It must be noted that Juhász obtains his result in [Juh82] using the best estimate for the degree of commutativity that was available at the time of his writing. We have corrected the formulation of Theorem 1.3 to take account of the exact estimate later obtained by Fernández-Alcober in [FA95].

2. Derivations and automorphisms.

We conveniently extend to nonabelian groups (written multiplicatively) a piece of notation usually adopted for the endomorphism ring of an abelian group (traditionally written additively). For maps σ, τ from a set S to a (multiplicative) group G we define the map $\sigma + \tau \in G^S$ by setting $s(\sigma + \tau) = (s\sigma)(s\tau)$ for all $s \in S$. The "addition" operation thus defined, which is not commutative unless G is, makes G^S into a group, the cartesian product of copies of G indexed by the elements of S. We write as 0 and $-\sigma$ the identity element and the inverse of σ in G^S, and we write $\sigma - \tau$ for $\sigma + (-\tau)$. The identities $s0 = 1$ and $s(-\sigma) = (s\sigma)^{-1}$ for $s \in S$ would look more natural by using the exponential notation s^σ for $s\sigma$ which is traditional in similar contexts, but we avoid doing that to prevent proliferation of exponents. In the special case where $S = G$, another operation on G^G is given by composition, written here as (left-to-right) juxtaposition. It is left-distributive with respect to addition, but not right-distributive, in general.

Recall (cf. [LGM02, § 9.5]) that a derivation of a group G into a G-bimodule A is a map $\delta : G \to A$ satisfying $(gh)\delta = (g\delta)h + g(h\delta)$ for all $g, h \in G$. The set of derivations of G into A, denoted by $\mathrm{Der}(G, A)$, is an abelian group with operation induced by that on the codomain A, as described above. We define the *kernel* of a derivation δ as $\ker \delta = \{g \in G : g\delta = 0\}$, bearing in mind that this is a subgroup of G but need not be normal.

REMARK 2.1. If N is a normal subgroup of G for which A is the trivial bimodule then we can and will identify $\mathrm{Der}(G/N, A)$ with the subset of $\mathrm{Der}(G, A)$ consisting of the derivations whose kernel contains N.

Now let A be an abelian normal subgroup of a group G. Then it is customary to make A into a G-bimodule with the trivial action on the left and the conjugation action on the right, that is, $g \cdot a \cdot h = a^h$ for $a \in A$ and $g, h \in G$. In this context, writing the group operation in A multiplicatively, as in G, the condition for $\delta : G \to A$ being a derivation reads $(gh)\delta = (g\delta)^h (h\delta)$. In particular, this readily implies that $1\delta = 1$ and $g^{-1}\delta = ((g\delta)^{-1})^{g^{-1}}$. It follows that if $\delta \in \mathrm{Der}(G, A)$ then the map $\alpha = 1 + \delta$, given by $g\alpha = g(g\delta)$ according to notation introduced earlier, is an endomorphism of G. Conversely, if α is an endomorphism of G which sends A into itself and induces the identity map on the quotient G/A, then $-1 + \alpha$ belongs to $\mathrm{Der}(G, A)$. Since $(1 + \delta_1)(1 + \delta_2) = 1 + \delta_1 + \delta_2 + \delta_1\delta_2$ for $\delta_1, \delta_2 \in \mathrm{Der}(G, A)$, the operation "$\bullet$" on derivations given by $\delta_1 \bullet \delta_2 = \delta_1 + \delta_2 + \delta_1\delta_2$ makes $\mathrm{Der}(G, A)$ into a monoid, and the correspondence $\delta \mapsto 1 + \delta$ becomes a monomorphism of $\mathrm{Der}(G, A)$ into the monoid $\mathrm{End}(G)$ with respect to composition. We record part of these conclusions for later reference.

LEMMA 2.2. *Let A be an abelian normal subgroup of a group G. The map sending δ to $1+\delta$ is a monomorphism of the monoid* $\mathrm{Der}(G,A)$ *with the operation $\bullet$ into the monoid* $\mathrm{End}(G)$ *with respect to composition. Its image consists of the endomorphisms of G which send A into itself and induce the identity map on G/A.*

The endomorphism $1+\delta$ is injective provided δ maps no element g of G (or, equivalently, of A) to its inverse. A sufficient condition for this to occur is, for example, that δ is nilpotent, in the sense that some power δ^k vanishes, because then $1+\delta$ admits the inverse $1-\delta+\delta^2-\delta^3+\cdots$. (This occurs in Lemma 2.3 below, with $k=2$.) In case G is a finite group, a sufficient condition for $1+\delta$ to be an automorphism of G for all $\delta\in\mathrm{Der}(G,A)$, and hence for $1+\mathrm{Der}(G,A)$ to be a subgroup of $\mathrm{Aut}(G)$, is that A is contained in the Frattini subgroup of G. In fact, in that case the image of $1+\delta$ supplements the Frattini subgroup, whence $1+\delta$ is surjective, and thus injective by finiteness of G.

On the subset $\mathrm{Der}(G/A,A)$ of $\mathrm{Der}(G,A)$ the operation $\bullet$ coincides with addition, because $\delta_1\delta_2=0$ for $\delta_1,\delta_2\in\mathrm{Der}(G/A,A)$. In particular, it is commutative in this case. The properties of the correspondence $\delta\mapsto 1+\delta$ stated in Lemma 2.2 read as follows.

LEMMA 2.3. *Let A be an abelian normal subgroup of a group G. The map sending δ to $1+\delta$ is a monomorphism of the additive group* $\mathrm{Der}(G/A,A)$ *into* $\mathrm{Aut}(G)$*. Its image is the abelian subgroup consisting of the automorphisms which send A into itself and induce the identity map on G/A.*

A familiar instance of Lemma 2.3 is when A is the centre of G. Then derivations $\delta\in\mathrm{Der}(G/A,A)$ are the same thing as group homomorphisms of G/A into A, and the correspondence $\delta\mapsto 1+\delta$ maps $\mathrm{Der}(G/A,A)$ onto the group of *central* automorphisms of G.

We will need the following fact on derivations of a group G into an (abelian) term of its lower central series $G=\gamma_1(G)\geq\gamma_2(G)\geq\cdots$.

LEMMA 2.4. *Suppose that $\gamma_r(G)$ is abelian and let* $\delta\in\mathrm{Der}(G,\gamma_r(G))$*. Then $\gamma_i(G)\delta\subseteq\gamma_{i+r-1}(G)$ for all $i\geq 1$.*

In particular if G is nilpotent, with $\gamma_n(G)=1$, then we have, according to Remark 2.1,

$$\mathrm{Der}(G,\gamma_r(G))=\mathrm{Der}(G/\gamma_{n-r+1}(G),\gamma_r(G)).$$

PROOF. Since $1+\delta$ is an endomorphism of G, for all $g,h \in G$ we have

$$[g,h]([g,h]\delta) = [g,h](1+\delta) = [g(1+\delta), h(1+\delta)] = [g(g\delta), h(h\delta)],$$

and hence $[g,h]\delta = [g,h]^{-1}[g(g\delta), h(h\delta)]$. The commutator identity

$$[gu, hv] = [g,v]^u [g,h]^{vu} [u,v][u,h]^v$$

shows that

$$[g,h]^{-1}[gu,hv] \in \gamma_{i+j+r-1}(G)$$

if $g \in \gamma_i(G)$, $u \in \gamma_{i+r-1}(G)$, $h \in \gamma_j(G)$ and $v \in \gamma_{j+r-1}(G)$. Since $\gamma_{i+1}(G)$ is generated by all commutators $[g,h]$ with $g \in \gamma_i(G)$ and $h \in G = \gamma_1(G)$, our claim follows by induction on i by taking $u = g\delta$ and $v = h\delta$. □

3. Automorphisms of p-groups of maximal class.

We take [LGM02] as a reference for p-groups of maximal class, but see also [Hup67, III.14], and Blackburn's original paper [Bla58].

In this section, G will be a p-group of maximal class of order p^n, with $p \geq 5$ and $n \geq 4$. As usual, write $G_i = \gamma_i(G)$ for $i \geq 2$, and define a maximal subgroup G_1 of G by

$$G_1 = C_G(G_2/G_4) = \{g \in G \mid [G_2, g] \subseteq G_4\}.$$

In particular, $G_{n-1} > G_n = 1$.

The degree of commutativity l of G is defined as $n-3$ if G_1 is abelian, and otherwise as the largest integer l such that $[G_i, G_j] \leq G_{i+j+l}$ for all $i,j \geq 1$. Since $[G_1, G_1] = [G_1, G_2]$ we have $l \leq n-3$ in all cases. One can show ([LGM02, Theorem 3.3.5], [Hup67, Hauptsatz III.14.6]) that for $n > p+1$ the degree of commutativity of a group G of maximal class of order p^n is positive, that is,

$$G_1 = C_G(G_i/G_{i+2})$$

for all $i = 2, \ldots, n-2$. From now on we take $n > p+1$. Choose $s_1 \in G_1 \setminus G_2$ and $s \in G \setminus G_1$, and define $s_{i+1} = [s_i, s]$ for $i \geq 1$. We then have $G_i = \langle s_i, G_{i+1} \rangle$ for $i = 1, \ldots, n-1$.

LEMMA 3.1. *Let $r \geq (n-l)/2$ and $\delta \in \mathrm{Der}(G, G_r)$. Then*

$$G_{n-r+1} \leq \ker(\delta),$$

and hence δ can be viewed as an element of

$$\mathrm{Der}(G/G_{n-r+1}, G_r).$$

PROOF. Note that G_r is abelian, as $[G_r, G_r] \leq G_{2r+l} \leq G_n = 1$. The conclusion follows at once from Lemma 2.4. □

Let now G' be a group of maximal class which is metabelian, that is, with the obvious notation, $[G'_2, G'_2] = 1$. A result of [CS91] (see also [CM96]) guarantees that G' has plenty of automorphisms.

THEOREM 3.2. *Let $M = \langle x, y \rangle$ be a metabelian, 2-generator finite nilpotent group, and let $M_2 = [M, M]$ be its derived subgroup. Then for all $u, v \in M_2$ there is an automorphism of M such that*

$$\begin{cases} x \mapsto x \cdot u, \\ y \mapsto y \cdot v. \end{cases}$$

With the terminology introduced in the previous section we can rephrase the conclusion of Theorem 3.2 as follows: for all $u, v \in M_2$ there is a derivation $\delta \in \mathrm{Der}(M, M_2)$ such that $x\delta = u$ and $y\delta = v$.

We intend to exploit this in the following way. Given an arbitrary p-group G of maximal class of order p^n, with $n > p + 1$, we will show that there are a suitable r, and a suitable metabelian p-group G' of maximal class, of the same order as G, such that

- G_r is abelian,
- G/G_{n-r+1} is isomorphic to G'/G'_{n-r+1}, and
- the G/G_{n-r+1}-module G_r is similar to the G'/G'_{n-r+1}-module G'_r.

It will follow that

$$\mathrm{Der}(G/G_{n-r+1}, G_r) \cong \mathrm{Der}(G'/G'_{n-r+1}, G'_r). \tag{3.1}$$

Now Theorem 3.2 tells us that G' has many automorphisms, that is, the set at the right hand side of 3.1 is large, so that the set at the left hand side is also large, and G in turn has many automorphisms.

We begin by defining G', following [Bla58, p. 83-84], by the presentation

$$\begin{aligned} G' = \langle s', s'_i, i = 1, \ldots, n-1 :\ & s'^p = 1, \\ & [s'_i, s'] = s'_{i+1} \text{ for } i = 1, \ldots, n-2, \\ & [s'_i, s'_j] = 1 \text{ for } i,\ j = 1, \ldots, n-1, \\ & s'^p_i s'^{\binom{p}{2}}_{i+1} \cdots s'_{i+p-1} = 1 \text{ for } i = 1, \ldots, n \rangle. \end{aligned}$$

(We assume $s'_i = 1$ for $i \geq n$.) This group may be constructed in the following way. One starts with the abelian group

$$M = \langle s'_i, i = 1, \dots, n-1 : [s'_i, s'_j] = 1 \text{ for } i,\ j\ =\ 1, \dots, n-1,$$
$$s'^p_i s'_{i+1}{}^{\binom{p}{2}} \cdots s'_{i+p-1} = 1 \text{ for } i\ =\ 1, \dots, n-1\rangle.$$

This has order p^{n-1}, and its structure can be understood by reading the last group of relations backwards. Now M admits an automorphism $\sigma : s'_i \mapsto s'_i s'_{i+1}$, as σ preserves the defining relations. Moreover, for $i \geq 2$ one has, by [Hup67, Hilfssatz 10.9(b)] or [LGM02, Corollary 1.1.7],

$$\sigma^p(s'_{i-1}) = s'_{i-1} \cdot s'^p_i s'_{i+1}{}^{\binom{p}{2}} \cdots s'_{i+p-1} = s'_{i-1},$$

so that σ has order p. Now G' above can be constructed as the cyclic extension of M by a cyclic group $\langle s' \rangle$ of order p, where s' induces σ on M.

Take $r = n - l - 1$, and $A = G_r$. We have $[G_1, A] = 1$, so that in particular A is abelian. Note that $r > (n-l)/2$ because $l \leq n-3$, and hence $\mathrm{Der}(G, A) = \mathrm{Der}(G/G_{l+2}, A)$ according to Lemma 3.1.

It is time to recall some basic facts about p-groups of maximal class, valid under our assumption $n > p+1$. If $g \in G \setminus G_1$, then $g \notin C_G(G_i/G_{i+2})$, for $i = 1, \dots, n-2$. Thus $C_G(g) \cap G_1 = G_{n-1} = Z(G)$. It follows that $C_G(g) = \langle g, G_{n-1} \rangle$, so that $g^p \in G_{n-1}$. Also, the conjugacy class g^G of g has order $G : C_G(g) = p^{n-2}$, so that $g^G = gG_2$.

As $s \notin G_1$, we obtain in particular that s and ss_i are conjugate, for $i \geq 2$, and hence the elements s^p and $(ss_i)^p$, which by the above lie in the centre of G, do coincide. If $i \geq r$, we have that s_i commutes with all of the elements $s_j = [s_i, \underbrace{s, \dots, s}_{j-i}]$, for $j \geq i$. Consequently, we have

$$1 = s^{-p}(ss_i)^p = s_i^p s_{i+1}^{\binom{p}{2}} \dots s_{i+p-1}, \tag{3.2}$$

again by [Hup67, Hilfssatz 10.9(b)] or [LGM02, Corollary 1.1.7]. These relations define G_r as an abelian group generated by the s_i, for $i \geq r$, so that G_r is isomorphic to G'_r.

Because $[G_1, G_1] \leq G_{l+2} = N$, the quotient G_1/N is abelian. As above, we have

$$s_i^p s_{i+1}^{\binom{p}{2}} \dots s_{i+p-1} \equiv 1 \pmod N \tag{3.3}$$

for all $i \geq 2$. Since $s^p \equiv (ss_1)^p \equiv 1 (\operatorname{mod} N)$, equation 3.3 also holds for $i = 1$. We thus see that $G/N = G/G_{l+2}$ is isomorphic to the corresponding factor group G'/G'_{l+2} of G'. Finally, the action of G on G_r is given by $[s_i, s_1] = 1$ and $[s_i, s] = s_{i+1}$, for $i \geq r$. Thus, the G/G_{l+2}-module G_r is similar to the G'/G'_{l+2}-module G'_r.

According to Theorem 3.2 and Lemma 2.2, we conclude that for all $u, v \in A$ there is an automorphism of G determined by

$$\varphi_{u,v} : \begin{cases} s \mapsto s \cdot u, \\ s_1 \mapsto s_1 \cdot v. \end{cases}$$

A comment following Lemma 2.2 shows that these automorphisms form a subgroup of Aut (G). In particular, the automorphisms among these which fix s form a subgroup $\mathcal{H} = \{ \varphi_{1,v} : v \in A \}$, of order p^{n-r}.

Suppose that $\varphi_{1,v} \in \mathrm{Inn}\,(G)$ for some $v \neq 1$, so that $s^g = s$ and $s_1^g = s_1 v \neq s_1$ for some $g \in G$. Since $C_G(s) = \langle s, G_{n-1} \rangle$, we have that $g \equiv s^i$ modulo the centre G_{n-1} for some i, and $v = [s_1, s^i] \in G_2 \setminus G_3$. Thus the group $\mathcal{H}$ intersects Inn (G) trivially if $r > 2$. We will verify below that the metabelian case $r = 2$, that is $l = n - 3$, is covered by Theorem 3.2, so in the following we assume $l < n - 3$.

It follows that Aut (G) contains the subgroup $\mathcal{H}\,\mathrm{Inn}\,(G)$, with $\mathcal{H} \cap \mathrm{Inn}\,(G) = \{ 1 \}$. Note that all automorphisms $\varphi_{u,v}$ belong to $\mathcal{H}\,\mathrm{Inn}\,(G)$. In fact, for $u \in A$ one has that $su \in sA \subseteq sG_2$, so su is conjugate to s. If $su = s^g$ for some $g \in G$, then composing $\varphi_{u,v}$ with conjugation by g^{-1} one obtains an element of $\mathcal{H}$.

So we get that Aut (G) has a subgroup of size at least p^c, with

$$c \geq n - 1 + n - r = n + l.$$

Using the estimate $2l \geq n - 2p + 5$ of [FA95] (see [LGM02, Theorem 3.4.11] for a version of this bound weakened by one) we conclude that

$$c \geq \frac{3n - 2p + 5}{2},$$

thus completing a proof of Theorem 1.3 except for the case where G is metabelian. However, in that case Theorem 3.2 provides $p^{2(n-2)}$ distinct automorphisms of G, and this number exceeds $p^{(3n-2p+5)/2}$ for $p \geq 3$.

REMARK 3.3. The spirit of the times suggests an alternative description of G', as in [LGM02, Examples 3.1.5], which we sketch here. Let θ be a primitive pth root of unity over the rational field $\mathbb{Q}$. Then the abelian group M can be realised (in additive notation) as the additive group of the quotient ring $\mathbb{Z}[\boldsymbol{\theta}]/(\boldsymbol{\theta}-1)^{n-1}$, where the residue class of $(\boldsymbol{\theta}-1)^{i-1}$ plays the role of s'_i. (The defining relations in Blackburn's presentation for M are then all consequences of the relation $(1+(\boldsymbol{\theta}-1))^p-1=0$.) We construct G' as the cyclic extension of M by a cyclic group $\langle s' \rangle$ of order p, where s' acts on M by multiplication by $\boldsymbol{\theta}$. Now, the derivations $\delta \in \mathrm{Der}(G', M)$ such that $s'\delta = 0$ correspond to the endomorphisms of M as $\langle s' \rangle$-module, which are clearly given by all multiplications by polynomials in $\boldsymbol{\theta}$. In particular, this gives an explicit description of $\mathrm{Der}(G', A)$ which allows one to construct $\mathcal{H}$ without recourse to Theorem 3.2.

In order to prove Theorem 1.4, consider the subgroup G_t of G_r, where $t = \max\left(n-l-1, \left\lceil \frac{n+1}{2} \right\rceil\right)$. According to Lemma 3.1, and because $n-t+1 \leq t$, we have

$$\mathrm{Der}(G, G_t) = \mathrm{Der}(G/G_{n-t+1}, G_t) = \mathrm{Der}(G/G_t, G_t).$$

Lemma 2.3 implies that $\{ \varphi_{u,v} : u, v \in G_t \}$ is an abelian subgroup of $\mathrm{Aut}(G)$ isomorphic with the additive group $\mathrm{Der}(G/G_t, G_t)$, and hence of order $|G_t|^2 = p^{2(n-t)}$. This abelian subgroup of $\mathrm{Aut}(G)$ is normal, again according to Lemma 2.3, because G_t is a characteristic subgroup of G. Because $2l \geq n-2p+5$ and $p \geq 5$ we see that $2(n-t) \geq n-2p+7$, thus proving Theorem 1.4.

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