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On the Boundedness of the Set of Ample Vector Bundles with Fixed Sectional Genus.

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Let X be a projective variety (say, smooth) and $L \in \operatorname{Pic}(X)$ with L ample. Set $n := \dim(X)$ and $K := K_X$. Recall that the sectional genus $g(L)$ of L can be defined by the formula: $2g(L) - 2 = (K + L)L^{n-1}$. It is known ([F2]) that $g(L)$ is an integer and that $g(L) \geq 0$ if L is ample. If $C \subset X$ is a curve which is scheme-theoretically the intersection of $n-1$ divisors in $|L|$, then $g(L) = p_a(C)$ by the adjunction formula. In [F2], Th. 13.1, it was proved that for evert integer g the set $T(n, g)$ of pairs (X, L) with X smooth complete variety of dimension n (in characteristic 0) and $L \in \operatorname{Pic}(X)$ with L ample and with $g(L) = g$ is bounded. Recall that here «bounded» means the existence of an algebraic scheme T (a parameter space), a flat morphism $f\colon \boldsymbol{X} \to T$ and $\boldsymbol{L} \in \operatorname{Pic}(\boldsymbol{X})$ such that for every $t \in T$ we have $\boldsymbol{L}(f^{-1}(t))$ and viceversa every pair $(X, L) \in T(n, g)$ arises (up to isomorphism) in this way; the important thing here is that T is algebraic, not only locally algebraic (and in particular it has only finitely many irreducible components).

Now fix a rank r vector bundle E on X. In [F1] (or see [F2], p. 174 and p. 175) T. Fujita gave two definitions of sectional genus for the vector bundle E. First we describe the less interesting one (in our opinion); take as sectional genus the integer $g(\boldsymbol{O}_{\mathbf{P}(E)}(1))$ where $\boldsymbol{O}_{\mathbf{P}(1)}(1)$ is the tautological quotient line bundle on $\mathbf{P}(E)$; this integer was called the $\boldsymbol{O}(1)$-sectional genus (see [F2], p. 175); with this definition it is easy to work (obtaining for instance lists of pairs (X, E) with low genera and E ample) using properties proven for polarized manifolds. Here is the second definition: set $L := c_1(E)$ and call c_1-sectional genus of E the integer $g(L)$. Both definitions have two very big advantages: they are very general (i.e. they are always defined) and they are useful. However they are not very natural for the following reason; if for instance C is a curve which is scheme-theoretically the zero locus $(s)_0$ of $s \in H^0(X, E)$, then in general $p_a(C)$ is neither the $\boldsymbol{O}(1)$-sectional genus

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nor the c_1-sectional genus of E; indeed, if $E \cong M^{\oplus r}$ with $M \in \mathrm{Pic}(X)$ then in general the c_1-genus of E is not $g(M)$, since $L = M^{\oplus r}$ (which is seldom isomorphic to M).

In [B] we gave another definition of sectional genus for the vector bundle E, but only under extremely strong assumptions: E spanned by its global sections and $\mathrm{rank}(E) + 1 = \dim(X)$. Under these assumptions, for general $s \in H^0(X, E)$ the zero locus $(s)_0$ is a curve (even smooth in characteristic zero) and it is natural to call $p_a(C)$ «the» (or «a») sectional genus of E. Note that by the adjunction formula we have $2p_a(C) - 2 = (K + L)\, c_{n-1}(E)$. By [FL] it is easy to check that if E and $K + L$ are ample, then $p_a(C) < g(L)$.

Here is the main result of this note.

THEOREM 0.1. *Assume that the algebraically closed base field* $\mathbf{K}$ *has* $\mathrm{char}(\mathbf{K}) = 0$. *Then the set* $A(n, r, g) := \{(X, E)$: *X is a smooth projective manifold and E is a* rank r *ample vector bundle on X with* c_1-*sectional genus* $g\}$ *is bounded.*

Then we prove the following result 0.3 about the boundedness for the sectional genus in the sense of [B]. To state 0.3 we need the following definition.

DEFINITION 0.2. A pair (X, E) with X n-dimensional projective manifold and E rank r vector bundle on X is called a scroll over a curve (or a classical scroll or, briefly, a scroll) if there is a smooth curve C, a rank n vector bundle F on C and a morphism $\pi: X \to C$ such that π is isomorphic to the projection $\mathbf{P}(F) \to C$ and the restriction of E to every fiber of π (which is isomorphic to $\mathbf{P}^{n-1}$) is the direct sum of r line bundles of degree 1.

By the base change theorem and the projection formula, the last condition of 0.2 is equivalent to the existence of $H \in \mathrm{Pic}(X)$ and a rank r vector bundle A on C with $E \cong \pi^*(A)(H)$ and $\deg(H|\pi^{-1}(x)) = 1$ for every $x \in C$; note that such a bundle is π-uniform in the sense of Ishimura ([Is]).

THEOREM 0.3. *Assume* $\mathrm{char}(\mathbf{K}) = 0$. *Set* $Y(3, 2, g) := \{(X, E)$: *X is a smooth projective threefold and E a* rank 2 *ample and spanned vector bundle on X with g as sectional genus in the sense of* [B], *and* (X, E) *is not a scroll*$\}$. *Then* $Y(3, 2, g)$ *is bounded.*

Theorems 0.1 and 0.3 will be proved (using heavily the rank 1 case proved in [F2], Th. 13.1) in the first section. In the second (and last) section we turn to the positive characteristic case, and prove the boundedness of $A(2,2,g)$ (Theorem 2.1). The proof of 2.1 will use in an essential way [Ek]. It is very easy to show that in 0.3 we have to exclude the scrolls (X, E). Indeed, with the notations of 0.3, we may find (exactly as in [Fu2], Remark 13.3, for the rank 1 case) E with sectional genus C and determinant of arbitrarly high self-intersection: just take a very positive H. We think that the existence of these exceptional cases shows that the definition of sectional genus given in [B] is a very natural one (although it is very restrictive). One defines the sectional genus $g(E)$ of a vector bundle E on a complete variety X with $\dim(X) = \operatorname{rank}(E) + 1 = n$, by the formula: $2g(E) - 2 = (K_X + c_1(E))\, c_{n-1}(E)$ (assuming K_X not too bad). However we were unable to prove much about $g(E)$ without assuming strong restrictions on E.

1. Proofs of 0.1 and 0.3.

We work over an algebraically closed base field $\mathbf{K}$. In this section we will assume always $\operatorname{char}(\mathbf{K}) = 0$. This section contains exactly the proofs of 0.1 and 0.3. In this paper we will use both additive and multiplicative notations for line bundles and divisors, and make standard abuses of notations switching from line bundles to divisors or viceversa.

Here X will be a projective manifold with $\dim(X) = n$ and E a rank r ample vector bundle on X. Take $L := c_1(E) \in \operatorname{Pic}(X)$ with L ample; set $g := g(L)$.

DEFINITION 1.1. Let $B(X, L, r)$ be the set of rank r ample vector bundles on X with L as determinant.

Unless otherwise stated we will assume $L := c_1(E)$.

First we need two easy and very well-known remarks.

REMARK 1.2. Since E is ample, we have $L \cdot C > 0$ for every curve C of X (no restriction on the characteristic of the base field or the singularities of X).

REMARK 1.3. By 1.2, the smoothness of X and Mori theory, we have $(K + tL) \cdot C > 0$ for every curve C in X and every integer t with $rt > n + 1$ (no restriction on the characteristic of these base field). Note that if $K + tL$ is nef, then $K + (t+1)L$ is ample by Kleiman ampleness criterion. In particular, checking directly the case $n = r = 2$ and $X = \mathbf{P}^2$ we obtain that $K + nL$ is ample if $n \geqslant 2$ and $r \geqslant 2$.

LEMMA 1.4. *Fix an ample vector bundle F on a complete manifold V and an integer $t > 0$. Then $F(\det(F)^{\otimes t})$ is Nakano positive. Furthermore $H^i(V, F(\det(F)^{\otimes t})K_V) = 0$ for every $i > 0$.*

PROOF. The first part (which is linear algebra) was proved (jointly) by Demailly and Skoda (see [D1] or [D2] or [Sk1] or [Sk2]). The second part is a theorem of Nakano (see either [N] or [D2], Cor. 0.5, or [Sk1]). ■

LEMMA 1.5. *Fix an integer m such that mL is very ample. Then $E(kL + K)$ is spanned by its global sections for every integer $k \geqslant (n - 1)m + 1$.*

PROOF. Note that the restriction of a Nakano positive bundle to a submanifold is again Nakano positive (hence satisfies the Nakano's vanishing theorem whose content is the last assertion of 1.4). Fix $x \in X$; take a smooth $D \in |mL|$ with $x \in D$ and apply the observation just made, the adjunction exact sequence for the pair (X, D) and induction on n. ■

Theorem 0.1 will follow rather easily from a result of Fujita ([Fu2], Th. 13.1) and the next proposition.

PROPOSITION 1.6. *For every X, L, and r, the set $B(X, L, r)$ is bounded.*

PROOF. By 1.5 there is an integer m' (depending only on (X, L), not on E) such that $E(kL + K)$ is spanned by global sections for every $k \geqslant m'$. Fix any such integer k. By a theorem of Bertini type due to Kleiman ([Kl2]), there are $r - 1$ sections of E which induce an exact sequence:

$$(1) \qquad 0 \to (r-1)\boldsymbol{O} \to E(kL + K) \to \boldsymbol{I}_Z \otimes \boldsymbol{O}((rk+1)L + rK) \to 0$$

with $Z \subset S$, Z reduced (and even smooth, since we are in characteristic 0), Z of pure codimension 2, Z representing the class of $c_2(R(kL + K))$; note that the class of $c_2(R(kL + K))$ is represented by the class of $c_2(E) + (r-1)(kL + K)L + (r/(r-1)/2)(kL + K))$. Now we use that $L^2 \cdot L^{n-2} > c_2 \cdot L^{n-2}$ ([BG] or [FL]), the existence of an integer $m > 0$, m depending only on X and L, such that $H := mL$ is very ample, and the existence of the Chow variety (with respect to the projective space $\mathbf{P}(H^0(X, H))$) to obtain the boundedness of $\dim(\operatorname{Ext}(\boldsymbol{I}_Z \otimes \boldsymbol{O}((rk + 1)L + rK), (r-1)\boldsymbol{O}))$ (in terms of X and L). Thus the set $B(X, L, r)$ is bounded, proving 1.1. ■

PROOF OF 01. By [Fu], Th. 13.1, the set $A(n,g)$ of pairs (X,L) with X n-dimensional projective manifold, $L \in \mathrm{Pic}(X)$, L ample and of sectional genus g with (X,L) not a classical scroll (i.e. a scroll in the sense of [Fu2], not in the sense of Sommese) is bounded. Note that by 1.2 if $L = \det(E)$ with E ample, (X,L) is never a scroll since $r := \mathrm{rank}(E) >$ > 1. By 1.6 for every $(X, L) \in A(n, g)$ the set $B(X,L)$ is bounded. From this, [Ma2], and «general principles» (as in [Kl1], in particular [Kl1], Th. 3.13) (or heavy use of semicontinuity theorems) it would be easy to deduce 0.1. But there is an alternative path: use the proof of 1.6 in a relative setting; this is possible and 0.1 follows from [Fu], Th. 13.1, and the exact sequence (1) for families, using either [BPS] or [L]. ■

PROOF OF 0.3. By 1.2 the pair (X,L) has no exceptional divisor in the sense of [Io] (i.e. no divisor $J \cong \mathbf{P}^2$ with $\deg(L \mid J),$), i.e. (X,L) is its own reduction. By 1.2, [Io], part (1.7) of the theorem in § 1, and the fact (X,E) is not a $\mathbf{P}^2$-bundle (again by 1.2), we obtain that $K+L$ is nef. With the notations just introduced (with $n = 3$) we have again the exact sequence (1) with Z smooth curve of genus g. Fix $x \in X$ and an integral surface $D \in |L|$. By the ampleness of $K + L$, the vanishing of Kodaira, the classification of surfaces and the exact sequence

$$0 \to tK + (t-1)L \to tK + tL \to \omega_D^{\otimes t} \to 0 \tag{2}$$

we see that $tK + tL$ is spanned by global sections if, say, $t \geq 12$. Fix a smooth surface $S \in |12K + 12L|$; set $x := c_2(12K + 12L) := 24(g-1)$ and $M := L|S$. If $M^2 \leq 4x$, then $L^3 \leq 8(g-1)$, i.e. L^3 is bounded only in term of g and we conclude by 0.1. Thus we may assume $M^2 > 4x$. Hence $E|S$ is Bogomolov unstable. Hence there is a filtration

$$0 \to A \to E|S \to B \otimes I_T \to 0 \tag{3}$$

with T_{red} finite and $AM \geq BM$ (i.e. $A^2 \geq B^2$ because $A \otimes B = M$). Note that $B^2 > 0$ by the ampleness of E. Since $BM > 0$, if $BA < 0$, then $A^2 > |BA|$; the contradiction comes from Hodge index theorem. Since $B^2 \geq 1$, $AB \geq 0$, $AB \leq x$, $A^2 \geq B^2$ and $(A^2)(B^2) \leq (AB)^2$, we obtain $M^2 \leq 2x^2 + 2x$, proving 0.3. ■

2. Positive characteristic.

Here we consider the case char$(\mathbf{K}) > 0$, but only for $n = 2$ and $r = 2$; the restriction «$n = 2$» will be extemely important for the proof. Hence here X will be a smooth projective surface. Here we will prove the following result.

PROPOSITION 2.1. *Assume* $p := \mathrm{char}(\mathbf{K}) > 0$. *Fix an integer* $g \geqslant 0$. *Then the set* $A(2,2,g)$ *of all pairs* (X, E) *with* X *smooth,* E *ample and spanned,* $\mathrm{rank}(E) = 2$, E *with* c_1*-sectional genus* g (*over* $\mathbf{K}$) *is bounded.*

PROOF. The proof will be divided into 5 steps.

(*a*) By the adjunction formula we have:

$$K \cdot L + L^2 = 2g - 2. \tag{4}$$

Hence if $K \cdot L \geqslant 0$ (or $-K \cdot L$ has an upper bound depending only on g), L^2 is bounded in terms of g. Note that $K \cdot L \geqslant 0$ if X has Kodaira dimension $\kappa(X) \geqslant 0$, Now take $E \in B(X, L, 2)$. Since E is ample, we have $L^2 > c_2$ and $c_2 > 0$ ([BG] or [FL]). Hence if L^2 is bounded only in terms of g there are only finitely many possibilities for $c_2(E)$ (for fixed g).

(*b*) Here we check that 1.6 holds in our situation under the assumption that $K \cdot L \geqslant 0$ (or $-K \cdot L$ is bounded in terms of g), and in particular tha it holds if X has Kodaira dimension $\kappa(X) \geqslant 0$. Fix L with $g(L) = g$ and $E \in B(X, L, 2)$. By part (*a*) $c_2(E)$ can take only finitely many values. We will give a few estimates depending only on X and $L := \det(E)$. If E is L-stable, it is sufficient to note that the set of L-stable rank 2 vector bundles on X with $c_1 = L$ and fixed c_2 is bounded ([M1] and [M2]). Hence we may assume that E is not L-stable, i.e. we may assume the existence of the exact sequence (3) with $AL \geqslant BL$ (i.e. $A^2 \geqslant B^2$). Note that $B^2 > 0$ by the ampleness of E. Since $BL > 0$, if $BA < 0$, then $A^2 > |BA|$; the contradiction comes from Hodge index theorem. Since $B^2 \geqslant 1$, $AB \geqslant 0$, $AB \leqslant c_2$, $A^2 \geqslant B^2$ and $(A^2)(B^2) \leqslant$ $\leqslant (AB)^2$, we obtain $L^2 \leqslant 2(c_2)^2 + 2c_2$, (exactly the same words as in the proof of 0.3), concluding this part.

(*c*) Here we assume $\kappa(X) = 2$ and give the full Proof of 2.1. By steps (*a*) and (*b*) it is sufficient to check the boundedness of the set of all pairs (X, L) with fixed $g := g(L)$. Fix any such X. Let U be the minimal model of X and let c be the number of blowing-ups of points needed to pass from U to X (hence $(K_U)^2 = K^2 + c$). By 1.2 we have $KL > 2c$; since $L^2 > 0$, c is bounded only in terms of g by eq. (4). Since both L^2 and KL have $2g - 2$ as upper bound, by Hodge index theorem K^2 has an upper bound depending only on g. Thus $(K_U)^2$ is bounded only in terms of g. It is known (see for instance [Ko]) that this implies the boundedness of the set of all possible minimal models U, hence of the possible surfaces X. Hence we may fix X; it remains only to show that the set of possible L on a fixed X is bounded (or equivalently the set of all possible $3K + 6L$). Consider the possible Hilbert polynomials $p(t) :=$

$:= \chi(O(t(3K+6L)) = at^2 + a't + \chi(O_X))$. We have $2a = (3K+6L)^2$ (which, for fixed X, is bounded only in terms of g). By general principles (see for instance again [Kl1], Th. 3.13) it is sufficient to give a bound for $|a'|$ in terms of g, or equivalently to bound $|p(1)|$. It was checked in [AB] (using the theorem stated in the introduction of [Ek] as fundamental tool) that $h^1(3K+6L) = 0$. Fix a point x of X. Note that we may find $D \in |3K+6L|$ with high multiplicity, m at x, and m increasing with $h^0(3K+6L)$. By the easy part of a criterion of ampleness due to Seshadri ([Ha]), p. 37), and the fact that $(3K+6L)L \leq 12g$, we see that m is bounded by a function of g; the same holds true for $h^0(3K+6L)$. Since $h^2(3K+6L) = 0$, we obtain that $|p(1)|$ is bounded only in terms of g.

(*d*) Here we prove the full statement of 2.1 under the assumption that $\kappa(X) \leq 1$, except for the class of quasi-elliptic surfaces with $p = 2$ or $p = 3$ and $\kappa(X) = 1$ considered as exceptional cases in [Ek], Th. 1.6 of § 2. By assumption we may apply Ekedhal's vanishing theorem ([Ek], Th. 1.6 of § 2). Thus $0 \leq h^0(K+L) = g - q + p_g$ with $q := q(X) := := h^1(O_X)$. Hence q is bounded in terms of g if $k(X) \leq 0$; if $\kappa(X) = 1$, to prove the same assertion we need to bound p_g; this is done exactly as in the last part of step (*c*). Hence we may fix q. If $L^2 \leq 4q + t(g)$ with $t(g)$ any function depending only on g, the proof of part (*b*) applies. Thus we may assume $L^2 > 4q + t(g)$ for suitable $t(g)$. By [BCM], Th. 0.1 and Remark 1.1, for the case «$k = 1$», the line bundle $K + L$ is very ample for suitable $t(g)$ (in the case $\kappa(X) = 1$ we have to use that $h^0(2K)$ is bounded by a function of g, again by the ampleness criterion of Seshadri ([Ha], p. 37) exactly as in the last part of step (*c*)). Let h be the embedding of X into a projective space $\mathbf{P}$ determined by $|K+L|$. Note that since $(K+L)^2 = K^2 + KL + 2g - 2 < 8 + 2g - 2$, the set of possible X is bounded (e.g. by the existence of the Chow varieties of $\mathbf{P}$); thus (as in the first sketch of the proof of 0.1) we may fix X. Fix a general integral $C \in |K+L|$. Since $-2 \leq 2p_a(C) - 2 = 2k^2 + 3KL + L^2 \leq 18 + 3KL + + L^2$, we have $-KL \leq 7 + (L^2)/3$. Thus by eq. (4) the integer L^2 is bounded in terms of g and we may apply the proof in part (*b*).

(*e*) Here we check the exceptional cases with $p = 2$ or $p = 3$ left open in step (*d*). We have only to find a suitable ample line bundle to whom to apply Ekedhal vanishing theorem, to obtain the boundedness of $h^1(O_X)$. Let $\phi: X \to C$ be the quasi-elliptic fibration (i.e. the Albanese map). Since by 1.2 $K+L$ is nef, we have $(K+L)^2 \geq 0$, i.e. L^2, KL, and the number of blowing-ups needed to obtain X from its minimal model are bounded by a function of g. Look at parts (*a*) and (*b*) of the statement of [Ek], Th. 1.6 in § 2; we will use the notations of that statement. We do not need Kodaira vanishing for L; it is sufficient to have it for, say, $L := L^{\otimes 3}$. With the notations of loc. cit. it is sufficient to check that $h^0(N \otimes L^{-1}) = 0$ if $p = 2$ and $h^0(M \otimes L^{-1}) = 0$ if $p = 3$. Note that the degree of the restriction to a general fiber T of ϕ of the line bundle N (if $p = 2$) or M (if $p = 3$) depend only on p, not on X, because it can be calculated after completion at one sufficiently general fiber, furthermore, this degree depends only on what happens to the restriction of such a line bundle near the cusp of T. Thus, for instance for $p = 3$, $\deg(L^2)$ will be greater than such a degree; hence $h^0(M \otimes L^{-1}) = 0$ as wanted. ■.

REFERENCES

[AB] M. ANDREATTA - E. BALLICO, *On the adjunction process over a surface in char p*, Manuscripta Math., **62** (1988), pp. 227-244.

[B] E. BALLICO, *On vector bundles on threefolds with sectional genus, I*, Trans. Amer. Math. Soc., **324** (1991), pp. 145-147.

[BCM] E. BALLICO - L. CHIANTINI - V. MONTI, *On the adjunction mapping for surfaces of Kodaira dimension* $\leqslant 0$ *in char p*, Manuscripta Math., **73**(1991), pp. 313-318.

[BPS] C. BANICA - M. PUTINAR - G. SCHUMACHER, *Variation der globalen Ext in Deformationen kompakter komplexer Raume*, Math. Ann., **250** (1980), pp. 135-155.

[BG] S. BLOCH - D. GIESEKER, *The positivity of the Chern classes of an ample vector bundle*, Invent. Math., **12** (1971), pp. 112-117.

[D1] J. P. DEMAILLY, *Relations entre les différences notions de fibrés et de courant positifs*, in Seminaire Lelong-Skoda, pp. 56-76, Lect. Notes in Math., **919**, Springer- Verlag (1982).

[D2] J. P. DEMAILLY, *Estimations* L^2 *pour l'operateur* $\bar{\partial}$ *d'un fibré vectoriel holomorphe semi-positif au-dessous d'une varieté Kahlérienne complète*, Ann. Scient. Ec. Norm. Sup., **15** (1982), pp. 457-511.

[Ek] T. EKEDHAL, *Canonical models of surfaces of general type in positive characteristic*, Publ. I.H.E.S., **67** (1988), pp. 97-144.

[F1] T. FUJITA, *Ample vector bundles of small* c_1*-sectional genus*, J. Math. Kyoto Univ., **29** (1989), pp. 1-16.

[F2] T. FUJITA, *Classification theories of polarized varieties*, London Math. Soc. Lect. Notes Series, **155**, Cambridge University Press (1990).

[FL] W. FULTON - R. LAZARSFELD, *The numerical positivity of ample vector bundles*, Ann. of Math., **118** (1983), pp. 35-60.

[Ha] R. HARTSHORNE, *Ample subvarieties of algebraic varieties*, Lect. Notes in Math., **156**, Springer-Verlag (1970).

[Io] P. IONESCU, *Generalized adjunction and applications*, Math. Proc. Cambridge Philos. Soc., **99** (1986), pp. 457-472.

[Is] S. ISHIMURA, *On π-uniform vector bundles*, Tokyo J. Math., **2** (1979), pp. 337-342.

[Kl1] S. KLEIMAN, *Les théorèmes de finitude pour le foncteur de Picard*, exposé XIII in SGA 6, pp. 616-666, Lect. Notes in Math., **225**, Springer-Verlag (1971).

[Kl2] S. KLEIMAN, *On the transversality of a general translate*, Compositio Math., **38** (1974), pp. 287-297.

[Ko] J. KOLLAR, *Toward moduli of singular varieties*, Compositio Math., **56** (1985), pp. 369-398.

[L] H. LANGE, *Universal families of extensions*, J. Algebra, **83** (1983), pp. 101-112.

[M1] M. MARUYAMA, *Moduli of stable sheaves, II*, J. Math. Kyoto Univ., **18** (1978), pp. 557-614.

[M2] M. MARUYAMA, *On boundedness of families of torsion free sheaves*, J. Math. Kyoto Univ., **21** (1981), pp. 673-701.

[N] S. NAKANO, *Vanishing theorems for weakly* 1*-complete manifolds, II*, Publ. RIMS, **10** (1974), pp. 101-110.

[Sk1] M. SKODA, *Morphisms surjectifs et fibrés lineaire semi-positifs*, in: Seminaire Lelong-Skoda, pp. 290-324, Lect. Notes in Math., **694**, Springer-Verlag (1978).

[Sk2] H. SKODA, *Remarques a propos des théorèms d'annulation pour les fibrés semi-positifs*, in: Seminaire Lelong-Skoda, pp. 252-268, Lect. Notes in Math., **822**, Springer-Verlag (1980).

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