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Some sporadic groups as Galois groups II

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H. PAHLINGS (*)

The purpose of this note is to show, that the sporadic simple groups J_3, McL, Ru, and Ly and their automorphism groups are Galois groups over $\boldsymbol{Q}$, and what is more over the field $\boldsymbol{Q}(t)$ of rational functions over $\boldsymbol{Q}$. Taking into account the results of various authors ([2, 3, 4, 5, 9, 10]; see [6] for an exposition and summary of known results) this shows, that all the sporadic simple groups are Galois groups over $\boldsymbol{Q}$ (or $\boldsymbol{Q}(t)$) with the possible exception the Mathieu group M_{23}. For this group the methods of this paper seem to be insufficient.

We use the notation and definitions of the first part of this paper [9]. In particular we use the notation of a GAR-realization, which is of importance for the extension problem, and state our main result as

THEOREM. The sporadic simple groups J_3, Mcl, Ru, Ly have GAR-realizations over $\boldsymbol{Q}(t)$.

As in [9] the proof uses the Rationality Criteria of Belyi, Matzat and Thompson and follows from the following lemmas.

LEMMA 1. For the rational class structure

a) $\mathcal{C} = (2B, 3B, 8B)$ of $\mathrm{Aut}(J_3)$,

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b) $\mathcal{C} = (2B, 3A, 10B)$ of Aut (McL),

c) $\mathcal{C} = (2A, 4A, 13A)$ of Ru,

one has $l^i(\mathcal{C}) = n(\mathcal{C}) = 1$.

Thus the groups Aut (J_3), Aut (McL), Ru are rationally rigid in the sense of Thompson [10].

LEMMA 2. *Ly has a rational class structure*

$$\mathcal{C} = (2A, 5A, 14A) \quad \text{with} \quad l^i(\mathcal{C}) = 1 \quad \text{and} \quad n(\mathcal{C}) = \tfrac{3}{2}\,.$$

Here, concerning the conjugacy classes, the notation of the ATLAS [1] is used; in particular $2B$ is the class of outer involutions in Aut (J_3) or Aut (McL). The normalized structure constant of a triple $\mathcal{C} = (C_1, C_2, C_3)$ of classes of the groups G is denoted by $n(\mathcal{C})$ and $l^i(\mathcal{C})$ is the number of orbits of G on

$$\{(g_1, g_2, g_3)\colon\ g_i \in C_i\ (i = 1, 2, 3),\ g_1 g_2 g_3 = 1\,,\ \langle g_1, g_2, g_3\rangle = G\}\,.$$

PROOF OF LEMMA 1. *a*) It is easily verified that $n(2B, 3B, 8B) = 1$. Let $g \in 2B$, $h \in 3B$ be such that $gh \in 8B$ and let $H = \langle g, h\rangle$. We show that H is not contained in a maximal subgroup of Aut (J_3). The maximal subgroups of Aut (J_3) are (cf. [1]) J_3 and (up to isomorphism)

$$H_1 = L_2(16){:}4\,, \qquad H_2 = 19{:}18\,, \qquad H_3 = 2^4{:}(3 \times A_5)\cdot 2\,,$$

$$H_4 = L_2(17) \times 2\,, \qquad H_5 = (3 \times M_{10}){:}2\,, \qquad H_6 = 3^2\cdot(3^{1+2}){:}8\cdot 2\,,$$

$$H_7 = 2_-^{1+4}\cdot S_5\,, \qquad H_8 = 2^{2+4}{:}(S_3 \times S_3)\,.$$

Using the CAS-system (cf. [8]) the table of all primitive permutation characters of Aut (J_3) has been computed. This is useful for other applications, too. The result is reproduced in the Appendix. It shows, that H_1 does not contain elements of $2B$, the groups H_3, H_4, H_6 contain no elements of $8B$ and H_7 no elements of $3B$. Obviously H cannot be contained in H_2. H_5 has three normal subgroups of index 2, one being $3 \times M_{10}$; this group contains the elements of $H_5 \cap 8B$, the cubes of elements of order 24, but no outer involutions, i.e. elements of $2B$. So products of elements of $H_5 \cap 8B$ with those of $H_5 \cap 2B$ cannot be contained in $3 \times A_6$ and hence do not have order 3. Finally the character table of H_8 has been computed and its fusion into Aut (J_3) has been determined. It is reproduced in the appendix. An easy computation shows that products of elements of $H_8 \cap 2B$ with elements of $H_8 \cap 3B$ are not in $H_8 \cap 8B$. This shows that H is not contained in any maximal subgroup of Aut (J_3) and so $H = \mathrm{Aut}(J_3)$; thus

$$l^i(2B, 3B, 8B) = n(2B, 3B, 8B) = 1 .$$

b) In Aut (McL) we have $n(2B, 3A, 10B) = 1$. Let $g \in 2B$ $h \in 3A$ be such that $gh \in 10B$ and let $H = \langle g, h\rangle$. The maximal subgroups of Aut (McL) are (cf. [1]) McL and up to conjugation

$$H_1 = U_4(3){:}2\,, \qquad H_2 = U_3(5){:}2\,, \qquad H_3 = 3_+^{1+4}{:}4S_5\,,$$

$$H_4 = 3^4{:}(M_{10} \times 2)\,, \qquad H_5 = L_3(4){:}2^2\,, \qquad H_6 = 2\cdot S_8\,,$$

$$H_7 = 2^{2+4}{:}(S_3 \times S_3)\,, \qquad H_8 = M_{11} \times 2\,, \qquad H_9 = 5_+^{1+2}{:}3{:}8\cdot 2\,.$$

The table of primitive permutation characters of Aut (Mcl) (cf. Appendix) shows, that the only maximal subgroups of Aut (McL) which contain elements of the classes $2B$, $3A$ and $10A$ simultaneously are H_1 and H_4. The intersection of $2B$, $3A$ and $10B$ with H_1 are the classes $2F$, $3A$ and $10C$, respectively, of the group $U_4(3){:}2_3$ in Atlas notation ([1], pp. 54-55) and an easy computation shows that the structure constant of this class triple vanishes.

Finally, since the elements of $3A$ intersect with H_4 in a class contained in the elementary abelian normal subgroup of order 3^4 it is quite clear that a product of elements of order 2 with elements of order 10 in H_4 is not in $3A \cap H_4$. Thus $H = \mathrm{Aut}(McL)$ and $l^i(2B, 3B, 8B) = 1$.

c) Consider the classes $2B$, $4A$, $13A$ of the simple group Ru and let $g \in 2B$, $h \in 4A$ be such that $gh \in 13A$. For the normalized structure constant one gets $n(2B, 4A, 13A) = 1$. Put $H = \langle g, h\rangle$. The only maximal subgroups of Ru, which contain elements of order 13 are up to conjugation

$$H_1 = {}^2F_4(2)\,, \qquad H_2 = (2^2 \times Sz(8)){:}3\,,$$

$$H_3 = L_2(25)\cdot 2^2\,, \qquad H_4 = L_2(13){:}2\,.$$

H_1 contains no elements of $2B$ and H_2 and H_4 no elements of $4A$, as the primitive permutation characters (see the Appendix) show. The character tables of some maximal subgroups of Ru and the fusion maps have been computed by S. Mattarei [7].

The class $2B$ of Ru is the class of involutions which are not third powers of elements of order 6. H_3 contains just one such class ($2B$ in the notation of the ATLAS [1], p. 17) and this is in a coset of $L_2(25)$, which does not contain an element of order 4. So H cannot be contained in H_3, hence $H = G$ and $l^i(2B, 4A, 13A) = n(2B, 4A, 13A) = 1$.

Proof of Lemma 2. We consider the classes $2A$, $5A$, $14A$ of Ly; the normalized structure constant $n(2A, 5A, 15A)$ is $\frac{3}{2}$. The maximal subgroups of Ly are (up to conjugation)

$$H_1 = G_2(5)\,, \qquad H_2 = 2\cdot McL{:}2\,, \qquad H_3 = 5^3\cdot L_3(5)\,,$$

$$H_4 = 2\cdot A_{11}\,, \qquad H_5 = 5^{1+4}_{+}{:}4S_6\,, \qquad H_6 = 3^5{:}(2\times M_{11})\,,$$

$$H_7 = 3^{2+4}{:}2A_5\cdot D_8\,, \qquad H_8 = 67{:}22\,, \qquad H_9 = 37{:}18\,.$$

The only maximal subgroups, which contain elements of order 14 are H_2 and H_4. The class $5A$ (resp. $14A$) of Ly intersects with H_4 in the class $5a$ (resp. $14a$) of $2\cdot A_{11}$, whereas both involution classes $2a$ amd $2b$ of $2\cdot A_{11}$ fuse into the class $2A$ of Ly. But the structure constants $n(2a, 5a, 14a)$ and $n(2b, 5a, 14a)$ are both *zero*.

The class $5A$ (respectively $14A$) of Ly intersected with H_2 gives one conjugacy class $5a$ (respectively $14a$) of H_2, whereas $2A \cap H_2$ consists of two classes $2a$ and $2b$, the latter being the outer involution class. One finds $n(2a, 5a, 14a) = \frac{1}{2}$ and, obviously $n(2b, 5a, 14a) = 0$. It follows that the number of triples (g, h, gh) with $g \in 2A$, $h \in 5A$, $gh \in 14A$ which generate a proper subgroup of Ly is at most (and in fact equal to) $\frac{1}{2}|Ly|$. Thus, since the center of Ly is trivial, Ly has one regular orbit on $\{(g, h)\colon g \in 2A, h \in 5A, gh \in 14A, \langle g, h\rangle = Ly\}$ and $l^i(2A, 5A, 14A) = 1$.

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j3.2primitiv

2		8	8	4	1	6	1	4	4	1	1	1	1
3		5	1	3	5	1	1	1	.	3	3	3	.
5		1	1	1	.	.	1	.	.	.	.	.	1
17		1	.	.	.	.	.	.	.	.	.	.	.
19		1	.	.	.	.	.	.	.	.	.	.	.
		1a	2a	3a	3b	4a	5a	6a	8a	9a	9b	9c	10a
2P		1a	1a	3a	3b	2a	5a	3a	4a	9b	9c	9a	5a
3P		1a	2a	1a	1a	4a	5a	2a	8a	3b	3b	3b	10a
5P		1a	2a	3a	3b	4a	1a	6a	8a	9c	9a	9b	2a
Y.1	PSL(2,16):2	6156	76	36	.	12	1	4	.	.	.	.	1
Y.2	2^4:(3×A5).2	17442	50	72	27	6	2	8	.	.	.	.	.
Y.3	PSL(2,17)×2	20520	120	.	27	12	.	.	2	3	3	3	.
Y.4	(3×M10):2	23256	136	81	9	4	1	1	2	.	.	.	1
Y.5	3^2.(3^1+2):8.2	25840	80	10	1	8	.	2	4	1	1	1	.
Y.6	2—^1+4:S5	26163	131	45	.	7	3	5	1	.	.	.	1
Y.7	2^2+4:(S3×S3)	43605	85	90	27	5	.	10	1	.	.	.	.
Y.8	19:18	293760	.	.	54	.	.	.	.	6	6	6	.

2	3	.	1	1	.	5	5	1	5	5	2	1	1	1	3	3	1	1
3	1	1	.	.	.	2	1	2	1	.	1	2	2	2	1	1	.	.
5	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
17	.	.	1	1	.	1	.	.	.	.	.	.	.	.	.	.	1	1
19	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.
	12a	15a	17a	17b	19a	2b	4b	6b	8b	8c	12b	18a	18b	18c	24a	24b	34a	34b
2P	6a	15a	17a	17b	19a	1a	2a	3b	4a	4a	6a	9a	9c	9b	12a	12a	17b	17a
3P	4a	5a	17b	17a	19a	2b	4b	2b	8b	8c	4b	6b	6b	6b	8b	8b	34b	34a
5P	12a	3a	17b	17a	19a	2b	4b	6b	8b	8c	12b	18b	18c	18a	24a	24b	34b	34a
Y.1	.	1	2	2	.	.	8	.	.	8	2	.	.	.	.	.	.	.
Y.2	.	2	.	.	.	102	18	3	.	4	.	.	.	.	.	.	.	.
Y.3	.	.	1	1	.	154	6	1	.	4	.	1	1	1	.	.	1	1
Y.4	1	1	.	.	.	102	10	3	4	.	1	.	.	.	1	1	.	.
Y.5	2	.	.	.	.	136	8	1	.	.	2	1	1	1	.	.	.	.
Y.6	1	.	.	.	.	153	13	.	7	3	1	.	.	.	1	1	.	.
Y.7	2	.	.	.	.	255	15	3	3	3	.	.	.	.	.	.	.	.
Y.8	.	.	.	.	1	272	.	2	.	.	.	2	2	2	.	.	.	.

The primitive permutation characters of J3.2

2^2+4:(S3×S3)

2	8	8	5	6	1	5	5	4	3	3	2	2	2	2	4	5	5	3	5	5	4	1
3	2	1	1	.	2	1	1	.	2	1	2	1	1	1	1	.	.	.	.	.	.	1
	1a	2a	2b	4a	3a	2c	4b	8a	3b	6a	3c	6b	6c	12a	2d	2e	4c	4d	8b	8c	8d	6d
2P	1a	1a	1a	2a	3a	1a	2a	4a	3b	3b	3c	3c	3b	6a	1a	1a	2a	2b	4a	4a	4a	3a
3P	1a	2a	2b	4a	1a	2c	4b	8a	1a	2a	1a	2b	2c	4b	2d	2e	4c	4d	8b	8c	8d	2d
Subgroup fusion into j3.2																						
	1a	2a	2a	4a	3b	2a	4a	8a	3a	6a	3a	6a	6a	12a	2b	2b	4b	4b	8b	8c	8c	6b
Subgroup fusion into mcl.2																						
	1a	2a	2a	4a	3b	2b	4b	8c	3b	6b	3b	6b	6c	12c	2b	2a	4a	4a	8c	8b	8a	6c
Y.1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Y.2	1	1	1	1	1	−1	−1	−1	1	1	1	1	−1	−1	1	−1	−1	−1	1	1	−1	1
Y.3	2	2	2	2	−1	.	.	.	2	2	−1	−1	.	.	2	.	.	.	2	2	.	−1
Y.4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1
Y.5	1	1	1	1	1	−1	−1	−1	1	1	1	1	−1	−1	−1	1	1	1	−1	−1	1	−1
Y.6	2	2	2	2	−1	.	.	.	2	2	−1	−1	.	.	−2	.	.	.	−2	−2	.	1
Y.7	2	2	2	2	2	2	2	2	−1	−1	−1	−1	−1	−1	.	.	.	.	.	.	.	.
Y.8	2	2	2	2	2	−2	−2	−2	−1	−1	−1	−1	1	1	.	.	.	.	.	.	.	.
Y.9	4	4	4	4	−2	.	.	.	−2	−2	1	1	.	.	.	.	.	.	.	.	.	.
Y.10	6	6	2	−2	.	.	.	.	.	.	3	−1	.	.	.	−2	−2	.	.	.	2	.
Y.11	6	6	2	−2	.	.	.	.	.	.	3	−1	.	.	.	2	2	.	.	.	−2	.
Y.12	9	9	−3	1	.	3	3	−1	.	.	.	.	.	.	3	1	1	−1	−1	−1	1	.
Y.13	9	9	−3	1	.	3	3	−1	.	.	.	.	.	.	−3	−1	−1	1	1	1	−1	.
Y.14	9	9	−3	1	.	−3	−3	1	.	.	.	.	.	.	3	−1	−1	1	−1	−1	−1	.
Y.15	9	9	−3	1	.	−3	−3	1	.	.	.	.	.	.	−3	1	1	−1	1	1	1	.
Y.16	12	12	4	−4	.	.	.	.	.	.	−3	1	.	.	.	.	.	.	.	.	.	.
Y.17	12	−4	.	.	.	2	−2	.	3	−1	.	.	−1	1	.	2	−2	.	−2	2	.	.
Y.18	12	−4	.	.	.	2	−2	.	3	−1	.	.	−1	1	.	−2	2	.	2	−2	.	.
Y.19	12	−4	.	.	.	−2	2	.	3	−1	.	.	1	−1	.	−2	2	.	−2	2	.	.
Y.20	12	−4	.	.	.	−2	2	.	3	−1	.	.	1	−1	.	2	−2	.	2	−2	.	.
Y.21	24	−8	.	.	.	4	−4	.	−3	1	.	.	1	−1	.	.	.	.	.	.	.	.
Y.22	24	−8	.	.	.	−4	4	.	−3	1	.	.	−1	1	.	.	.	.	.	.	.	.

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2		8	8	4	3	6	2	1	4	3	1	4	.	2	1	1
3		6	2	6	5	1	1	.	2	2	.	.	3	1	.	.
5		3	1	1	.	.	3	2	1	.	.	.	.	1	.	.
7		1	1	.	.	.	.	.	.	.	1	.	.	.	.	.
11		1	.	.	.	.	.	.	.	.	.	.	.	.	1	1
		1a	2a	3a	3b	4a	5a	5b	6a	6b	7a	8a	9a	10a	11a	11b
2P		1a	1a	3a	3b	2a	5a	5b	3a	3b	7a	4a	9a	5a	11b	11a
3P		1a	2a	1a	1a	4a	5a	5b	2a	2a	7a	8a	3a	10a	11a	11b
5P		1a	2a	3a	3b	4a	1a	1a	6a	6b	7a	8a	9a	2a	11a	11b
Y.1	PSU(4,3):2	275	35	5	14	7	.	5	5	2	2	1	2	.	.	.
Y.2	PSU(3,5):2	7128	168	.	27	12	3	3	.	3	2	2	.	3	.	.
Y.3	3+^1+4:4.S5	15400	280	10	37	20	.	5	10	1	.	2	1	.	.	.
Y.4	3^4:(M10×2)	15400	56	91	10	12	25	.	11	2	.	2	1	1	.	.
Y.5	PSL(3,4):2^2	22275	435	.	54	15	.	5	.	6	1	1	.	.	.	.
Y.6	2.S8	22275	211	81	27	7	25	.	1	7	1	1	.	1	.	.
Y.7	2^2+4:(S3×S3)	779625	3465	.	189	33	.	.	.	9	.	1	.	.	.	.
Y.8	M11×2	113400	840	.	54	12	.	5	.	6	.	2	.	.	1	1
Y.9	5+^1+2:3:8.2	299376	336	486	.	8	1	1	6	.	.	4	.	1	.	.

2	3	1	1	1	5	5	2	5	5	1	2	2	2	2	1
3	1	.	1	1	2	2	2	1	.	.	2	2	.	.	.
5	.	.	1	1	1	1	.	.	.	1	.	.	1	1	.
7	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.
11	.	.	.	.	1	.	.	.	.	.	.	.	.	.	1
	12a	14a	15a	30a	2b	4b	6c	8b	8c	10b	12b	12c	20a	20b	22a
2P	6a	7a	15a	15a	1a	2a	3b	4a	4a	5b	6a	6b	10a	10a	11b
3P	4a	14a	5a	10a	2b	4b	2b	8b	8c	10b	4b	4b	20a	20b	22a
5P	12a	14a	3a	6a	2b	4b	6c	8b	8c	2b	12b	12c	4b	4b	22a
Y.1	1	.	.	.	11	15	2	1	5	1	3	.	.	.	.
Y.2	.	.	.	.	66	6	3	12	.	1	.	3	1	1	.
Y.3	2	.	.	.	66	70	3	.	4	1	4	1	.	.	.
Y.4	3	.	1	1	110	26	2	4	.	.	5	2	1	1	.
Y.5	.	1	.	.	121	45	4	3	7	1	.	.	.	.	.
Y.6	1	1	1	1	165	1	3	7	3	.	1	1	1	1	.
Y.7	.	.	.	.	495	15	9	3	3	.	.	3	.	.	.
Y.8	.	.	.	.	166	90	4	.	4	1	.	.	.	.	1
Y.9	2	.	1	1	396	36	.	8	.	1	.	.	1	1	.

The primitive permutation characters of McL.2

ruprimitiv

2		14	14	8	4	9	8	10	9	3	2	4	2	5
3		3	1	.	3	1	1	.	.	.	1	1	.	1
5		3	1	1	1	1	1	.	.	3	2	.	.	.
7		1	.	1	.	.	.	.	.	.	.	.	1	.
13		1	.	1	.	.	.	.	.	.	.	.	.	.
29		1	.	.	.	.	.	.	.	.	.	.	.	.
		1a	2a	2b	3a	4a	4b	4c	4d	5a	5b	6a	7a	8a
2P		1a	1a	1a	3a	2a	2a	2a	2a	5a	5b	3a	7a	4a
3P		1a	2a	2b	1a	4a	4b	4c	4d	5a	5b	2a	7a	8a
5P		1a	2a	2b	3a	4a	4b	4c	4d	1a	1a	6a	7a	8a
Y.1	^2F4(2)	4060	92	.	10	32	20	4	8	10	.	2	.	6
Y.3	(2^2 × Suz(8)):3	417600	320	456	72	.	40	.	16	.	5	8	1	.
Y.7	PSL(2,25).2^2	4677120	3584	1120	45	160	160	.	32	20	.	5	.	16
Y.13	PSL(2,13):2	66816000	10240	4160	180	.	320	.	.	.	.	4	6	.

2	6	5	3	2	3	2	2	2	2	2	.	4	4	2	2	2	3	3	
3	.	.	.	.	1	1	.	.	.	.	1	.	.	.	.	.	1	1	
5	.	.	1	1	.	.	.	.	.	.	1	.	.	1	1	1	.	.	
7	.	.	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.	.	
13	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	
29	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	
	8b	8c	10a	10b	12a	12b	13a	14a	14b	14c	15a	16a	16b	20a	20b	20c	24a	24b	2
2P	4c	4d	5a	5b	6a	6a	13a	7a	7a	7a	15a	8b	8b	10a	10a	10a	12a	12a	1
3P	8b	8c	10a	10b	4a	4b	13a	14b	14c	14a	5b	16b	16a	20a	20c	20b	8a	8a	2
5P	8b	8c	2a	2b	12a	12b	13a	14c	14a	14b	3a	16a	16b	4a	4b	4b	24a	24b	2
Y.1	4	2	2	.	2	2	4	.	.	.	.	2	2	2	.	.	.	.	
Y.3	.	.	.	1	.	4	1	1	1	1	2	.	.	.	.	.	.	.	
Y.7	.	.	4	.	1	1	6	.	.	.	.	.	.	.	.	.	1	1	
Y.13	.	.	.	.	.	2	4	2	2	2	2	.	.	.	.	.	.	.	

2	2	2	.	.
3	3	.	.	.
5	5	.	.	.
13	1	1	.	.
29	.	.	1	1
	26b	26c	29a	29b
2P	13a	13a	29b	29a
2P	26a	26b	29b	29a
5P	26b	29c	29a	29b
Y.1	.	.	.	.
Y.3	1	1	.	.
Y.7	2	2	.	.
Y.13	.	.	.	.

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