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Generators of Hyperbolic Heat Equation in Nonlinear Thermoelasticity.

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1. Introduction.

Let $\mathcal{B}$ a region of three-dimensional Euclidean space, referred to an orthonormal basis $\{\boldsymbol{e}_i\}$. Let $\boldsymbol{x} \equiv \boldsymbol{x}(\boldsymbol{X}, t)$ be the position of the generic particle in the deformed configuration at time t and $\boldsymbol{X}$ the position in the undeformed reference configuration, $\boldsymbol{u} = \boldsymbol{x} - \boldsymbol{X}$ the displacement vector. The field equations of continuum mechanics with finite deformations may be written as a first order quasi-linear systems:

conservation of momentum

$$\varrho_0 \partial_t v_i - \partial_A T_{iA} = 0 \tag{1.1}$$

$$\partial_t F_{iA} - \partial_A v_i = 0 , \tag{1.2}$$

balance of energy

$$\partial_t \varepsilon + \partial_A (Q_A - v_i T_{iA}) = 0 \tag{1.3}$$

where $\boldsymbol{T} \equiv (T_{iA})$ is the first Piola-Kirchoff stress tensor, $\boldsymbol{v} = \partial_t \boldsymbol{u} \equiv (v_i)$ is the velocity, $\boldsymbol{F} \equiv (F_{iA} = \partial_A x_i)$ is the gradient tensor, e is the spe-

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cific internal energy, S the specific entropy, ϱ_0 is the reference mass density, $\boldsymbol{Q} \equiv (Q_A)$ is related to the usual heat flux vector $\boldsymbol{q}$: $Jq_i = F_{iA}Q_A$, $J = \det \boldsymbol{F}$, and ε is the total energy:

$$(1.4) \quad \varepsilon = \varrho_0(v^2/2 + e); \qquad \partial_t = \partial/\partial t\,, \quad \partial_A = \partial/\partial X_A\,, \quad (i, A = 1, 2, 3)\,.$$

To eqs. (1.1)-(1.3), the costitutive equations must be added; in particular one has to determine the constitutive dependence of the heat flux. In the classical approach this is the Fourier equation:

$$(1.5) \qquad \partial_A\theta = -Q_A/\chi$$

satisfying the entropy inequality:

$$(1.6) \qquad \varrho_0\partial_t S + \partial_A(Q_A/\theta) = s$$

with the entropy source (for an hyperelastic material)

$$(1.7) \qquad s = Q^2/(\theta^2\chi) > 0\,,$$

provided the scalar function χ (thermal conductivity) is positive (θ is the absolute temperature).

As well known the presence of (1.5) destroys the hyperbolicity of the field system with the consequence of infinite wave propagation speed. Several authors proposed alternative equations in order to eliminate the paradox and hyperbolize the system. Starting from Maxwell's idea and from the well known paper by Cattaneo [1] (in the case of a rigid heat conductor), a large body of literature exists to which one can objectively refer. Substantially two point of view are predominant (in particular in the context of fluid dynamics) one is the « extended irreversible thermodynamics » (Muller [2]) and the other one is the point of view of « rational thermodynamics » (see i.e., Coleman-Noll [3], Müller [4], Gurtin-Pipkin [5], Green-Lindsay [6], Grioli [7]). A comparison of the two points of view is in the review article by Hutter [8]. In [9] we have exposed some critical considerations on the models alternative to Fourier system, in particular on the question of hyperbolicity (generally not garanteed for all times) and on the lack of conservative form for the new heat equation that prevents us from establishing a weak formulation for the Cauchy problem, therefore preventing us from studying shock waves. Moreover, we have made an attempt to unify this two type of approach (« extended » and « rational » thermodynamics) in the sense that we have taken the heat flux as a field component that appears in the nonequilibrium entropy as the extended thermodynamics approach, but we have determined the evolution equation for $\boldsymbol{q}$ directly from an « entropy principle » in the frame of « rational thermodynamics ». The entropy principle we have used is however more restrictive than the usual one since « a priori » we require to select those constitutive equations only, which give partial differential equations in « conservative » from (dissipation is present through the source terms). The tecnique used is based on some recent results for the quasi-linear partial differential systems compatible with a supplementary equation (see [9] and references quoted there).

Let us shortly sketch the methodology. One writes the indefinite field equations and the heat equation as a one quasi-linear, conservative first order system of N scalar equations of the type:

(1.8) $$\partial_\alpha \boldsymbol{F}^\alpha = \boldsymbol{f}$$

where $\boldsymbol{F}^\alpha$ and $\boldsymbol{f}$ are column vectors of R^N, while the entropy law can be put in the form of a scalar equation also quasi-linear and conservative

(1.9) $$\partial_\alpha h^\alpha = g;$$

$\boldsymbol{F}^\alpha, \boldsymbol{f}, h^\alpha$ and g are dependent on the field $\boldsymbol{U} \equiv \boldsymbol{U}(x^\beta)$, the unknown R^N-vector in the differential system (1.8); $\alpha, \beta = 0, 1, 2, 3$; $\partial_0 = \partial_t$; $i, A = 1, 2, 3$.

The system (1.8) compatible with (1.9) (which plays the role of « entropy principle » once a definite sign for the source term is taken) have a peculiar structure since it is possible to define for it the « generators » of the system, *i.e.* a set of $2N + 4$ quantities: $\{\boldsymbol{U}', h'^\alpha, \boldsymbol{f}\}$ named so in [9] since once they are know it is possible to identify both the differential system (1.8) and the supplementary law (1.9): $\boldsymbol{U}'$ (main field) is defined as the sets of multipliers such that

(1.10) $$\boldsymbol{U}' \cdot d\boldsymbol{F}^\alpha \equiv dh^\alpha; \qquad \boldsymbol{U}' \cdot \boldsymbol{f} = g$$

and h'^α (four-vector generator) as

(1.11) $$h'^\alpha = \boldsymbol{U}' \cdot \boldsymbol{F}^\alpha - h^\alpha .$$

From (1.10) and (1.11), if the components of $\boldsymbol{U}'$ are independent, we obtain

$$\boldsymbol{F}^{\alpha} = \partial h'^{\alpha}/\partial \boldsymbol{U}' \,. \tag{1.12}$$

If we suppose that the generators are known, we obtain from (1.12) the system and from (1.11) and the latter of (1.10) the quantities h^{α} and g, i.e. the supplementary law.

The methodology presented in [9] (to which we refer for more details) consists of the following steps: *a*) evaluation of the generators in the classical theory by the Fourier equation; *b*) modification of the generators by substituting the entropy density and the related thermodynamic quantities with new functions to be determined, playing the role of the same quantities but evaluated at non-equilibrium and to be taken as dependent on the heat flux beside the usual variables; *c*) from the modified generators determination of the new system and the new entropy law; *d*) prove that $h_N'^0$ (new h'^0) is a convex function of the vector $\boldsymbol{U}_N'$ (new $\boldsymbol{U}'$); such a convexity condition is sufficient to guaranted that the new system obtained in this way is a « symmetric » one in the sense of Friedrichs (see for example [10]) and then a conservative hyperbolic one for any field (see [11]-[13], [9]). The procedure here sketched has been proved an efficient one for a thermoviscous fluid and in [9] we have proved that it is possible to obtain a symmetric hyperbolic conservative system form a unique constitutive function. The aim of this short paper is to extend this methodology to the case of a hyperelastic solid under finite deformations and to show that even in this case where the equations in Lagrangian form are matematically different, it is possible to obtain a heat equation giving finite propagation speeds and well-posed Cauchy problem.

2. – Generators of Fourier system.

The system (in the following named as Fourier system) given by the equations (1.1), (1.2), (1.3), (1.5) to which we add the constitutive relations for a hyperalstic material,

$$T_{iA} = \varrho_0 \,\partial\Psi/\partial F_{iA} \,, \qquad S = -\,\partial\Psi/\partial\theta \,, \tag{2.1}$$

where Ψ is the free energy

$$\Psi = e - \theta S \,, \tag{2.2}$$

assumes the form (1.8) and the entropy law (1.6) the form (1.9) when we choose:

$$\boldsymbol{F}^0 \equiv (\varrho_0 v_i, F_{iB}, \varepsilon, O_B)^T \tag{2.3}$$

$$\boldsymbol{F}^A \equiv (-T_{iA}, -v_i\delta_{AB}, Q_A - v_i T_{iA}, \theta\delta_{AB})^T \tag{2.4}$$

$$\boldsymbol{f} \equiv (O_i, O_{iB}, O, -Q_B/\chi)^T \tag{2.5}$$

$$h_0 = -\varrho_0 S \tag{2.6}$$

$$h^A = -Q_A/\theta \tag{2.7}$$

$$g = -s = -Q^2/(\theta^2\chi) \tag{2.8}$$

(where O, O_i, O_{ij} $(i, j = 1, 2, 3)$ respectively the null elements of a scalar, a vector and a tensor; the free index i, $B = 1, 2, 3$ and T indicated the transposition).

From (2.1)-(2.2) we obtain the Gibbs relation:

$$\theta\, dS = de - \frac{1}{\varrho_0} T_{iB}\, dF_{iB} \,, \tag{2.9}$$

and from (1.4)

$$\varrho_0 de = d\varepsilon - \varrho_0 \boldsymbol{v}\cdot d\boldsymbol{v} \,. \tag{2.10}$$

Taking into account (2.6), (2.7), (2.9) and (2.10) we have

$$dh_0 = d(-\varrho_0 S) = \frac{1}{\theta}\{v_i d(\varrho_0 v_i) + T_{iB} dF_{iB} - d\varepsilon\} \tag{2.11}$$

$$\begin{aligned} dh^A &= d(-Q_A/\theta) = \theta^{-2} Q_A d\theta - \theta^{-1} dQ_A = \\ &= \frac{1}{\theta}\{v_i d(-T_{iA}) + T_{iB} d(-v_i\delta_{AB}) - d(Q_A - v_i T_{iA}) + \theta^{-1} Q_B d(\theta\delta_{AB})\} \,. \end{aligned} \tag{2.12}$$

By comparison with (2.11), (2.12) and (1.10), (2.3), (2.4), we get immediately the main field $\boldsymbol{U}'$:

$$\boldsymbol{U}' \equiv \frac{1}{\theta}(v_i, T_{iB}, -1, Q_B/\theta)^T . \tag{2.13}$$

From (1.11) instead we obtain the four-vector generator h'^α:

$$h'^0 = \frac{\varrho_0}{\theta}(v^2/2 - G) \tag{2.14}$$

$$h'^A = \frac{1}{\theta}(Q_A - v_i T_{iA}) \tag{2.15}$$

where G is the specific chemical potential:

$$G = \Psi - \frac{1}{\varrho_0} T_{iA} F_{iA} . \tag{2.16}$$

The generators of the Fourier system are then $\boldsymbol{U}'$ given by (2.13), h'^α by (2.14)-(2.15) and the source $\boldsymbol{f}$ given by (2.5).

3. – The new system, deduced via « generators ».

In agreement with what said in the introduction, we assume now, in the frame of extended thermodynamics, the entropy density at non-equilibrium S_N as dependent on the lagrangean heat flux $\boldsymbol{Q}$, besides on F_{iA} and θ.

Since we want to modify the Fourier system as less as possible and because only the thermodynamical variables G and θ appear in the « generators » deduced before, we assume that only the non-equilibrium chemical potential G is affected by the dependence of S_N also on the heat flux. Therefore, we are looking for a new system such that the new generators are the same as before except that G must be replaced by a new function G_N, yet to be determined and to be interpreted as the non-equilibrium chemical potential. In other words we assume as new «generators» (1):

$$(3.1)\qquad \boldsymbol{U}'_N = \boldsymbol{U}'; \quad h'^0_N = \frac{\varrho_0}{\theta}(v^2/2 - G_N); \quad h'^A_N = h'^A; \quad \boldsymbol{f}_N = \boldsymbol{f},$$

while quantities without an index are those of the Fourier system.

If we call

$$(3.2)\qquad \mathcal{S} = (G - G_N)/\theta$$

we have

$$(3.3)\qquad h'^0_N = h'^0 + \varrho_0 \mathcal{S}\,.$$

From (1.12) we obtain

$$(3.4)\qquad \boldsymbol{F}^\alpha_N \cdot d\boldsymbol{U}' = dh'^\alpha_N; \quad \boldsymbol{F}^\alpha \cdot d\boldsymbol{U}' = dh'^\alpha$$

and then when $\alpha = 0$

$$(3.5)\qquad \varrho_0 d\mathcal{S} = d(h'^0_N - h'^0) = (\boldsymbol{F}^0_N - \boldsymbol{F}^0)\cdot d\boldsymbol{U}'$$

Since the conserved quantities in the indefinite equations must remani the same, $\boldsymbol{F}^0_N$ turns out to be of the form:

$$(3.6)\qquad \boldsymbol{F}^0_N = \boldsymbol{F}^0 + (O_i, O_{iA}, O, \varrho_0 w_A)^T$$

with w_A as unknown vector.

Substituting (3.6) in (3.5), we obtain

$$(3.7)\qquad d\mathcal{S} = w_A dQ^*_A; \quad Q^*_A \doteq Q_A/\theta^2$$

that implies for the function $\mathcal{S}$ a dependence only on $\boldsymbol{Q}^*$ and

$$(3.8)\qquad w_A = \frac{\partial \mathcal{S}}{\partial Q^*_A}\,.$$

Observing the equation before last in (3.1) and the linear independence

(1) If we make a comparison with the case of a fluid [9], we notice a relevant difference with the one of a solid since our approach gives there $\boldsymbol{U}'_N \neq \boldsymbol{U}'$ and $h'^\alpha = h'^\alpha$, being G in the main field.

for the components of $d\boldsymbol{U}'$, we obtain from (3.4) when $\alpha = A$:

$$\boldsymbol{F}^A_N = \boldsymbol{F}^A . \tag{3.9}$$

The new system is then determined and given by the same field equations (1.1)-(1.3) unchanged from the classical formulation, and by the new equations substituting the Fourier equation

$$\varrho_0 \partial_t(\partial \mathcal{S}/\partial Q^*_A) + \partial_A \theta = - Q_A/\chi . \tag{3.10}$$

Let's evaluate now the new supplementary law, *i.e.* the inequalities for entropy in the new system. From (1.11) we obtain $h^\alpha_N = \boldsymbol{U}'_N \cdot \boldsymbol{F}^\alpha_N - h'^\alpha_N$ and from the second of (1.10) $g_N = \boldsymbol{U}'_N \cdot \boldsymbol{f}_N$. Taking into account (3.1), (3.6), (3.8) and (3.2), we have

$$h^0_N = \boldsymbol{U}' \cdot \boldsymbol{F}^0_N - h'^0_N = \boldsymbol{U}' \cdot \boldsymbol{F}^0 - h'^0 + \boldsymbol{U}' \cdot (\boldsymbol{F}^0_N - \boldsymbol{F}^0) - (h'^0_N - h'^0) =$$

$$h^0 + \boldsymbol{U}' \cdot (\boldsymbol{F}^0_N - \boldsymbol{F}^0) - (h'^0_N - h'^0) = - \varrho_0 S + \varrho_0 (Q^*_A \partial \mathcal{S}/\partial Q^*_A - \mathcal{S}) ,$$

and since h^0_N has to be interpreted as $- \varrho_0 S_N$ for analogy with (2.6), we have the new entropy density at non-equilibrium:

$$S_N = S + \mathcal{S} - Q^*_A \frac{\partial \mathcal{S}}{\partial Q^*_A} ; \tag{3.11}$$

Furthermore,

$$h^A_N = h^A = - Q_A/\theta \tag{3.12}$$

$$g_N = g = - Q^2/(\theta^2 \chi) . \tag{3.13}$$

The new system gives than the following entropy balance:

$$\varrho_0 \partial_t S_N + \partial_A (Q_A/\theta) = Q^2/(\theta^2 \chi) > 0 \tag{3.14}$$

with S_N given by (3.11).

In conclusion, having fixed the constitutive function $\mathcal{S} \equiv \mathcal{S}(\boldsymbol{Q}^*)$, for a hyperelastic medium, heat conducting and under finite deformations, one obtain the following differential system:

$$(3.15)\qquad \begin{cases} \partial_t(\varrho_0 v_i) - \partial_A T_{iA} = 0 \\ \partial_t F_{iA} - \partial_A v_i = 0 \\ \partial_t \varepsilon + \partial_A(Q_A - v_i T_{iA}) = 0 \\ \partial_t(\varrho_0\, \partial \mathcal{S}/\partial Q_A^*) + \partial_A \theta = -Q_A/\chi \end{cases}$$

with

$$T_{iA} = \varrho_0\, \partial\Psi/\partial F_{iA}\,, \quad S = -\,\partial\Psi/\partial\theta; \qquad \Psi \equiv \Psi(F_{iA}, \theta)\,.$$

The entropy density and the chemical potential at non-equilibrium are given respectively by (3.11) and (3.2):

$$(3.16)\qquad G_N = G - \theta\mathcal{S}\,.$$

4. – Convexity and hyperbolicity.

It has to be noticed that the system (3.15) is in conservative form; since it is of the form (1.8) with $\boldsymbol{F}_N^\alpha = \partial h_N'^\alpha/\partial \boldsymbol{U}'$, choosing as field the main field $\boldsymbol{U}'$, it becomes:

$$(4.1)\qquad \frac{\partial^2 h_N'^\alpha}{\partial \boldsymbol{U}'\,\partial \boldsymbol{U}'}\,\partial_\alpha \boldsymbol{U}' = \boldsymbol{f}\,.$$

As a consequence all the matrices multiplying the derivatives of $\boldsymbol{U}'$ are hessian ones and then symmetrical; in order to show that the system (4.1) is «symmetric» in the sense of Friedrichs, it has to be verified that $\partial^2 h_N'^0/\partial \boldsymbol{U}'\,\partial \boldsymbol{U}'$ is positive definite. For this reason we prove the following proposition:

STATEMENT 1. *Necessary and sufficient condition for the system* (3.15) *to be a symmetric hyperbolic one is that the specific internal energy* $e \equiv e(F_{iA}, S)$ *and the constitutive function* $\mathcal{S} \equiv \mathcal{S}(Q_A^*)$ *are convex functions on a convex domain* $\mathfrak{D}$.

PROOF. We evaluate the quadratic form

$$(4.2)\qquad K = (\partial^2 h_N'^0/\partial \boldsymbol{U}'\,\partial \boldsymbol{U}')\,d\boldsymbol{U}'\cdot d\boldsymbol{U}'$$

and we check under which condition it is positive for any $d\mathbf{U}'$, with $\mathbf{U}'$ belonging at convex domain $\mathfrak{D} \subseteq R^N$. We observe that $\mathbf{F}^0_N = \partial h'^0_N / \partial \mathbf{U}'$ and then $K = d\mathbf{F}^0_N \cdot d\mathbf{U}'$, so that from (3.6), (2.3) and (2.13), after simple manipolations we obtain:

$$K = \frac{\varrho_0}{\theta}(dv)^2 + \frac{1}{\theta} dT_{iA} dF_{iA} + d(1/\theta)\{d(\varrho_0 v^2/2) + T_{iA} dF_{iA} - d\varepsilon)\} + \\ + \varrho_0 d\left(\frac{\partial \mathcal{S}}{\partial Q^*_A}\right) dQ^*_A .$$

Taking into account (2.10) first and then (2.9) one obtains

$$K = \frac{\varrho_0}{\theta}(dv)^2 + \frac{1}{\theta} dT_{iA} dF_{iA} + \frac{\varrho_0}{\theta} d\theta\, dS + \varrho_0 d\left(\frac{\partial \mathcal{S}}{\partial Q^*_A}\right) dQ^*_A .$$

At least noticing that from $T_{iA} = \varrho_0 \partial e / \partial F_{iA}$, $\theta = \partial e / \partial S$, it follows

$$K = \frac{\varrho_0}{\theta}(dv)^2 + \frac{\varrho_0}{\theta}\left\{d\left(\frac{\partial e}{\partial F_{iA}}\right) dF_{iA} + d\left(\frac{\partial e}{\partial S}\right) dS\right\} + \varrho_0 d\left(\frac{\partial \mathcal{S}}{\partial Q^*_A}\right) dQ^*_A , \tag{4.3}$$

and then the statement. This prove in the purely mechanical case it has already been obtained by a different procedure in [14].

The physical meaning of convexity of $\mathcal{S}$ is the same of the one in fluid dynamics and is equivalent to the following result on thermodynamical stability:

STATEMENT 2. *The convexity condition for* $\mathcal{S} \equiv \mathcal{S}(\mathbf{Q}^*)$ *with* $\mathcal{S}(0) = 0$ *is equivalent to the condition of thermodynamic stability, i.e. that the entropy density at equilibrium is maximum*:

$$S_N < S \qquad \forall \mathbf{Q} \neq 0 .$$

PROOF. The prove is straitforward, noticing that for the choice $\mathcal{S}(0) = 0$, the convexity condition for $\mathcal{S}$ can be written also in the following way $Q^*_A \partial \mathcal{S} / \partial Q^*_A - \mathcal{S}(\mathbf{Q}^*) > 0$ $\forall \mathbf{Q}^* \neq 0$ and then from (3.11) the proposition follows.

5. – Consequences and conclusions.

Interesting consequences, both on mathematical and physical standpoint, arise from the conservative and symmetric hyperbolic structure of our system. Let us examine some problems:

i) *Cauchy problem.* For a symmetric system a general theorem given by Fischer and Marsden [15] on the well-position (locally) of the Cauchy problem, holds, ensuring existence and uniqueness of the solution with the same regularity of the initial data in a neighborhood of the initial manifold, when the initial data are chosen in a Sobolev space H^s, with $s \geqslant 4$.

ii) *Shock waves.* The conservative form of our system makes possible to define weak solutions in the usual way and, in particular, to study shock waves. Moreover the properties of shock waves shown in [11], [16], [12], hold: *a*) entropy increases across the shock wavefront, *b*) the jump of entropy determines the knowledge of the jumps of all the main field variables, *c*) the shock propagation speeds are bounded between the smallest and the largest characteristic speeds.

iii) *Acceleration waves.* Acceleration waves (weak discontinuities) propagate with real and finite speeds. Furthermore it is possible to show that anisotropy appears because of the presence of heat flow; anisotropy disappears when propagation takes place across a constant state. The latter property is not present when Cattaneo's type heat equation is employed, while it is present in the model proposed by Grioli [7], which, even if deduced starting from a different view point, formally looks like the model exposed in the present paper.

iv) *Special case.* Since $\mathcal{S} \equiv 0$ gives Fourier equation, it is reasonable to assume $\mathcal{S}$ very small. But $\mathcal{S}$ must be convex with $\mathcal{S}(0) = 0$; then it is natural to analyze the special case: $\mathcal{S} = aQ^{*2}/2$, $a = \text{const.}$ Then the heat equation becomes:

$$\varrho_0 a \partial_t(\boldsymbol{Q}/\theta^2) + \boldsymbol{\nabla}_{\mathbf{X}}\theta = -\boldsymbol{Q}/\chi \cdot$$

which is different form Cattaneo's type equation. The mentioned anisotropy in propagation is introduced because of the term $\partial_t\theta$ in the l.h.s.

v) *Objectivity principle.* The heat equation (3.10), as the analogous law for the fluid (in Eulerian form), do not fulfil the objectivity principle, generally required for constitutive equations; however our equations are invariant under Galilean transformations. It is known that such circumstance characterizes the models of the extended thermodynamics. Several authors introduced « ad hoc » terms to restore objectivity (e.g. [17]). We think that, to be coherent with our approach the principle must not be fulfilled by the heat equation. In fact, by requiring that the heat equation is conservative, and the heat flux is a field variable, in order to have a hyperbolic conservative system, we require that the heat law is a field equation, playing the some role of the remaining equations of the system. This claim is equivalent to the assumption for the conservative heat equation to be of the same type, e.g., of momentum equation (but, with a dissipative source), i.e. of the form:

$$\varrho_0 \partial_t \boldsymbol{w} + \mathrm{Div}\, \Theta = -\boldsymbol{Q}/\chi$$

in which $\varrho_0 \boldsymbol{w}$ plays the role of « thermal momentum » and Θ that of « thermal stress ». This starting hypothesis is equivalent to consider the heat equation as a balance equation (which is reasonable to think, coming it from a principle); the quantities $\boldsymbol{w}$ and Θ are related to the fields variables through constitutive relations which must fulfil the principle of objectivity. The entropy principle here adopted shows the special structure: $\boldsymbol{w} = \partial \mathcal{S}/\partial \boldsymbol{Q}^*$, $\Theta = \theta I$. The only claim is that $\mathcal{S}$ is a scalar function of $\boldsymbol{Q}^*$, i.e. a function of its square modulus.

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