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Boundary values of holomorphic functions and Cauchy problem for $\bar{\partial}$ operator in the polydisc

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Boundary Values of Holomorphic Functions and Cauchy Problem for $\bar{\partial}$ Operator in the Polydisc.

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0. Introduction.

In the first part of the paper (§ 2) the boundary values of holomorphic functions on the polydisc Δ and continuous up to the boundary are studied: in particular we give the necessary and sufficient conditions in order that a continuous function on $\check{S}$ (prop. 2.4) or on $\partial\Delta$ (prop. 2.6) be the boundary value of an analytic function on Δ.

Then (§ 3) it is proved the existence of a solution of the $\bar{\partial}$-problem,

$$\bar{\partial}u = f\,, \qquad f \in C^{\infty}_{0,1}(\bar{\Delta})\,, \qquad \bar{\partial}f = 0\,,$$

which is uniformly bounded with its derivatives (theorem 3.1): (in the proof of the theorem the integral representation formula stated in [5] is basic).

The careful study of this solution (necessarily unique) allows to study, instead of Cauchy problem for $\bar{\partial}$ operator, another equivalent problem (see § 4). By this method it is possible to give, for the Cauchy problem, an existence theorem with estimates and furthermore the integral representation of the solution.

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1. Notations.

Let Δ be the unitary polydisc in C^n centred in $(0, \ldots, 0)$,

$$\Delta = \{z \in \mathbf{C}^n \colon |z_i| < 1,\, i = 1, \ldots, n\} .$$

We shall denote with $\partial\Delta$ its boundary, i.e.

$$\partial\Delta = \bigcup_{i=1}^{n} \{|z_1| \leqslant 1, \ldots, |z_{i-1}| \leqslant 1,\, |z_i| = 1, \ldots, |z_n| \leqslant 1\}$$

and with $\check{S}$ its Chilov boundary,

$$\check{S} = \{z \in \mathbf{C}^n \colon |z_i| = 1,\, i = 1, \ldots, n\} .$$

If $u \in L^2(\check{S})$ and $I = (i_1, \ldots, i_n)$ is a multindex, $\hat{u}(I)$ will be the Fourier coefficient of u of index I, i.e.

$$\hat{u}(I) = \int_{\check{S}} u(z) \bar{z}_1^{i_1+1} \ldots \bar{z}_n^{i_n+1} \, dz_1 \ldots dz_n = \int_{\check{S}} u(z) \bar{z}^{I+1} \, dz$$

$I \geqslant 0$ will mean $i_k \geqslant 0$ for $k = 1, \ldots, n$.

If u is a differentiable function $D_z^I D_{\bar{z}}^J u$ will be as usual

$$\frac{\partial^{|I|+|J|} u}{\partial z_1^{i_1} \ldots \partial z_n^{i_n} \partial \bar{z}_1^{j_1} \ldots \partial \bar{z}_n^{j_n}} \qquad \text{and} \quad |u|_{L^\infty, s} = \sup_{\Delta, |I|+|J| \leqslant s} |D_z^I D_{\bar{z}}^J u| .$$

$C_{m,t}(\Delta)$ = {space of forms of type (m, t) with coefficients in $C^\infty(\Delta)$}

$W^{2,s}(\Delta)$ = {space of functions $L^2(\Delta)$ with derivatives of order $\leqslant s$ in $L^2(\Delta)$}

$$\|u\|_{W^{2,s}} = \sum_{|I|+|J| \leqslant s} \|D^I D^J u\|_{L^2(\Delta)}$$

$\mathcal{O}(\Delta)$ = {holomorphic functions on Δ}.

2. Boundary values of holomorphic functions.

DEF. 2.1. Let $u \in L^2(\partial\Delta)$ and f a $(0,1)$ form whose coefficients are in $L^2(\Delta)$. We say that

$$\bar{\partial}_b u = f \qquad \textit{in weak sense on } \partial\Delta$$

if

$$\int_{\Delta} f \wedge \varphi = \int_{\partial\Delta} u\varphi$$

for every $\varphi \in C^{\infty}_{n,n-1}(\bar{\Delta})$ s.t. $\bar{\partial}\varphi = 0$.

DEF. 2.2. Let $u \in L^2(\check{S})$. We say that u satisfies

$$\bar{\partial}_b u = 0 \qquad \textit{in weak sense on } \check{S}$$

if $\hat{u}(I) = 0$ for every $I \leqslant 0$.

LEMMA 2.3. Let $u \in C(\partial\Delta)$ such that $\bar{\partial}_b u = 0$ in weak sense on $\partial\Delta$. Then, for every $j = 1, \dots, n$ and $|z_1| \leqslant 1, \dots, |z_{j-1}| \leqslant 1, |z_{j+1}| \leqslant 1, \dots, |z_n| \leqslant 1$, we have

$$\hat{u}(z_1, \dots, z_{j-1}, -k, z_{j+1}, \dots, z_n) = 0 \qquad \forall k \geqslant 0$$

In particular $\bar{\partial}_b u = 0$ on $\check{S}$ (in weak sense).

PROOF. Consider

$$\varphi = \psi(z_2, \dots, z_n) z_1^k d\bar{z}_2 \wedge \dots \wedge d\bar{z}_n] \wedge dz$$

with $k \geqslant 0$ and $\psi \in C^{\infty}\big(|z_2| \leqslant 1, \dots, |z_n| \leqslant 1\big)$.

Then $\bar{\partial}\varphi = 0$ for every ψ and by hypothesis,

$$0 = \int_{\substack{|z_1|=1 \\ |z_2| \leqslant 1 \\ \vdots \\ |z_n| \leqslant 1}} u(z_1, \dots, z_n)\psi(z_2, \dots, z_n) z_1^k d\bar{z}_2 \wedge \dots d\bar{z}_n \wedge dz$$

But then $F(z_2, \dots, z_n) = \int\limits_{|z_1|=1} u(z_1, z_2, \dots, z_n) z_1^k \, dz_1 = 0$ a.e.: hence, as F is continuos, $F = 0$.

Iterating the argument, with convenient φ, we get the thesis. For continuous boundary values of holomorphic function the following proposition holds

PROP. 2.4. Let $u \in C(\check{S})$ then u has an analytic extension to Δ if and only if $\bar{\partial}_b u = 0$ in weak sense on $\check{S}$.

Furthermore the analytic extension Φ is defined and continuos on $\partial\Delta$ and satisfies $\bar{\partial}_b \Phi = 0$ in weak sense on $\partial\Delta$.

PROOF. *Nec.*: obvious.

Suff.: take

$$\Phi(w) = \frac{1}{(2\pi i)^n} \int\limits_{\check{S}} \frac{u(z)}{(z_1 - w_1) \dots (z_n - w_n)} dz_1 \wedge \dots \wedge dz_n$$

and we immediatly have

$$\Phi|_{\check{S}} = u$$

(it is sufficient to use Fejer's theorem, see for example [2]). For the second part of the proposition if is sufficient to observe that, in virtue of Lemma 2.3 and the previous part, if $|w_1| = 1, \dots, |w_s| = 1$, $|w_{s+1}| < 1, \dots, |w_n| < 1$ we have

$$\Phi(w) = \frac{1}{(2\pi i)^{n-s}} \int\limits_{|z_{s+1}| = \dots = |z_n| = 1} \frac{u(w_1, \dots, w_s, z_{s+1}, \dots, z_n)}{(z_{s+1} - w_{s+1}) \dots (z_n - w_n)} dz_{s+1} \wedge \dots \wedge dz_n .$$

So Φ is defined and continuous on $\partial\Delta$ and satisfies $\bar{\partial}_b \Phi = 0$: in fact on each component of $\partial\Delta$, $|w_1| \leq 1, \dots, |w_i| = 1$, $w_{i+1} \leq 1, \dots, |w_n| \leq 1$ it is possible to expand u in power serie of w_i.

The following basic lemma holds

LEMMA 2.5. Let $v \in C(\partial\Delta)$ s.t.

1) $v = 0$ on $\check{S}$

2) $\bar{\partial}_b v = 0$ in weak sense on $\partial\Delta$

Then $v = 0$ on $\partial\Delta$.

PROOF. Let $\bar{\partial}\beta$ any $(n-1, n-1)$ differential form in $\mathbf{C}^{n-1}$, where β has compact support on $|z_1| < 1, \dots, |z_{n-1}| < 1$, and α the $(n, n-1)$

differential form defined by,

$$\alpha = -\beta \wedge d\bar{z}_n \wedge dz_n + \bar{z}_n \bar{\partial}\beta \wedge dz_n$$

We have $\bar{\partial}\alpha = 0$, for every β, and hence, by hypothesis

$$\int_{\partial\Delta} v\alpha = 0 .$$

This means,

$$0 = \int_{\substack{|z_1|\leqslant 1 \\ \vdots \\ |z_{n-1}|\leqslant 1 \\ |z_n| = 1}} v\bar{z}_n \bar{\partial}\beta \wedge dz_n = \sum_j \int_{\substack{|z_1|\leqslant 1 \\ \vdots \\ |z_{n-1}|\leqslant 1 \\ |z_n| = 1}} \hat{v}(z_1, \ldots, z_{n-1}, j) z_n^j \bar{z}_n \bar{\partial}\beta \wedge dz_n$$

that is

$$\int_{\substack{|z_1|\leqslant 1 \\ \vdots \\ |z_{n-1}|\leqslant 1}} \hat{v}(z_1, \ldots, z_{n-1}, 0)\, \bar{\partial}\beta = 0$$

but then $\hat{v}(z_1, \ldots, z_{n-1}, 0)$ is a holomorphic function of $z_1, \ldots, z_{n-1}$ and continuous on $|z_1| \leqslant 1, \ldots, |z_{n-1}| \leqslant 1$; so, as by hypothesis it is identically 0 on $|z_1| = 1, \ldots, |z_{n-1}| = 1$, we have

$$\hat{v}(z_1, \ldots, z_{n-1}, 0) \equiv 0 \qquad |z_j| \leqslant 1 \quad j = 1, \ldots, n-1$$

Analogously, taking

$$\alpha = -2\bar{z}_n \beta \wedge d\bar{z}_n \wedge dz_n + \bar{z}_n^2 \bar{\partial}\beta \wedge dz_n$$

we get $\hat{v}(z_1, \ldots, z_{n-1}, 1) = 0$ and so on.

Finally we deduce $v = 0$ on $|z_n| = 1$, $|z_i| \leqslant 1$, $i = 1, \ldots, n-1$. With a suitable choice of α we have that v is 0 also on the other components of $\partial\Delta$ and hence the thesis.

An important consequence of the lemma is

PROP. 2.6. $u \in C(\partial\Delta)$ has an analytic extension to Δ if and only if $\bar{\partial}_b u = 0$ in weak sense on $\partial\Delta$.

PROOF.

Nec.: obvious.

Suff.: By Lemma 2.3 we have $\bar{\partial}_b u = 0$ in weak sense on $\check{S}$ so the holomorphic function

$$\Phi(w) = \frac{1}{(2\pi i)^n} \int_{\check{S}} \frac{u(z)}{(z_1 - w_1) \dots (z_n - w_n)} dz_1 \wedge \dots \wedge dz_n$$

is such that

$$\Phi(w)|_{\check{S}} = u|_{\check{S}} .$$

In virtue of Prop. 2.4 Φ is defined and continuos on $\partial\Delta$ and satisfies $\bar{\partial}_b \Phi = 0$ in weak sense on $\partial\Delta$: from this the continuous function $v = \Phi - u$ fulfils the hypothesis of Lemma 2.5 and we have

$$\Phi|_{\partial\Delta} = u|_{\partial\Delta}$$

3. Some estimates for the $\bar{\partial}$-problem in the polydisc.

From now on, Δ will be the unitary polydisc in $\mathbf{C}^2$ centered in $(0, 0)$. We have:

THEOREM 3.1. Let $f \in C^\infty_{0,1}(\bar{\Delta})$, $\bar{\partial}$-closed.
Then the $\bar{\partial}$-equation

$$\bar{\partial} u = f$$

has a solution u which is $C^\infty(\bar{\Delta})$ and satisfies

$$(a) \quad \int_{\check{S}} \frac{u(z)}{(z_1 - w_1)(z_2 - w_2)} dz_1 \wedge dz_2 = 0 \qquad (w_1, w_2) \in \Delta$$

$$(b) \quad |u|_{L^\infty,s} \leqslant C(\Delta, s) |f|_{L^\infty,s}$$

PROOF. Consider the $\bar{\partial}$-problem,

$$(1) \qquad \bar{\partial} v_\varepsilon = f_\varepsilon$$

where $\varepsilon \in (0,1)$ and $f_\varepsilon = f(\varepsilon z)$ (since f is $\bar\partial$-closed, f_ε is still $\bar\partial$-closed).

As f_ε is smooth in a neighborhood of $\bar\Delta$, (1) always admits solution v_ε which is smooth in a neighborhood of $\bar\Delta$ (see for example [4]).

In [5] has been showed that, if $v \in C^2(\bar\Delta)$, the following integral representation holds,

$$(2) \qquad v(w) = I(\bar\partial v)(w) + c\int_{\check S} \frac{v(z)}{(z_1 - w_1)(z_2 - w_2)}\, dz_1 \wedge dz_2 \qquad \forall (w_1, w_2) \in \Delta$$

where $I(\bar\partial v)(w)$ is a function bounded uniformly by $|\bar\partial v|_{L^\infty,0}$.

Hence, appling (2) to v_ε, we get that

$$u_\varepsilon(w) = v_\varepsilon(w) - c\int_{\check S} \frac{v_\varepsilon(z)}{(z_1 - w_1)(z_2 - w_2)}\, dz_1 \wedge dz_2$$

is still a $C^\infty(\bar\Delta)$ solution of (1), bounded uniformly by $|f|_{L^\infty,0}$ (because $|f_\varepsilon|_{L^\infty,s} \leqslant |f|_{L^\infty,s}$ for every s) which satisfies the fundamental relation (see also [5])

$$(3) \qquad \int_{\check S} \frac{u_\varepsilon(z)}{(z_1 - w_1)(z_2 - w_2)}\, dz_1 \wedge dz_2 = 0$$

for every $(w_1, w_2) \in \Delta$.

To finish the proof of the theorem, we need the following lemma,

Lemma 3.2. If $v(z) \in C^\infty(\bar\Delta)$ and satisfies

$$(4) \qquad \int_{\check S} \frac{v(z)}{(z_1 - w_1)(z_2 - w_2)}\, dz_1 \wedge dz_2 = 0$$

for every $(w_1, w_2) \in \Delta$, then

$$|v|_{L^\infty,s} \leqslant C(\Delta, s)|\bar\partial v|_{L^\infty,s}$$

for every integer $s \geqslant 0$.

Proof. For $s = 0$ see [5].

Let $s = 1$: if we apply (2) respectively to $\partial v/\partial w_1$ and $\partial v/\partial w_2$

we get

$$\frac{\partial v}{\partial w_1} = I\left(\bar\partial\left(\frac{\partial v}{\partial z_1}\right)\right)(w) + c\int_{\check S} \frac{\partial v}{\partial z_1}\frac{1}{(z_1 - w_1)(z_2 - w_2)}\, dz_1 \wedge dz_2 = J_1 + J_2$$

and

$$\frac{\partial v}{\partial w_2} = I\left(\bar\partial\left(\frac{\partial v}{\partial z_2}\right)\right)(w) + c\int_{\check S} \frac{\partial v}{\partial z_2}\frac{1}{(z_1 - w_1)(z_2 - w_2)}\, dz_1 \wedge dz_2 = J_3 + J_4$$

Hence to get the thesis it is sufficient to have the estimates

$$|J_2|_{L^\infty,0} \leqslant K_2 |\bar\partial v|_{L^\infty,1}\,, \qquad |J_4|_{L^\infty,0} \leqslant K_4 |\bar\partial v|_{L^\infty,1}\,. \tag{5}$$

We shall do it for J_2 (the proof for J_4 is completely analogous).

By (4) we have that

$$J_2 = c\int_{\check S} \frac{\partial}{\partial z_1}\left[\frac{v(z)}{(z_1 - w_1)}\right]\frac{1}{(z_2 - w_2)}\, dz_1 \wedge dz_2 =$$

$$= - c\int_{\check S} \frac{\partial v}{\partial \bar z_1}\frac{1}{(z_1 - w_1)(z_2 - w_2)}\, d\bar z_1 \wedge dz_2 = c\int_{\check S} \frac{\partial v}{\partial \bar z_1}\frac{\bar z_1^2}{(z_1 - w_1)(z_2 - w_2)}\, dz_1 \wedge dz_2$$

(we have used that, on $|z_1| = 1$, $-\bar z_1^2 dz_1 = d\bar z_1$); now, (2) applied to $-\bar z_1^2(\partial v/\partial \bar z_1)$ gives the first estimate in (5). To get the thesis for every s, it is sufficient to iterate the argument.

Let us come back to the proof of theorem 3.1.

Lemma 3.2 is applicable to u_ε: then we have,

$$|u_\varepsilon|_{L^\infty,s} \leqslant C(\Delta, s)|f|_{L^\infty,s}\,.$$

Setting $\varepsilon = 1 - 1/k$, we have obtained a sequence $\{u_k\}$ such that

$$|u_k|_{W^{2,s}} \leqslant C'(\Delta, s) \tag{6}$$

where C' is independent of k.

By Banach-Saks theorem (see [1]), we can extract a subsequence of $\{u_k\}$, again denoted with $\{u_k\}$, s.t.

$$U_n = \frac{1}{n}\sum_0^{n-1}{}_k u_k \xrightarrow{W^{2,2}} u \qquad n \to +\infty .$$

So u is a solution of $\bar{\partial}u = f$ and satisfies, in virtue of (6), the estimate (b).

In particular, as $u(z)$ has all the derivatives bounded it is $C^\infty(\bar{\Delta})$.

Let $V(z)$ be a $C^2(\bar{\Delta})$ function, then an elementary argument shows that

$$\|V\|_{L^2(\check{S})} \leqslant K\|V\|_{W^{2,2}(\Delta)}$$

then we have

$$U_n \xrightarrow{L^2(\check{S})} u .$$

But this means, by (4),

$$\int_{\check{S}} \frac{u(z)}{(z_1 - w_1)(z_2 - w_2)}\, dz_1 \wedge dz_2 = 0$$

for every $(w_1, w_2) \in \Delta$. Q.D.E.

REMARK. The argument used in the proof gives the same result also in the case of f bounded with its derivatives.

4. Cauchy problem for $\bar{\partial}$-operator.

Let us consider the Cauchy problem for $\bar{\partial}$ in Δ, that is to find a function $\alpha \in C^s(\bar{\Delta})$ such that

$$(A) \qquad \begin{cases} \bar{\partial}\alpha = f \\ \alpha|_{\partial\Delta} = g \end{cases}$$

where $f \in C^\infty_{0,1}(\bar{\Delta})$, $\bar{\partial}f = 0$, and $g \in C^s(\partial\Delta)$ satisfies the necessary compatibility condition $\bar{\partial}_b g = f$ in weak sense on $\partial\Delta$.

Instead of problem (A), we want to study the following problem (B), of finding a function $\beta \in C^s(\bar{\Delta})$, such that

$$(B) \qquad \begin{cases} \bar{\partial}\beta = f \\ \int\limits_{\check{S}} \frac{\beta(z)}{(z_1 - w_1)(z_2 - w_2)} dz_1 \wedge dz_2 = h(w) \,, \qquad h(w) \in C(\Delta) \end{cases}$$

As in the case of strictly pseudoconvex domains, where Henkin's kernel takes the place of Cauchy kernel (see [6]), we have equivalence between problem (A) and (B). In fact the following lemma holds,

LEMMA 4.1. Solvability of problem (A) is equivalent to solvability of problem (B).

PROOF. $(A) \Rightarrow (B)$.

It is enough to solve the Cauchy problem

$$\begin{cases} \bar{\partial}\beta = f \\ \beta|_{\partial\Delta} = h + u|_{\partial\Delta} \end{cases}$$

where u is the solution of the $\bar{\partial}$-problem $\bar{\partial}u = f$ given by theorem 3.1: β is then solution of (B).

$(B) \Rightarrow (A)$.

Consider the problem (B) given by

$$\begin{cases} \bar{\partial}\alpha = f \\ \int\limits_{\check{S}} \frac{\alpha(z)}{(z_1 - w_1)(z_2 - w_2)} dz_1 \wedge dz_2 = \int\limits_{\check{S}} \frac{g(z)}{(z_1 - w_1)(z_2 - w_2)} dz_1 \wedge dz_2 \end{cases}$$

Observe that

$$\int\limits_{\check{S}} \frac{g(z)}{(z_1 - w_1)(z_2 - w_2)} dz_1 \wedge dz_2$$

is continuous with its derivatives in $\bar{\Delta}$ because it is equal to

$$\int\limits_{\check{S}} \frac{g - u}{(z_1 - w_1)(z_2 - w_2)} dz_1 \wedge dz_2$$

and g-u is the boundary value of a holomorphic function (see prop. 2.6).

Since α satisfies, for every $(w_1, w_2) \in \Delta$, the identity

$$\int_{\check{S}} \frac{\alpha(z) - g(z)}{(z_1 - w_1)(z_2 - w_2)}\, dz_1 \wedge dz_2 = 0$$

and furthermore

$$\bar{\partial}_b(\alpha - g) = 0$$

we immediatly have

$$\alpha|_{\check{S}} = g|_{\check{S}}$$

and this implies, by lemma 2.5,

$$\alpha|_{\partial\Delta} = g$$

and this finishes the proof.

For problem (B) the following theorem of existence and unicity holds,

THEOREM 4.2. Problem (B) always admits a unique solution β which is $C^s(\bar{\Delta})$ and satisfies the estimate

$$|\beta|_{L^\infty,k} \leq C(\Delta, k)[|f|_{L^\infty,k} + |h|_{L^\infty,k}] \tag{7}$$

for every $k \leq s$ (where $C(\Delta, k)$ is exactly given by thm 3.1 part (b)).

PROOF. Let u the solution of $\bar{\partial} u = f$ given by theorem 3.1.

Then if we take,

$$\beta = u + h$$

by part (a) of theorem 3.1 we have

$$\int_{\check{S}} \frac{\beta(z)}{(z_1 - w_1)(z_2 - w_2)}\, dz_1 \wedge dz_2 = h(w)$$

and by part (b) the estimate (7).

From theorem 4.2, in virtue of lemma 4.1, we get

THEOREM 4.3. The Cauchy problem for $\bar\partial$ in the polydisc $\Delta \subset \mathbf{C}^2$

$$\begin{cases} \bar\partial\alpha &= f \\ \alpha|_{\partial\Delta} &= g \end{cases}$$

where $f \in C^\infty_{0,1}(\bar\Delta)$, $\bar\partial f = 0$ and $g \in C^s(\partial\Delta)$ s.t. $\bar\partial_b g = f$ in weak sense on $\partial\Delta$, has unique solution $\alpha \in C^s(\bar\Delta)$ which satisfies the estimate,

$$|\alpha|_{L^\infty,k} \leqslant C(\Delta, k)[2|f|_{L^\infty,k} + |g|_{L^\infty(\partial\Delta),k}]$$

for every $k \leqslant s$.

REMARK. The estimate follows by maximum's principle, observing that

$$\int\limits_{\check S} \frac{g(z)}{(z_1 - w_1)(z_2 - w_2)} dz_1 \wedge dz_2 = \int\limits_{\check S} \frac{g(z) - u(z)}{(z_1 - w_1)(z_2 - w_2)} dz_1 \wedge dz_2$$

and $g - u$ is the boundary value of a holomorphic function.

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