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HERBERT DIETZSCH

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ON THE COMPUTATION OF THE ORDER OF JANKO'S FIRST SIMPLE GROUP $\mathfrak{J}_1$

HERBERT DIETZSCH *)

Janko's computation of the order of $\mathfrak{J}_1$ contains a mistake as the generalized character φ'_4 of [1; p. 156] does not vanish on non special classes of $N(U)$ in case (i) of [1; Lemma 3.1]. The purpose of this note is to correct this mistake.

We use terminology and notation of [1].

Case (i) **of Lemma 3.1.**

Here $N(U)\subset C(t)$. Thus, $T=\langle t, t_1\rangle$ is a S_2-subgroup of $N(U)$, where t_1 is an involution of F which acts invertingly on U.

We have $U\langle t_1\rangle \times \langle t\rangle = N(U)$.

The group $N(U)/\langle t\rangle$ has two linear characters: ξ'_0 (the principal character), β', and two irreducible characters ξ'_1, ξ'_2 of degree 2.

Let ξ' be the nontrivial linear character of $N(U)/(N(U)\cap F)$. Put $U=\langle \mu\rangle$. We get the following character-table of $N(U)$.

Let the set of « special classes » of $N(U)$ in the sense of Suzuki [2] consist of the conjugate classes of μ, μ^2, $t\mu$, $t\mu^2$ in $N(U)$..

The module (over the ring of integers) of the generalized characters of $N(U)$ vanishing on non special classes of $N(U)$ has the following basis:

$$\varphi'_1=\xi'_2-\xi'_1$$
$$\varphi'_2=\xi'\cdot\xi'_2-\xi'\cdot\xi'_1$$
$$\varphi'_3=\xi'_0+\beta'-\xi'_1$$
$$\varphi'_4=\xi'+\xi'\cdot\beta'-\xi'\cdot\xi'_1\,.$$

*) Indirizzo dell'A.: Mathematisches Institut der Universität, 65 Mainz, Germania Occ.

	ξ'_0	β'	ξ'_1	ξ'_2	ξ'	$\xi'\cdot\beta'$	$\xi'\cdot\xi'_1$	$\xi'\cdot\xi'_2$
1	1	1	2	2	1	1	2	2
μ	1	1	$-\frac{1+\sqrt{5}}{2}$	$-\frac{1-\sqrt{5}}{2}$	1	1	$-\frac{1+\sqrt{5}}{2}$	$-\frac{1-\sqrt{5}}{2}$
μ^2	1	1	$-\frac{1-\sqrt{5}}{2}$	$-\frac{1+\sqrt{5}}{2}$	1	1	$-\frac{1-\sqrt{5}}{2}$	$-\frac{1+\sqrt{5}}{2}$
t_1	1	-1	0	0	1	-1	0	0
t	1	1	2	2	-1	-1	-2	-2
$t\mu$	1	1	$-\frac{1+\sqrt{5}}{2}$	$-\frac{1-\sqrt{5}}{2}$	-1	-1	$\frac{1+\sqrt{5}}{2}$	$\frac{1-\sqrt{5}}{2}$
$t\mu^2$	1	1	$-\frac{1-\sqrt{5}}{2}$	$-\frac{1+\sqrt{5}}{2}$	-1	-1	$\frac{1-\sqrt{5}}{2}$	$\frac{1+\sqrt{5}}{2}$
tt_1	1	-1	0	0	-1	1	0	0

We have the following decomposition of the induced characters $\varphi_i'^*$:

$$\varphi_1'^* = \bar{\varepsilon}_1\bar{x}_1 + \bar{\varepsilon}_2\bar{x}_2$$

$$\varphi_2'^* = \bar{\eta}_1\bar{y}_1 + \bar{\eta}_2\bar{y}_2$$

$$\varphi_3'^* = 1_G + \bar{\varepsilon}_1\bar{x}_1 + \bar{\zeta}_2\bar{z}_2$$

$$\varphi_4'^* = \bar{\eta}_1\bar{y}_1 + \bar{\theta}_2\bar{v}_2 + \bar{\theta}_3\bar{v}_3$$

where $\bar{x}_i$, $\bar{y}_i$, $\bar{z}_2$, $\bar{v}_i$ are nontrivial, different irreducible characters of G and $\bar{\varepsilon}_i$, $\bar{\eta}_i$, $\bar{\zeta}_2$, $\bar{\theta}_i$ are all ± 1.

If χ is an irreducible character of G which appears in $\varphi_i'^*$ with the multiplicity $n_i (1 \leq i \leq 4)$, then

$$\chi(\sigma) = n_1 \xi'_2(\sigma) + (n_2 - n_4) \xi'(\sigma) \cdot \xi'_2(\sigma) + n_3 - n_4 \xi'(\sigma) \cdot \xi'_1(\sigma)$$

where σ is an arbitrary element of the set of « special classes » of $N(U)$.

Hence we get:

$$\bar{x}_1(t\mu) = \frac{1}{2} \bar{\varepsilon}_1 (1 + \sqrt{5}) \text{ and } \bar{z}_2(t\mu) = \bar{\zeta}_2$$

$$\bar{x}_1(t\mu^2) = \frac{1}{2} \bar{\varepsilon}_1 (1 - \sqrt{5}) \text{ and } \bar{z}_2(t\mu^2) = \bar{\zeta}_2 .$$

In Case (1) of [1; p. 155] we get $\bar{x}_1 \in \{y_1, y_2, y_3, y_4\}$ and $\bar{z}_2 \in \{x_3, v_1, v_2\}$. Since $0 = \varphi_3'^*(t)$, we get $\bar{x}_1 \in \{y_1, y_2, y_3, y_4\}$ and $\bar{z}_2 \in \{v_1, v_2\}$.

In Case (2) of [1; p. 156] we get $\bar{x}_1 \in \{x_1, x_2, y_3, y_4\}$ and $\bar{z}_2 \in \{z_4, v_1, v_2\}$. Since $0 = \varphi_3'^*(t)$, we get $\bar{x}_1 \in \{y_3, y_4\}$ and $\bar{z}_2 \in \{v_1, v_2\}$.

Therefore, in both cases we get: $\bar{x}_1(t) = \pm 3$ and $\bar{z}_2(t) = \pm 4$. We shall apply the Suzuki order formula of [2] for the generalized characters $\varphi'_3 = \xi'_0 + \beta' - \xi'_1$, $\varphi_3'^* = 1_G + \bar{\varepsilon}_1 \bar{x}_1 + \bar{\zeta}_2 \bar{z}_2$ and the subgroup $N(U)$ of G.

Put $f = \bar{\varepsilon}_1 \bar{x}_1(1)$, thus $f + 1 = -\bar{\zeta}_2 \bar{z}_2(1)$ as $0 = \varphi_3'^*(1)$.

Denoting $|G|$ by g we obtain

$$\frac{1}{g} \left\{ \frac{g^2}{120^2} \left(1 + \frac{3^2}{f} - \frac{4^2}{f+1} \right) \right\} = \frac{1}{20} \left\{ \frac{11^2}{1} + \frac{9^2}{1} - \frac{2^2}{2} \right\} = 10 .$$

Hence, $g \cdot (f-3)^2 = 2^7 \cdot 3^2 \cdot 5^3 \cdot f \cdot (f+1)$.

We know that both f and $f+1$ divide g and that f and $f+1$ are coprime. This implies that $f-3$ is a divisor of $2^3 \cdot 3 \cdot 5$.

On the other hand a S_2-subgroup of G has order 8 and U is a S_5-subgroup of G. This implies $2^2 \cdot 5$ divides $f-3$.

Therefore we have the following possibilities:

$$f-3=\pm 20, \ \pm 40, \ \pm 60, \ \pm 120.$$

It is easy to see that all eight possibilities lead to a contradiction. For example

$$f-3=20 \text{ implies } g\cdot 20^2=23\cdot 24\cdot 2^7\cdot 3^2\cdot 5^3$$

which is impossible as 7 divides g.

Similarly $f-3=-60$ implies $g=2^6\cdot 3\cdot 5\cdot 7\cdot 19$ which is against the order of a S_2-subgroup of G.

Thus, we have ruled out Case (i) of Lemma 3.1.

REFERENCES

[1] JANKO, Z.: *A new finite simple group with abelian Sylow-2-subgroups and its characterization,* Journal of Algebra 3 (2) (1966), 147-186.

[2] SUZUKI, M.: *Applications of group characters,* Proc. Symp. Pure Math. 1 (1959), 88-89.

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