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THE TRANSYLVANIAN PROBLEM OF RENEWABLE RESOURCES (*) (1)

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Abstract. — *This paper deals with a typical problem of renewable resources described in terms of an optimal control model. The differences in the analysis of the three cases of concave, linear, and convex utility functions are pointed out and optimal solutions are obtained. It is also demonstrated that the size of the discount rate can determine the structure of the optimal policy.*

Keywords: maximum principle; phase-plane analysis; bang-bang control; economics of human resources; vampire myth.

Résumé. — *Cet article traite d'un problème typique de ressources renouvelables, modélisé sous forme de commande optimale. On met en relief les différences d'analyse dans les trois cas d'une fonction d'utilité concave, linéaire, ou convexe, pour lesquels on obtient les solutions optimales. On démontre aussi que la valeur du taux d'actualisation peut déterminer la structure de la politique optimale.*

But first on earth, as Vampyre sent
Thy corpse shall from its tomb be rent
Then ghastly haunt thy native place
And suck the blood of all thy race
Lord BYRON, The Giaor

1. INTRODUCTION

There is no doubt that the vampire myth has a central place in our consciousness which makes its appeal and its attraction awesome. An exact definition of the vampire, called also by the Transylvanians *strigoiu*, may be found in Chamber's Encyclopedia, the work so highly regarded by Victorian writers (*see* also Haining [3] for a comprehensive analysis). Vampirism research (Rohr [10], Summers [12, 13]), works of scholarship on the vampire legend (MacNally and Florescu [7, 8], Wolf [14]) and studies of the vampire in film (*see* the excellent filmography in Michel [9]) are only a few remarkable facets of the interest in this subject.

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Our aim is to solve this problem without getting in touch with it, thus avoiding the fate which uncountably many vampirologists have undergone.

A mathematical model is presented, which is both sufficiently easy to be solved completely and general enough in order to demonstrate several methods which are of importance for solving many economic control problems (e. g. problems of renewable resources). In particular we discuss the assumptions of linear, convex and concave utility (objective) functions and emphasize the differences in the analysis of these three cases, as well as the common properties of the resulting optimal solutions.

2. OPTIMAL BLOODSUCKING STRATEGIES FOR DYNAMIC CONTINUOUS VAMPIRES

The isolated Transylvanian community Mamadracului may be found on any map giving the exact location of Dracula's Castle. In this community, let $v(t)$ denote the expected number of vampires at time t. The expected resource of human beings at time t will be denoted by $h(t)$. Every person who, when alive, has had his blood sucked by a vampire, will, after his immediate death, deal with other persons in like manner. Without loss of generality the following dynamic system equations may be derived:

$$\dot{v} = -av + cv; \qquad \dot{h} = nh - cv, \tag{1}$$

where n denotes the growth rate of the human population, a denotes the failure rate of vampires due to contact with sunlight, crucifixes, garlic, and vampire hunters. The bloodsucking rate per vampire at time t is given by $c(t)$. Blood is measured in units defined by the average capacity of the human body.

The instantaneous utility of blood is $U(c)$ with marginal utility $U' > 0$. A few words are in order on the subject of concave, linear, and convex utility functions. We shall consider the three cases:

(*a*) the asymptotically satiated vampire: $U'' < 0$;

(*b*) the blood maximizing vampire: $U(c) = \bar{U}c$;

(*c*) the unsatiable vampire: $U'' > 0$;

in which a vampire consuming two human beings rather than one derives a utility, which is smaller than, exactly equal, and larger than twice as large, respectively. In comparable economic control models where the decision makers are human beings (*cf.* Arrow [1], Intriligator [5]) usually a concave utility function is considered. In what follows we shall explore the implications and consequences of the assumptions (*a*), (*b*), and (*c*) on the structure of the optimal policy. For reasons of convenience we will assume $U(0)=0$ and restrict c to the bounded interval $[0, \bar{c}]$ in cases (b) and (c). In case (a) this is not necessary since a concave utility function implies that extremely high values of consumption are suboptimal.

Future utilities are discounted by r being the rate of time preference. Thus the vampire's objective is to maximize the present value of the utility stream:

$$\int_0^{\infty} e^{-rt}\, U(c(t))\, dt, \tag{2}$$

subject to (1), and the nonnegativity condition $h \geqq 0$. This optimal control problem with two state variables (v, h) and one control instrument c can be reduced to the following problem:

$$\text{maximize (2)} \quad \text{s. t.} \qquad \dot{x}=(n+a-c)\,x-c, \qquad x \geqq 0, \tag{3}$$

where x denotes the *humans/vampires ratio* $x=h/v$. Thus we are facing a typical consumption-resource trade off. The vampire society derives utility from consumption of blood but in sucking the blood of a human being and in turning him to a vampire the resource of human beings is reduced whereas the number of vampires is increased. Both of these effects diminish the resource of humans per vampire curtailing future possibilities of consumption. Applying standard control theory (Pontrjagin's Maximum Principle in current value formulation) we obtain necessary optimality conditions which in turn are:

$$H=U(c)+p\,[(n+a-c)\,x-c], \qquad \text{and} \qquad \operatorname*{Max}_{c \geqq 0} H. \tag{4}$$

The shadow price p satisfies the adjoint equation:

$$\dot{p}=(r+c-n-a)\,p. \tag{5}$$

Note that $x \geqq 0$ is equivalent with $\lim_{t \to \infty} x(t) \geqq 0$ as can be seen from the state equation (3). The transversality conditions are:

$$\lim_{t \to \infty} e^{-rt} p(t) \geqq 0, \qquad \lim_{t \to \infty} e^{-rt} p(t)\, x(t)=0. \tag{6}$$

This leads to sufficient conditions for the optimality of the solution (x, c) if the derived Hamiltonian $\max H$ is concave in x.

We now address ourselves to the different implications of the maximum condition (4) in the three cases of concave, linear and convex utility functions (and Hamiltonians, respectively):

TABLE I

(a) $U''<0$	(b) $U(c)=\bar{U}c$		(c) $U''>0$
$H'=0 \Rightarrow$ $U'(c)=p(1+x)$ provided that $c>0$ i. e. $U'(0)>p(1+x)$	$c(t)=\begin{cases}0\\ \bar{c}\end{cases}$ if $\bar{U}\{\lessgtr\}p(1+x)$ and		
	$c(t)$ undefined ε $[0, \bar{c}]$		$c(t)=0$ or $\bar{c}$
	if $\bar{U}=p(1+x)$		

The optimal policies in cases (b) and (c) seem to be almost the same. However, it will be seen later that the differences between the two are important for the existence of an optimal solution. In the concave case marginal utility always equals the value $p(1+x)$ of the $1+x$ units of marginal resource depletion. In the other cases the resource is depleted with zero and maximum effort if the marginal utility is smaller and larger than this value, respectively.

It is now convenient to analyse the three cases separately.

3. THE ASYMPTOTICALLY SATIATED VAMPIRE

The standard way that concave control models (i. e. H is concave in c) are treated is using the method of phase plane analysis. This could be done by analyzing the canonical system (3,5) directly (*cf.* Arrow [1]) or by obtaining a set of differential equations for (x, c) (*cf.* Intriligator [5]). Since we are mainly interested in the behavior of c we choose the latter approach. Thus we differentiate the $H_c=0$ condition in table I with respect to time to obtain:

$$\dot{p}(1+x)+\dot{x}p=U''\dot{c} \quad \Rightarrow \quad \dot{c}=\frac{U'(c)}{U''(c)}\left[r-\frac{n+a}{1+x}\right], \tag{7}$$

which may be interpreted in terms of the elasticity of marginal utility $\sigma\equiv U''c/U'$. In order to determine the behavior of the optimal solution in the (x, c)-diagram we investigate the (curves $\dot{x}=0$ and $\dot{c}=0$ called) isoclines:

$$\dot{x}=0 \quad \Leftrightarrow \quad c=(n+a)x/(1+x);$$

$$\dot{c}=0 \quad \Leftrightarrow \quad c=0 \qquad \text{or} \qquad x=(n+a-r)/r, \tag{8}$$

where we assume $\lim_{c \to 0} U'(c)/U''(c) = 0$ which is satisfied for any "good" utility function such as $U(c) = c^{\alpha}$; $0 < \alpha < 1$ or $U(c) = \ln(c)$. Thus the rest points of the system (3,7) are:

TABLE II

Case	Rest Points
(A) $n+a>r$	$x^{\infty} = \frac{n+a-r}{r}$, $c^{\infty} = n+a-r$ and $x=c=0$
(B) $n+a<r$	$x=c=0$

Computing the elements of the Jacobian matrix:

$$\frac{\partial \dot{x}}{\partial x} = n+a-c, \qquad \frac{\partial \dot{x}}{\partial c} = -(1+x),$$

$$\frac{\partial \dot{c}}{\partial x} = \frac{U'(n+a)}{U''(1+x)^2}, \qquad \frac{\partial \dot{c}}{\partial c} = \frac{dU'/U''}{dc}\left[r - \frac{n+a}{1+x}\right].$$

We obtain that:

$$\det(\mathrm{J}) = \frac{\partial \dot{x}}{\partial x}\frac{\partial \dot{c}}{\partial c} - \frac{\partial \dot{x}}{\partial c}\frac{\partial \dot{c}}{\partial x}$$

equals $U'(n+a)/[U''(1+x)]$ for $c = c^{\infty}$, $x = x^{\infty}$ and equals:

$$(n+a)(r-n-a)\frac{dU'/U''}{dc} \qquad \text{for} \quad c = x = 0.$$

Hence, in the case (A) of a *small discount rate* the point (x^{∞}, c^{∞}) is a saddle point and (0,0) is an unstable node. Figure 1 gives a sketch of the phase plane in this case where the optimal solution is represented by the stable path converging towards the equilibrium. In fact this is the only solution of (3,7) which satisfies the transversality condition (6). In the case (B) of a *large discount rate* (future utilities are thus rather unimportant) there exist no positive equilibrium values and (0,0) is a saddle point. The phase plane diagram in this case is depicted in figure 2.

The results derived so far deserve formal expression as:

PROPOSITION 1: *The optimal solution is given by a feedback rule $c = c(x)$ with $c'(x) > 0$ and $c(0) = 0$:*

(A) *If the discount rate is small:* $r<n+a$ *then positive equilibrium values* x^{∞}, c^{∞} (cf. table II) *exist. Furthermore,* $\lim_{t\to\infty}(x(t),\ c(t))=(x^{\infty},\ c^{\infty})$ *and* $x(t)$ $\{\lesseqgtr\}\,x^{\infty}$ *implies* $c(t)\{\lesseqgtr\}\,c^{\infty}$ *and* $\dot{x}(t),\ \dot{c}(t)\{\gtreqless\}\,0$;

(B) *If the discount rate is large:* $r>n+a$ *then* $\lim_{t\to\infty}x(t)=\lim_{t\to\infty}c(t)=0$ *and* $\dot{x}(t),\ \dot{c}(t)<0$.

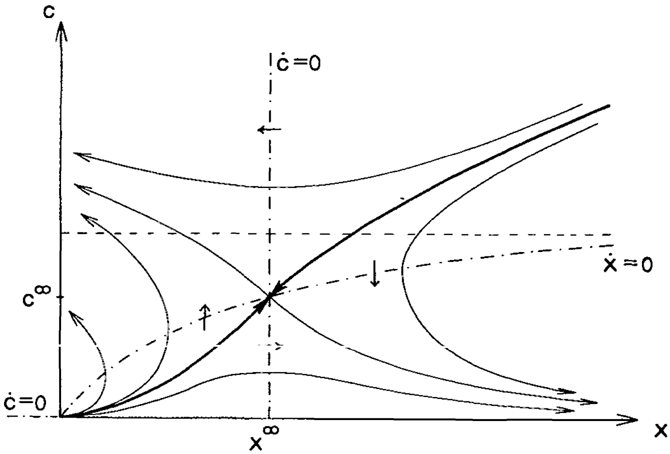

Figure 1. — **The phase plane in the case of** $n+a>r$

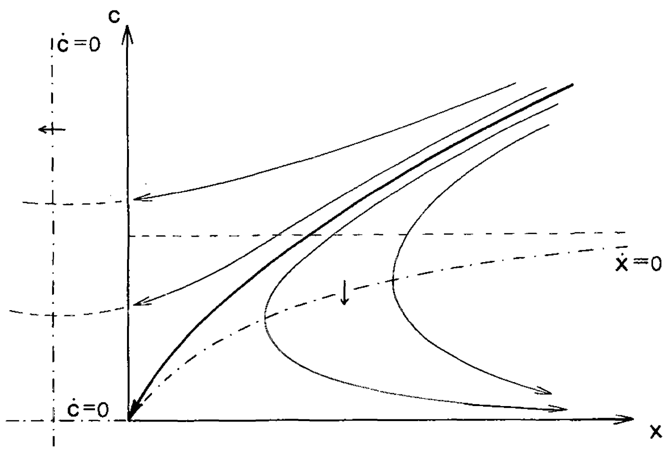

Figure 2. — **The phase plane in the case of** $n+a<r$

This result seems to be quite reasonable: if the future utilities are important (r small), then in the limit the resource is not used up completely ($x(t)\to x^{\infty}>0$). Otherwise (r large) the present consumption (extraction) is high and the resource approaches zero level.

In the equilibrium (x^∞, c^∞) the humans/vampires ratio x^∞ is constant. Then, by (1), it is easy to see that $\dot{v}/v = \dot{h}/h = n - r$. Thus both populations vampires and humans will grow (diminish) at the constant rate $n-r$ if the natural growth rate of the human population n exceeds (falls below) the discount rate r. The same result holds asymptotically for arbitrary values of x_0 as time tends to infinity.

4. THE BANG-BANG VAMPIRES

The linearity and convexity, of the Hamiltonians in the cases (b) and (c) of the blood maximizing and unsatiable vampire, respectively, implies that the optimal consumption is on the boundary of the control interval (*see* table I). The only exception is the singular solution in the linear case. Before we can determine the optimal solution by solving the problem of synthesis (i. e. patching together bang-bang and singular solution pieces) some preliminary results are needed.

Let us first analyze the case (A) of a small discount rate: $r < n + a$.

LEMMA : *Along an optimal solution path* $(x(t), c(t))$ *a switch from:*

$$c = 0 \quad \text{to} \quad c = \bar{c}; \qquad c = \bar{c} \quad \text{to} \quad c = 0,$$

at time t *is possible only if* $x(t) \left\{ \begin{matrix} \leq \\ \geq \end{matrix} \right\} x^\infty$ *where the "equilibrium" level* x^∞ *is defined as in table II.*

Proof : Switching from 0 to $\bar{c}$ at time τ implies, by table I, that $(1+x)$ $p \{ \gtrless \} \bar{U}$ for $t \{ \lesseqgtr \} \tau$ at least in the neighbourhood of the point τ. This, in turn, implies that $d/dt\,(1+x)\,p \leqq 0$ at time $t = \tau$. Using (3,5) we obtain:

$$\frac{d}{dt}(1+x)\,p = \dot{p}\,(1+x) + p\dot{x} = p\,[r\,(1+x) - (n+a)] = pr\,[x(t) - x^\infty], \tag{8}$$

which completes the proof, since p, being the shadow price of a stock of a resource, is always positive. □

Now it is easy to show, that

PROPOSITION 2: *Consider the* linear *case (of a blood maximizing vampire). If the vampire's discount rate is small:* $r < n + a$ [*case* (A)] *then there exist positive singular levels* x^∞ *and* c^∞ *which are identical with the equilibrium values* (x^∞, c^∞) *in table I for the concave case. It is optimal to approach the equilibrium* *level x^∞ as fast as possible by choosing either $c=0$ or $c=\bar{c}$ and then to maintain this level by consuming $c=c^\infty$. In particular, if:*

α) $\underline{x_0 < x^\infty}$ *then* $c(t)=\begin{cases} 0 \\ c^\infty \end{cases}$ *for* $\begin{cases} 0 \leqq \bar{t} < \\ t \geqq \bar{t} \end{cases}$. *The singular level x^∞ is reached at the time:*

$$\bar{t}=\frac{1}{n+a} \ln (x^\infty/x_0), \tag{9}$$

β) $\underline{x_0 = x^\infty}$ *then* $c(t) \equiv c^\infty$ *and* $x(t) \equiv x^\infty$;

γ) $\underline{x_0 > x^\infty}$ *then* $c(t)=\begin{cases} \bar{c} \\ c^\infty \end{cases}$ *for* $\begin{cases} 0 \leqq t < \bar{\bar{t}} \\ t \geqq \bar{\bar{t}} \end{cases}$;

where:

$$\bar{\bar{t}}=\frac{1}{\bar{c}-n-a} \ln \left[\left(x_0 + \frac{\bar{c}}{\bar{c}-n-a} \right) \Big/ \left(x^\infty + \frac{\bar{c}}{\bar{c}-n-a} \right) \right]. \tag{10}$$

Proof : A singular solution piece is characterized by the equations $H_c=0$ and $d/dt\, H_c=0$. From $0=\dot{p}(1+x)+p\dot{x}=[r(1+x)-n+a]\,p$, using (3) and (5), we easily obtain that $x=x^\infty$ and $c=c^\infty$.

Now let us turn to *case* α). Use $c(t)=0$ and $x(t)=x_0\, e^{(n+a)t}$ as long as $x(t)<x^\infty$ and show that this policy satisfies the optimality conditions (3-6). Obviously the level $x=x^\infty$ is reached at time $\bar{t}$ given by equation (9). For $t \geqq \bar{t}$ we have:

$$x(t)=x^\infty, \qquad c(t)=c^\infty, \qquad p(t)=p^\infty=\bar{U}/(1+x^\infty)=\bar{U}\,r/(n+a).$$

Now, for $t \leqq \bar{t}$, we have to solve (5) backwards in time using $p(\bar{t})=p^\infty$. This yields $p(t)=p^\infty e^{-(n+a-r)(t-\bar{t})}$. Thus it remains to show that $p(1+x)>\bar{U}$ for $t<\bar{t}$ in order for (3-6) to hold. The easiest way to do that, is to combine:

$$p(\bar{t})\,(1+x(\bar{t}))=p^\infty(1+x^\infty)=\bar{U} \qquad \text{and} \qquad \frac{d}{dt} p\,(1+x)=pr\,[x(t)-x^\infty]<0,$$

making use of (8). This implies $p(1+x)>p^\infty(1+x^\infty)=\bar{U}$ for $t<\bar{t}$ which completes the proof of part α). The same procedure can be used in order to prove parts β) and γ). □

Now we will show that the differences in the maximum conditions in table I for the cases (*b*) and (*c*) are, though small, of crucial importance:

PROPOSITION 3: *Consider the* convex *case* (*of an unsatiable vampire*). *If the vampire's discount rate is small: $r<n+a$, then there exists* no optimal policy. *However, there exists a series of* (*suboptimal*) *trajectories $\{c_n\}$ converging to the optimal solution of the linear model* (*proposition* 2) *such that:*

$$\lim_{n\to\infty}\int_0^\infty e^{-rt}U(c_n)\,dt=\operatorname{supr}_c\int_0^\infty e^{-rt}U(c)\,dt.$$

Proof : In the lemma above, we have already shown that a switch from 0 to $\bar{c}$ (from $\bar{c}$ to 0) can only be optimal if $x\leqq x^\infty$ (if $x\geqq x^\infty$), which implies that there is at most one switching point. Thus any solution with $x(\tau)\leqq x^\infty$ and $c(\tau)=\bar{c}$ for some $\tau\geqq 0$ will violate the constraint $x(t)\geqq 0$, since switching to $c=0$ is no longer possible since $x(t)\leqq x^\infty$ for $t\geqq\tau$. On the other hand any solution with the property "$c(t)=0$ if $x(t)\leqq x^\infty$" is non-optimal: In this case changing $c(t)=0$ to $c>0$ in some nonvanishing interval (t_1, t_2) will still give a feasible solution yielding a higher value of the total utility (2). In a similar way it is easy to show that any of the four possible policies ($c=0$, $c=\bar{c}$, switch $0\to\bar{c}$, switch $\bar{c}\to 0$) is either non-optimal or infeasible, which implies that there is no optimal solution.

The reason for this non-existence of an optimal policy is the fact that $c=c^\infty\in(0,\bar{c})$ is not possible along an optimal path if the Hamiltonian is convex. Now consider the linear utility function $\tilde{U}(c)=\bar{U}c$ where $\bar{U}$ is *defined as* $\bar{U}=U(\bar{c})$. Then obviously $U(c)<\tilde{U}(c)$ for $c\in(0,\bar{c})$ and $U(c)=\tilde{U}(c)$ for $c=0$, $\bar{c}$. Thus the optimal trajectory c^* of this linear model (given by proposition 2) establishes an upper bound for the value of the utility functional (2) for any consumption policy c of the convex vampire. Now define the policy $\{c_n\}$ by "*first approach the resource level x^∞ as fast as possible by either choosing $c=0$ (if $x<x^\infty$) or $c=\bar{c}$ (if $x>x^\infty$). Then proceed by following the rule: switch from* 0 *to $\bar{c}$ (from $\bar{c}$ to* 0) *if the resource x reaches the level $x^\infty+1/n$ ($x^\infty-1/n$)*". This leads to a cyclical "saw-toothed" bang-bang policy which is non-optimal. However, we have $U(c_n)\equiv\tilde{U}(c_n)$ (since $c_n=0$ or $\bar{c}$) and $\lim_{n\to\infty} c_n=c^*$, which, in turn, implies:

$$\lim_{n\to\infty}\int_0^\infty e^{-rt}\,U(c_n\,dt)=\lim_{n\to\infty}\int_0^\infty e^{-rt}\,\tilde{U}(c_n\,dt)$$

$$=\int_0^\infty e^{-rt}\,\tilde{U}(c^*)\,dt=\operatorname{supr}_c\int_0^\infty e^{-rt}\,U(c)\,dt,$$

using the well known Arzela-Osgood's theorem. □

The practical implication of proposition 3 for an unsatiable vampire with low discount rate is to behave nearly like the linear vampire. The only difference is that instead of choosing $c=c^{\infty}$ to maintain x^{∞} it is "optimal" to use a *chattering control* (switch from $c=0$ to $c=\bar{c}$ and back as fast as possible) in order to keep as close as possible to the equilibrium level x^{∞}. *See* also Clark [2], page 172.

Let us now turn to the case (B) where future utilities are rather unimportant. Bearing in mind that the optimal policies for the three types of vampires (*a*, *b*, *c*) are very similar to each other in case (A) [except that x^{∞} is reached in finite time in cases (*b*, *c*) and is approached asymptotically in case (*a*)] we can conjecture that in case (B) it will be optimal to exploit the resource as fast as possible by choosing $c=\bar{c}$ and then switching to $c=0$ if the resource is completely used up (*cf.* proposition 1 B). The following result shows that this is the case.

PROPOSITION 4: *For both, the linear and the convex vampires* (*b and c*) *the optimal policy in the case* (B) *of a high discount rate is:*

$$c(t) \doteq \begin{cases} \bar{c} \\ 0 \end{cases} \quad \text{for} \quad \begin{cases} 0 \leqq t < \tau, \\ t \geqq \tau, \end{cases}$$

where the resource x is driven to zero within finite time τ *given by:*

$$\tau = \frac{1}{\bar{c}-n-a} \ln \left[\left(x_0 + \frac{\bar{c}}{\bar{c}-n-a} \right) \Big/ \frac{\bar{c}}{\bar{c}-n-a} \right].$$

The proof is an easy repetition of the method used for proving proposition 2, which is why we omit it.

5. CONCLUDING REMARKS

We have obtained optimal policies for asymptotically satiated, blood-maximizing and unsatiable vampires. The applicability of the results derived should be obvious to any reader. It should be noted that after the first presentation of an earlier version of this paper (Hartl and Mehlmann [4]) the need for a defensive strategy against optimal behaving vampires became obvious. Therefore, Snower [11] provided the human beings with a decision rule of optimal splitting the output of the economy between consumption and production of defensive weapons against (non-optimal behaving) vampires.

Further research in this area by considering a differential game where both species can react on the decision of the other one is needed. Beside the undeniable practical relevance, the contribution of our study is mainly in pointing out the following results, which seem to be of general importance for many other economic control models:

1) The treatment of concave and linear (and convex, resp.) optimal control models requires completely different approaches. In particular, in sufficiently simple concave models the method of phase plane analysis is the appropriate tool, whereas in the latter models the problem of synthesis of bang-bang and singular solution pieces plays the crucial role in the analysis. We have also demonstrated, why in convex models there may be no optimal policy (no singular solution possible).

2) Although different in the analysis, the three types of objective functionals amount to similar optimal solutions. More specifically, there are equilibrium (singular) values of the resource stock which are to be approached either asymptotically or as fast as possible.

3) If the future utilities are quite important (i. e. the discount rate is small) then these equilibrium values are positive and for future generations approximately the same stock of the resource will be available as for the present one. Otherwise the resource is used up completely within finite time or asymptotically as $t \to \infty$, which can be regarded as policy of "Eat, drink, and be merry, for tomorrow we die".

4) We have also demonstrated the method of dividing one state variable by another one in order to reduce the dimension of the state space of an optimal control problem. This method is particularly useful in many resource problems. *See*, for instance, Kemp and Long [6].

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