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"PROBLEM SPECIALIZATION" IN MECHANICAL DECISION PROCESSES (*)

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Abstract. — *Management information systems are developed to aid managers' decision making capabilities. When a problem becomes highly structured and complex it is frequently suggested that the decision maker should resort to using mathematical models and decision rules to avoid the pitfalls that often are built into management information systems. This paper considers the notion that the pitfalls which one tries to avoid by resorting to purely mechanical processes like mathematical models and decision rules can also be present within the mechanical process itself.*

Résumé. — *Les systèmes d'information en administration sont destinés à aider les administrateurs dans leur capacité à prendre des décisions. Quand un problème se fait très complexe et structuré, il est souvent suggéré que celui qui prend les décisions devrait arriver à se servir de modèles de mathématiques et de lois en décision pour éviter les inconvénients qui sont compris dans les systèmes d'information en administration. Ce rapport prend en considération le point de vue que les inconvénients qu'on essaie d'éviter en arrivant à des procès purement mécaniques comme les modèles de mathématiques et les lois en décision, peuvent être présents dans le procès mécanique lui-même.*

1. INTRODUCTION

The use of management information systems (MIS) has developed rapidly in recent years and in establishing a working MIS it is important to remember the pitfalls of designing such a system. Shore [15] summarizes the *false* assumptions that often are used when designing a working MIS as:

A 1. Managers suffer from a lack of information;

A 2. If a manager has all of the necessary information he needs, his decision making will improve;

A 3. A working MIS should be based on the specific kinds of information that management needs;

A 4. A manager does not have to understand how a MIS works in order to use it.

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Theoretical and empirical studies have shown that these assumptions are frequently violated when decision makers are performing decision processes without depending entirely on mathematical models and decision rules [1 to 4, 12, 13, 14, 16].

Assumptions A 1 and A 2 are the primary concern of this paper so a closer look at them is in order. The violation of A 1 is the information overload phenomenon. Ackoff [1] first suggested that managers may become overloaded with information when attempting to make a decision and this excess information might actually be detrimental to reaching a good final decision. In a sense, the decision maker simply becomes confused by an overabundance of marginal information. Chervany and Dickson [3] conducted an experimental investigation which supported the conclusion that the information overload phenomenon can occur in practice.

A violation of A 2 is more counter intuitive than a violation of A 1 since A 2 limits the information given to the manager as being necessary information to the decision process. A violation of A 2 indicates that whenever information is provided to a manager it is necessary to evaluate the ability of the decision maker to use the data. The violation of A 2 will be referred to as the problem specialization phenomenon. This phenomenon partly results from the inability of individuals to be good problem solvers for every type of problem that is given to them. Another explanation for the violation of A 2 is that as problems become increasingly more complex it becomes increasingly more difficult for the decision maker to solve the problem without resorting to algorithms and purely mechanical methods.

Shore [15] suggests that when the problem being considered is highly structured and complex then the use of mathematical models and decision rules should be evaluated to remove these pitfalls. However, the question must now be asked, "Can A 1, A 2, A 3 and A 4 also be violated for decision processes which are made only on the basis of mathematical models and decision rules?"

Gehrlein and Fishburn [9] have shown that the information overload phenomenon *can* be observed in mathematical models and in decision rules based on purely mechanical processes. In this study it is seen that the same mechanical process, namely the sequel construction method of inducing weak orders from partial orders, also exhibits an extreme violation of A 2.

2. THE BASIC PROBLEM

The mechanical process which violates assumptions A 1 and A 2 is the sequel construction method of inducing weak orders from partial orders. To describe the type of situation where this procedure would be used consider the following problem. We have fragmentary information in the form of a partial order P on a set of elements X. A partial order is a binary relation that is irreflexive ($x P x$ for no $x \in X$), asymmetric ($x P y \Rightarrow$ not $y P x, \forall\ x, y \in X$) and transitive ($x P y$ and $y P z \Rightarrow x P z, \forall\ x, y, z \in X$). No restrictions are made on the type of binary relation that P might represent so that $x P y$ might mean that pencil x is longer than pencil y, beer brand x tastes better than beer brand y, or production plan x is estimated to cost more than production plan y. So we have a basic fragmentary information set about how some of the elements are ranked according to others under the P relation. Our information is fragmentary because P is a partial order so there can be pairs in the symmetric complement of P such that $x I y$ ($x I y \Rightarrow$ not $x P y$ and not $y P x$). If $x I y$ we then have no information about the P relation between x and y.

Suppose that we know that there is some actual underlying linear ordering relation, L_0, on X so that L_0 is a partial order that is complete (either $x L_0 y$ or $y L_0 x$ for all $x, y \in X$). The partial order P is taken from L_0 so $P \subseteqq L_0$ and therefore if $x L_0 y$ then it is not true that $y P x$. Given a pair $x, y \in X$ with $x I y$ in P we would like to determine whether or not it is more likely that $x L_0 y$ or $y L_0 x$ and we must base our decision only on the information contained in our fragmentary information set P.

Methods which can be used to induce relations on pairs in the symmetric complement of P that are likely to agree with the ordering in L_0 have been developed in [5 to 10]. All of these methods start with a partial order P and induce a weak order, W, that contains $P (P \subseteqq W)$. A weak order W is a binary relation on a set X that is asymmetric and negatively transitive (not $x W y$ and not $y W z \Rightarrow$ not $x W z, \forall\ x, y, z \in X$). We restrict $P \subseteqq W$ since the orders on pairs in P are in agreement with their orders in L_0 so there would be no reason to change or delete any of them in attempting to reconstruct L_0.

If W is a weak order then X can be partitioned into k equivalence classes E_1, $E_2, \ldots, E_k$ with $E_1\, W'\, E_2\, W' \ldots W'\, E_k$ with all pairs in the same equivalence class being in the symmetric complement of W. We stop inducing relations with W a weak order when L_0 is a linear order for there may be pairs in P which cannot be compared and resultingly should not have a relation induced on them. For example, consider an equivalence relation E defined by

$$x E y \quad \Leftrightarrow \quad [x P z \Leftrightarrow y P z, \forall\ z \in X] \qquad \text{and} \qquad [z P x \Leftrightarrow z P y, \forall\ z \in X]. \quad (1)$$

If $x E y$ in P, (1) tells us that there is no basis of comparison between x and y that uses only the information contained in P so there should be no attempt to induce a relation on them. Hence, we are willing to stop inducing relations when we have reached a weak order.

So the general notion is to begin with P and induce relations on pairs in the symmetric complement of P and construct a weak order W. The ordering of a pair in $W \setminus P$ is correct if it agrees with the ordering on the pair in L_0. Otherwise, the ordering on the pair is not correct. One construction method of inducing relations is better than another if it is expected to have a greater proportion of pairs induced correctly. To be admissible a construction method must induce significantly more than half of all pairs in the correct order or else it is not better than a random construction method. The sequel construction method has been shown to be a reliable method of inducing relations from P [5, 6, 7].

3. THE SEQUEL CONSTRUCTION METHOD

To describe the procedure by which the sequel construction method goes about inducing relations from P the following definitions are needed:

$$M(x)=\{y: y\,P\,x \text{ and } y\in X\};$$
$$L(x)=\{y: x\,P\,y \text{ and } y\in X\},$$
$$I(x)=\{y: x\,I\,y \text{ and } y\in X\},$$
$$S(x, y)=[I(x)\cap M(y)]\cup[L(x)\cap I(y)].$$

The first sequel $S^1(P)$ is defined by

$$(x, y)\in S^1(P) \iff \# S(x, y)>0 \quad \text{and} \quad \# S(y, x)=0, \tag{2}$$

where $\# A$ is the cardinality of a set A. So, from (2), a pair (x, y) with $x\,I\,y$ has a relation induced on it if there exists a z such that $x\,P\,z\,I\,y$ or $x\,I\,z\,P\,y$ and no similar relation exists for y over x. $S^1(P)$ is not necessarily a weak order but it must be a partial order [5]. The second sequel of P, $S^2(P)$, is obtained by substituting $S^1(P)$ for P in (2) and we recursively define the *i-th* sequel $S^i(P)$ in the same fashion. Eventually a point is reached where $S^{i+1}(P)=S^i(P)$ and no additional relations can be induced by the sequel construction method since $S^i(P)$ is a weak order. The minimum i such that $S^{i+1}(P)=S^i(P)$ is the degree, ρ, of P. For convenience define $S(P)=S^\rho(P)$. $S(P)$ is a weak order and in general let it have r equivalence classes $X_1, X_2, \ldots, X_r$ with $S(P)/I^*=X_1\,P^*\,X_2\,P^* \ldots P^*\,X_r$, where I^* is the symmetric complement of $S(P)$ and $X_j\,P^*\,X_k$ means that $x\,S(P)\,y$ for all $x\in X_j$ and for all $y\in X_k$.

To describe how the sequel construction method exhibits the information overload phenomenon it is necessary to define the sequel reduced partial order, R, of P.

R: *For a partial order P with* $\rho=1$ *and* $r \geqq 3$ *then* $R=P \setminus \{(x_i, x_{i+1}): x_i \in X_{i+1}, x_{i+1} \in X_{i+1}$, for $i=1, 2, \ldots, r-1\}$. *Otherwise,* $R=P$.

Therefore R is different than P only when $\rho=1$ and $r \geqq 3$ and in that case R is formed from P by removing from P all relations on pairs in adjacent equivalence classes of $S(P)$.

Having defined R we can now state the Sequel Reduction Theorem from [9] which is the main point of the argument which shows that the sequel construction method exhibits the information overload phenomenon:

THEOREM 1 (Sequel Reduction Theorem): *If P is a partial order on a set X with* $\rho=1$ *and* $r \geqq 3$, *then* $S(P) \subseteqq S(R)$.

This theorem tells us that if we use the sequel construction method on R instead of P then all ordered pairs in $S(P)$ will be in $S(R)$ and in addition $S(R)$ may contain extra relations that are not in $S(P)$. As a result, the information overload phenomenon is exhibited by the sequel construction method if the ordered pairs in $S(R) \setminus S(P)$ are significantly better than a random procedure of entering relations. This follows from the fact that we do better by giving the sequel construction method less information (R instead of P) to work with. Simulation analysis over a variety of partial order types indicate that in general we *can* expect significantly more than half of the ordered pairs in $S(R) \setminus S(P)$ to be entered in the correct order.

As a result of the Gehrlein and Fishburn [9] study we might say that for partial orders in general the sequel construction method does exhibit the information overload phenomenon (for partial orders with $\rho=1$ and $r \geqslant 3$) so use $S(R)$ in these cases instead of $S(P)$. However, a more extensive analysis shows that using $S(R)$ instead of $S(P)$ can be the wrong thing to do if P is a special type of partial order, more specifically when P is an interval order. The case of what happens in $S(R) \setminus S(P)$ for these special types of partial orders is considered in the next section.

4. SPECIAL TYPES OF PARTIAL ORDERS

There are two special types of partial orders that we wish to consider, namely, interval orders and semiorders. An interval order, P, is a partial order with the additional restriction that

$$x P y \quad \text{and} \quad w P z \;\Rightarrow\; x P z \quad \text{or} \quad w P y, \qquad \forall x, y, z, w \in X. \tag{3}$$

A semiorder is an interval order with the additional restriction that

$$x P y P z \;\Rightarrow\; x P w \quad \text{or} \quad w P z, \qquad \forall x, y, z, w \in X. \tag{4}$$

We begin our study by observing that the sequel reduced partial order of an interval order is an interval order when $\rho = 1$ and the sequel reduced partial order of a semiorder is a semiorder.

LEMMA 1: *If P is an interval order with $\rho = 1$ then R is an interval order.*

Proof: Three cases must be considered and all are proved by contradiction. (Assume P is an interval order and R is not an interval order.)

a. Assume P is an interval order with a, b, c, d such that $a P b$, $c P d$, $c P b$ and not $a P d$. Then assume R is not an interval order since it violates (3) with $a R b$, $c R d$, not $a R d$ and not $c R b$. Therefore, $c P b$ was removed in forming R but $c P d$ was not removed. By the definition of R this requires $b S(P) d$ since only pairs in neighboring equivalence classes are removed. However, this is clearly impossible since $a \in I(d) \cap M(b)$ so $\# S(d, b) > 0$ which precludes $b S(P) d$ with $\rho = 1$.

b. A symmetric argument holds if $a P b$, $c P d$, $a P d$ and not $c P b$.

c. Assume P is an interval order with $a P b$, $a P d$, $c P b$ and $c P d$. Then assume R is not an interval order since it violates (3) with $a R b$, $c R d$, not $a R d$ and not $c R b$. By the definition of R, a and d must be in neighboring equivalence classes of $S(P)$ and the same is true of c and b. It is easily shown that this violates $P \subseteq S(P)$ unless a and c are in the same equivalence class and b and d are in the same equivalence class which is impossible since $a R b$ and $c R d$.

Q.E.D.

Since all semiorders are interval orders the proof that P is a semiorder means R is a semiorder must only show that R must meet the second semiorder condition (4). Fishburn [5] has shown that when P is a semiorder then $\rho = 1$ so no further consideration of the degree of a semiorder is necessary.

LEMMA 2: *If P is a semiorder then R is a semiorder.*

Proof: Three cases must be considered and all are proved by contradictions [Assume P is a semiorder and R violates (4)].

a. Assume P is a semiorder with a, b, c, d such that $a P b P c$, $a P d$ and not $d P c$. Then assume R violates (4) with $a R b R c$, not $a R d$ and not $d R c$. By the definition of R this requires that $d S(P) b$ since all relations in neighboring equivalence classes are removed and $a R b$. However, $c \in L(b) \cap I(d)$ so $\# S(b, d) > 0$ which precludes $d S(P) b$ with $\rho = 1$.

b. A symmetric argument holds if $a P b P c$, $d P c$ and not $a P d$.

c. Assume P is a semiorder with $a P b P c$ and $a P d P c$. Then assume R is not a semiorder since it violates (4) with $a R b R c$, not $a R d$ and not $d R c$. By the definition of R, a and d must be in neighboring equivalence classes of $S(P)$ and the same is true of d and c. However, this is clearly impossible since $a R b R c$.

Q.E.D.

When P is a semiorder Gehrlein [8] has shown that $S(P/E)$ is a linear order so that a relation is induced on every pair in the symmetric complement of P for which not $x E y$. This is stated without proof as:

LEMMA 3: *For P a semiorder $S(P/E)$ is a linear order.*

It then follows that for P a semiorder that $S(P)=S(R)$ so nothing is gained by using the sequel reduction method of inducing relations from P beyond those given in $S(P)$.

COROLLARY 1: *For P a semiorder $S(P)=S(R)$.*

Proof: By theorem 1 $S(P) \subseteqq S(R)$ and by lemma 3 $S(R) \subseteqq S(P)$ so $S(P)=S(R)$.

Q.E.D.

When P is an interval order it is not necessarily true that $S(P)=S(R)$ but we do find an unexpected result which shows that the sequel construction method violates A 2 and exhibits problem specialization.

5. THE VIOLATION OF A 2

The way in which the sequel construction method violates A 2 and thereby exhibits problem specialization is observed by examining the relations induced on pairs in $S(R) \setminus S(P)$ when P is an interval order. When P is a partial order in general we expect the proportion of pairs in $S(R) \setminus S(P)$ that are in agreement with their order in L_0 to be better than random. When P is a semiorder the set $S(R) \setminus S(P)$ is empty. The surprising result is that when P is an interval order we expect the proportion of pairs in $S(R) \setminus S(P)$ that are in agreement with L_0 to be *worse* than random.

The fact that the pairs in $S(R) \setminus S(P)$ are generally induced incorrectly when P is an interval order does not preclude the use of sequel reduction method when P is an interval order. All that is necessary is to take the pairs in $S(R) \setminus S(P)$ and turn them upside down to form the weak order $S'(R)$ where

$$S'(R)=S(P) \cup [S(R) \setminus S(P)]^*, \tag{5}$$

where A^* is the dual of a set A.

So for partial orders in general we can use the sequel construction method to induce pairs that are expected to be correct in $S(R) \setminus S(P)$. As we expect from A 2, the sequel construction method cannot be expected to perform well on all problems. This is true to the extreme that when the initial partial order is an interval order then we expect the sequel construction method to be admissible only if the pairs in $S(R) \setminus S(P)$ are turned upside down before we use them.

To understand why this phenomenon takes place we must consider what must happen for a pair to be in $S(R) \setminus S(P)$ for P an interval order with $\rho=1$. Consider a pair $(x, y) \in [S(R) \setminus S(P)]$. For (x, y) in the symmetric complement of $S(P)$ for P an interval order with $\rho = 1$ either $L(x) \subset L(y)$ and $M(x) \subset M(y)$ or vice versa when not $x E y$. This fact follows directly from the interval order condition (3) and $x I y$ in P. Suppose $(x, y) \in S(R)$ so that by removing some relations from P (x, y) is no longer in the symmetric complement of the sequel. This can happen in a number of ways. Assume, without loss of generality, that it occurs as follows. We have $(x, y) \in [S(R) \setminus S(P)]$ because $L(x) \subset L(y)$ and $M(x) \subset M(y)$ in P with $L'(x) = L'(y)$ and $M'(x) \subset M'(y)$ in R. By the definition of R it must be true that there exist a and b in $M(y) \setminus M(x)$ with $a P b P y$ where $a \in M'(y)$ and $b \in I'(y)$ in R. Therefore, if there are r equivalence classes in $S(P)$, x and y must be in one of the equivalence classes $3, 4, \ldots, r$. For small $\# X$ it is therefore likely that x and y are below the central equivalence class of $S(P)$. That is, they are likely to be in an equivalence class E_i where $i > r/2$.

For small $\# X$ the pairs in $S(R) \setminus S(P)$ are expected to act like the following rule, defined as the AV Rule when P is an interval order with $\rho = 1$:

AV Rule: For (x, y) with not $x E y$ in the symmetric complement of $S(P)$ in equivalence class E_i induce a relation for x over y if $\# A(x) > \# A(y)$ for $i \leqq r/2$ or if $\# A(x) < \# A(y)$ for $i > r/2$.

Otherwise induce the relation y *over* x.

In this definition $A(z) = L(z) \cup M(z)$. For P an interval order we induce the relation for x over y in $S(R) \setminus S(P)$ with $A(x) \subset A(y)$ when x and y are expected to be below the central equivalence class as in the AV Rule. By symmetry the same argument holds for pairs in the equivalence classes of $S(P)$ above the central equivalence class. As a result for P an interval order with $\rho = 1$ we expect the pairs in $S(R) \setminus S(P)$ to be very similar to the pairs induced by the *AV* Rule when $\# X$ is small. However, previous studies [10] have shown that the *AV* Rule is inadmissible for interval orders. Since the pairs in $S(R) \setminus S(P)$ are similar to pairs entered by the *AV* Rule on interval orders of degree one we should expect the proportion of pairs in $S(R) \setminus S(P)$ that are in the correct order to be significantly worse than a random process. Resultingly the use of $[S(R) \setminus S(P)]^*$ should be admissible when P is an interval order.

To get an idea of the reliability of pairs entered in $[S(R) \setminus S(P)]^*$ simulation analysis was used.

6. SIMULATION RESULTS

The actual underlying linear ordering relation is assumed to be given by $L_0 = x_1 x_2 \ldots x_n$. During the simulation we wish to generate interval orders at random with $P \subseteq L_0$. This was done as follows:

On the interval $[0, m]$ generate n random numbers and assign them in increasing order to n points $\Sigma_1, \Sigma_2, \ldots, \Sigma_n$. Then for each Σ_i generate a number at random a number, t_i, from the interval $[0, t]$ and form the intervals $\Sigma_i \pm t_i$. To form P, enter the relation $x_i P x_j$ for $i<j$ if the interval $\Sigma_i \pm t_i$ does not overlap with the interval $\Sigma_j \pm t_j$.

As described in [6] and [7] the orders that result from this procedure must be interval orders. It certainly is possible to generate interval orders in other ways and the results presented here are related to the generation procedure used.

The results obtained are not a function of the specific m and t used but is only dependent on the ratio $\beta = t/m$. The simulation generated interval orders as described and for each order determined if $\rho = 1$. If ρ was greater than one the interval order was discarded and another was generated. Interval orders of degree one were generated until there were 300 ordered pairs induced in $S'(R) \setminus S(P)$.

The results in the table show the proportion of pairs in $S'(R) \setminus S(P)$ that were entered in the correct order. For each sample the hypothesis was tested that the proportion of pairs entered correctly was significantly different from .500. Samples were run for $n=10$ for each $\beta \in \{.1, .2, .3, .4\}$.

TABLE

Proportion of Pairs in $S'(R) \setminus S(P)$ That Were Correct

β	Proportion Correct
.10	.5640 (*)
.20	.5380
.30	.5600 (*)
.40	.5809 (*)

(*) Indicates that the proportion correct is significantly different (at .99) than .5000

The simulation results support the finding that the use of pairs in $S'(R) \setminus S(P)$ is admissible and as a result the sequel construction method does exhibit a violation of A 2. These findings indicate that the sequel construction method exhibits the problem specialization phenomenon in an extreme form.

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