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THE BUSY PERIOD OF A REPAIRMAN ATTENDING A $(n + 1)$ UNIT PARALLEL SYSTEM

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Résumé. — *The distribution function of the busy period of a single repairman attending a system of $(n + 1)$ parallel identical units subject to breakdowns has been derived explicitly by a Markov renewal branching argument. The method presented in this note is applicable to modified systems.*

INTRODUCTION

In this paper, we consider a system composed of $(n + 1)$ parallel identical units. A unit works continuously until it breaks down. If a unit fails, it goes immediately into repair unless the repairman is already repairing another unit. In this case, a waiting line is formed. It is supposed that the repairman is idle if all units are operative, otherwise we define the repairman to be busy. The repairtimes are independent positive identically distributed random variables with distribution function $R(x)$ with finite expectation. The failure time of a unit follows a negative exponential law with parameter λ. Statistical properties of the system have been derived by Takàcs [2] who used renewal theoretical arguments. In this paper, we study the stochastic law of the busy period of the repairman by a Markov renewal branching argument.

ANALYSIS OF THE SYSTEM

It is obvious that the order of repairing the units is irrelevant for the busy period of the repairman.

Let $B_n(t)$ be the distribution of the busy period $\underline{b}_n$ with $(n + 1)$ units; $n = 1, 2, ...$

For $n \geqslant 1$, let $\underline{\nu}$ be the number of failures during the repair of the unit starting the busy period at $t = 0$ and $\underline{\tau}$ the first repair time.

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Applying an argument of branching theory, we may write

$$\underline{b}_n \sim \underline{\tau} + \underline{b}_{n-\underline{\nu}+1} + \dots + \underline{b}_n \tag{1}$$

The basic idea of this method is due to Takàcs [2] who used it for the study of the busy period of the $M|G|1$ queue with infinite waitingroom and to Cohen [1] who applied it for the busy period of the $M|G|1$ queue with finite and infinite waitingroom.

For $k = 0, 1, 2, \dots, n$ we have

$$P[\underline{\tau} < t, \underline{\nu} = k] = \int_0^t \binom{n}{k} e^{-\lambda(n-k)x}(1 - e^{-\lambda x})^k \, dR(x)$$

Since the $\underline{b}_i$ are mutually independant, we have by (1)

$$B_n(t) = \int_0^t e^{-n\lambda x} \, dR(x) + \tag{2}$$

$$\sum_{k=1}^{k=n} \left\{ \int_0^t \binom{n}{k} e^{-\lambda(n-k)x}(1 - e^{-\lambda x})^k \, dR(x) \right\} * \varphi_k(t)$$

where $\varphi_k(t) = B_{n-k+1}(t) * B_{n-k+2}(t) * \dots * B_n(t)$
and $*$ denotes the convolution operation.

If we define the empty sum to be zero, then relation (2) is valid for $n = 0, 1, 2, \dots$

It is easy to verify that the set of integral equations (2) determine $B_n(t)$ uniquely.

Define for Re $s \geqslant 0$

$$\int_0^\infty e^{-st} \, dB_n(t) = b_n(s)$$

and

$$\int_0^\infty e^{-st} \, dR(t) = r(s)$$

Applications of the Laplace-Stieltjes transform to equation (2) yields for $n = 0, 1, 2, \dots$

$$b_n(s) = \int_0^\infty e^{-n\lambda x} e^{-sx} \, dR(x) + \tag{3}$$

$$\sum_{k=1}^{k=n} \int_0^\infty \binom{n}{k} e^{-(n-k)\lambda x}(1 - e^{-\lambda x})^k e^{-sx} \, dR(x) \prod_{i=n-k+1}^{i=n} b_i(s)$$

Define $\Delta_n(s)$ for $n = 0, 1, \dots$ by :

$$b_n(s) = \Delta_n(s)\Delta_{n+1}^{-1}(s) \qquad n = 0, 1, 2, \dots \tag{4}$$

(5) $$\Delta_0(s) \equiv 1\,;$$

and $q_{nj}(s)$ by :

(6) $$q_{nj}(s) = \int_0^\infty \binom{n}{j} e^{-\lambda jx}(1 - e^{-\lambda x})^{n-j} e^{-sx}\, dR(x)$$

By (3), (4), (5), (6) we have

(7) $$\Delta_n(s) = \sum_{j=0}^{j=n} q_{nj}(s)\Delta_{j+1}(s) \qquad n = 0, 1, 2, \ldots$$

In order to determine $\Delta_n(s)$ explicitly, we introduce

(8) $$\Lambda_j(s) = \sum_{l=0}^{l=j} (-1)^{j-l}\binom{j}{l}\Delta_l(s)$$

whence

(9) $$\Delta_n(s) = \sum_{j=0}^{j=n} \binom{n}{j}\Lambda_j(s)$$

Since

$$\binom{j}{l}\binom{l}{i} = \binom{j}{i}\binom{j-i}{j-l}$$

we obtain by (6)

(10) $$\sum_{l=i}^{l=j} (-1)^{j-l}\binom{j}{l} q_{li} = (-1)^{j-i}\binom{j}{i} r(s + j\lambda)$$

Furthermore

(11) $$\sum_{i=0}^{i=j} (-1)^{j-i}\binom{j}{i}\Delta_{i+1}(s) = \Lambda_{j+1}(s) - \Lambda_j(s)$$

Hence by (7), (8), (9), (10), (11) and (4)

$$b_n(s) = [1 + \Omega_n(s)][1 + \Omega_{n+1}(s)]^{-1}$$

where

$$\Omega_n(s) = \sum_{l=1}^{l=n} \binom{n}{l} \prod_{k=0}^{k=l-1} \frac{1 - r(s + k\lambda)}{r(s + k\lambda)} \qquad \textbf{Re } s \geqslant 0$$

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