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A REVIEW OF THE HISTORY OF JAPANESE MATHEMATICS

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ABSTRACT. — This review aims to introduce Japanese mathematics to a non-expert and a non-Japanese readership. It briefly characterizes mathematics in Japan, surveys its history, as it developed over the last century, and provides a large (if not exhaustive) bibliography of works in the primary European languages.

RÉSUMÉ. — APERÇU SUR L'HISTOIRE DES MATHÉMATIQUES JAPONAISES. Le but de cette note est de présenter les mathématiques japonaises à un public non spécialisé dans le domaine. Les mathématiques au Japon sont brièvement caractérisées, leur histoire, telle qu'elle s'est développée durant le dernier siècle, est passée en revue et finalement une importante bibliographie dans les principales langues européennes est proposée, même si elle ne peut prétendre à l'exhaustivité.

1. INTRODUCTION – NOT ONLY *SANGAKU*

The custom of hanging *sangaku* (算額), wooden plates on which are inscribed mathematical problems and their answers, under the roofs of shrines in the Edo period (1603–1867) in Japan is familiar enough to have been illustrated and described in *Scientific American*, May 1998 [Rothman 1998]. It is, however, obvious that sangaku is not all there is to Japanese mathematics. It would be fallacious to consider that the essence of Japanese mathematics reveals itself in sangaku.

As a beginning, this essay attempts briefly to introduce Japanese mathematics to a non-Japanese readership. It will answer the following questions :

1) What kind of mathematical disciplines developed in Japan ?
2) How were mathematical expressions written in vertical typesetting ?
3) How did the Japanese calculate ?
4) Is it true that, lacking proofs, Japanese mathematics was not logical ?

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5) Is Japanese mathematics just a poor imitation of Chinese mathematics?

6) Was Japanese mathematics useful in other fields?

7) How was mathematics learned in Japan?

It will then look back on 100 years of study of the history of Japanese mathematics, dividing it into six areas of study. The survey will be limited mainly to treatises written in European languages, and will close with a bibliography of articles written in European languages on the history of Japanese mathematics. Although it is not exhaustive, I hope it will be useful for all readers interested in Japanese mathematics.

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2. PECULIARITY OF JAPANESE MATHEMATICS

Japanese mathematics (*Wasan*, 和算) is defined as the mathematics developed in Japan before the Meiji Restoration in the latter half of the 19th century when Japan was forced to end its seclusion and was exposed to Western culture.

It flourished especially in the Edo period when Japan was a closed country[1]. This means that it was one of the last non-European mathematical traditions to westernize.

2.1. What kind of mathematical disciplines developed in Japan?

Various domains are represented in Japanese mathematics. Articles in the bibliography, for example, concern the plane geometry of polygons, circles, ellipses, number theory of indefinite equations and Pythagorean triangles, theory of determinants, problems concerning sums of progressions, and so on[2]. There are other disciplines represented such as solid

[1] [Hayashi 1903–1905, 1907b], [Mikami 1974], [Ogura 1993], and [Smith and Mikami, 1914] are complete histories. Although [Smith and Mikami 1914] is standard, [Ogura 1993] is also suitable as a primer. It was first published in 1940 and has only 100 pages, but it is illustrated and is easy to understand. The fact that these works first appeared 50 to 100 of years ago suggests that the historical study of Japanese mathematics has developed rather slowly during the last fifty years. However, the situation has changed considerably since the 1990's. The time is now ripe for a new complete history of Japanese mathematics.

[2] On plane geometry, see [Mikami 1915] and [Shinomiya and Hayashi 1917]. See,

geometry. One of the most brilliant achievements among these is *enri* (円理), or the circle principle, the general term for analytical methods of calculating lengths of circles, arcs, and other curves, or of computing volumes or surface areas of solids[3]. For example, Takebe Katahiro (建部賢弘, 1664–1739) calculated the value of π to 41 decimal places in 1722 using a numerical method similar to Richardson's extrapolation method[4] and also got an infinite series,

$$\left(\frac{s}{2}\right)^2 = cd\left\{1 + \frac{2^2}{3\cdot 4}\left(\frac{c}{d}\right) + \frac{2^2\cdot 4^2}{3\cdot 4\cdot 5\cdot 6}\left(\frac{c}{d}\right)^2 + \frac{2^2\cdot 4^2\cdot 6^2}{3\cdot 4\cdot 5\cdot 6\cdot 7\cdot 8}\left(\frac{c}{d}\right)^3 + \cdots\right\},$$

where d is the diameter of the given circle, c and s are the sagitta and the length of an arc respectively[5].

2.2. How were mathematical expressions written in vertical typesetting?

Today, the Japanese write sentences both horizontally and vertically, but in the Edo period, they only wrote vertically. Mathematics was also written vertically in Japan (as it was in China). But there were some improvements in Japanese mathematics. They were accomplished by Seki Takakazu (関孝和, ?–1708)[6] and his successor, Matsunaga Yoshisuke (松永良弼, ?–1744), in the second half of the 17th century and called *Tenzan Jutsu* (點竄術). They introduced characters and frac-

in particular, [Mikami 1912b] and [Yanagihara 1915] for ellipses. For a discussion of the solutions to the indefinite equation $x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2 = y^2$ by Ajima Naonobu (安島直円, 1732–1798) and Pythagorean triangles, see [Mikami 1912a] and [Yanagihara 1914–1915], respectively. [Hayashi 1909–1910] and [Mikami 1909–1910b, 1914–1919] deal with the theory of determinants and [Yanagihara 1918] analyzes various progressions that appear in Japanese mathematics.

[3] See [Harzer 1905] and [Mikami 1909–1910a, 1930] for an outline. For further details of enri before Takebe Katahiro, see [Horiuchi 1994a, 1994b]. [Mikami 1914b] analyzes the representation of π by an infinite product in *Suuri Shinpen* (数理神篇, 1860) by Saitou Noriyoshi (斎藤宜義, 1816–1889). See also [Mikami 1913a,b,c,d], [Mikami 1914a,b], and [Nakamura 1994]. See [Horiuchi 1994a] for a recent account in French.

[4] See [Horiuchi 1994a] or [Xu 1999] for more details. Takebe said in his treatise *Tetsujutsu sankei* (綴術算経, 1722) that he had calculated π^2. Both Horiuchi and Xu explain this calculation, but the values Takebe wrote down in it are, in fact, for calculating π itself.

[5] See [Kikuchi 1896] and [Horiuchi 1994a,b].

[6] Seki is one of the most famous and brilliant mathematicians in the history of Japanese mathematics, and many authors mention him. [Hayashi 1906b,c], [Jochi 1993], [Kikuchi 1896], [Mikami 1908], and [Ogawa 1995] contain the word 'Seki' in the title directly.

tional expressions. Matsunaga, for example, wrote expressions $ab/h + h$ and $a^2b^2/h^2+2ab+h^2$ as (a) and (b) in the next figure respectively.[7] These would be as (a') and (b') if we replace Chinese characters with alphabetic representations.

(a)	(b)	(a′)	(b′)
中勾 | 長短	中勾巾 | 長短巾巾	$h \mid ab$	$h^2 \mid a^2b^2$
| 中勾	‖ 長短	$\mid h$	$\Vert\, ab$
	| 中巾		$\mid h^2$

Figure 1*. Expressions in Japanese mathematics.*

In some cases, these formulae mean simple and quadratic equations $ab/h + hx = 0$ and $a^2b^2/h^2 + 2abx + h^2x^2 = 0$ in the unknown x, but no confusion arises in context.[8] *Tenzan Jutsu* had the potential to express any complicated calculation, but, since there were few symbols other than the ones in figure 1, they had to complement calculation with sentences. Takebe's formula in 2.1 was also described in this manner. This style was widely used until the end of Japanese mathematics. Today, it is somewhat difficult to follow such calculations, especially when they involve many unknowns.

2.3. How did the Japanese calculate?

Japanese mathematicians mainly used abaci for numerical computation. Abaci were tools imported from China that spread throughout the country. Although it is uncertain when they were imported into Japan, the Japanese used them for calculation in both private and business settings until the advent of the electric calculator in the latter half of the

[7] *Enchuu San'gen Tekitou* (円中三原適等). Some Chinese characters are omitted from the expressions.

[8] See [Hayashi 1905 p. 340–346] for further details.

20th century[9]. The abacus was imported from China together with books such as *Suanfa Tongzong* (算法統宗, Cheng Dawei in the Ming Dynasty, 1593). People in the Edo period could perform not only addition, subtraction, multiplication, and division but also square root extraction with the abacus. Moreover, they solved equations of any degree in one variable using multiple abaci simultaneously. This method had its origin in a Chinese method using computing rods.[10] Since the Japanese used the abacus, for calculation, they seldom wrote down the process of calculation.

Is it true that, lacking proofs, Japanese mathematics was not logical?

It is often said that Japanese mathematics was not constructed logically and that it contained no proofs. When a Chinese translation of Euclid's *Elements* by an Italian missionary in China, Matteo Ricci (1552–1610) who collaborated with Xu Guangqi (徐光啓), was introduced into Japan, Japanese mathematicians did not recognize that it aimed at building the edifice of mathematical knowledge[11]. It is certain that there were few definitions, postulates, and even proofs in Japanese mathematics. But this does not imply that Japanese mathematics represented a collection of illogical and unfounded assertions. People in the Edo period familiarized themselves with the famous theorem of Pythagoras and understood many kinds of proofs of it, though they were not interested in fundamental concepts like points, lines, and so on. After all, mathematics was regarded not as a deductive study, but as an inductive one. In the case of the calculation of π or the length of an arc, mathematicians in the Edo period investigated empirically and drew what seemed to be irrefutable conclu-

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[9] Today, few in Japan use abaci regularly, but there are still calculation contests in which the contestants manipulate abaci with great speed. They can answer 10 questions of adding or subtracting 15 numbers with 10 digits within 4 minutes, and 20 questions of multiplying or dividing two numbers with 6 digits within 3 minutes. Some of these virtuosi never actually handle the abacus, since they calculate by visualizing the abacus' movements. It is astonishing that they can calculate the next question as they write down the answer of the previous question. Their special skills are certainly superior to those of the people in the Edo period, but Takebe might have been able to calculate faster and more accurately than we expect.

[10] Hayashi [1903–1905, p. 313–324], explains how to use computing rods (*sangi* (算木), in Japanese). See also [Mikami 1911c].

[11] Ricci's *Elements*, that was translated from Clavius's, contained only the first six books. They were introduced into Japan as a part of *Tianxue Chuhan* (天学初函).

sions.

The computation of π by Takebe Katahiro is a typical example. He first calculated the circumferences σ_n of regular 2^{n+1}-polygons $(n = 1, 2, \dots, 9)$ inscribed in a given circle of diameter 1 *shaku*[12] :

$$\sigma_1 = 2.82842\ 71247\ 46190\ 09760\ 33774\ 48419\ 39615\ 71393\ 43750,$$
$$\sigma_2 = 3.06146\ 74589\ 20718\ 17382\ 76798\ 72243\ 19093\ 40907\ 56499,$$
$$\sigma_3 = 3.12144\ 51522\ 58052\ 28557\ 25578\ 95632\ 35585\ 48430\ 65884,$$
$$\sigma_4 = 3.13654\ 84905\ 45939\ 26381\ 42580\ 44436\ 53906\ 75563\ 73541,$$
$$\sigma_5 = 3.14033\ 11569\ 54752\ 91231\ 71185\ 24331\ 69013\ 21437\ 03233,$$
$$\sigma_6 = 3.14127\ 72509\ 32772\ 86806\ 20197\ 70788\ 21440\ 83796\ 63262,$$
$$\sigma_7 = 3.14151\ 38011\ 44301\ 07632\ 85150\ 59456\ 82230\ 79353\ 13815,$$
$$\sigma_8 = 3.14157\ 29403\ 67091\ 38413\ 58001\ 10270\ 76142\ 95336\ 37794,$$
$$\sigma_9 = 3.14158\ 77252\ 77159\ 70062\ 88542\ 62701\ 91873\ 93992\ 80858.$$

He then calculated the ratios of differences,

$$\delta_n = \frac{\sigma_{n+1} - \sigma_n}{\sigma_{n+2} - \sigma_{n+1}},$$

of this numerical sequence[13], where $n = 1, 2, \dots, 7$:

$$\delta_1 = 3.88545\ 00933\ 17747\ 65438\ 05476\ 61665\ 26636\ 07702\ 84087,$$
$$\delta_2 = 3.97115\ 47337\ 47624\ 22161\ 58561\ 42353\ 19888\ 01248\ 83468,$$
$$\delta_3 = 3.99277\ 56390\ 82541\ 67167\ 43801\ 62627\ 68131\ 04618\ 82098,$$
$$\delta_4 = 3.99819\ 30935\ 97580\ 83294\ 63029\ 04091\ 44508\ 81433\ 25777,$$
$$\delta_5 = 3.99954\ 82223\ 74493\ 30195\ 90586\ 32058\ 93864\ 55512\ 74018,$$
$$\delta_6 = 3.99988\ 70524\ 04346\ 82623\ 56273\ 66653\ 51865\ 66984\ 40970,$$
$$\delta_7 = 3.99997\ 17629\ 01753\ 48568\ 01506\ 89128\ 80751\ 52798\ 92518.$$

Observing that this sequence tends to 4, he accelerated to get 7 approximations of π as follows. He put $\delta_k = \delta_{k+1} = \cdots = 4$ in the expressions,

$$\sigma_k^{(2)} = \sigma_k + (\sigma_{k+1} - \sigma_k) + (\sigma_{k+1} - \sigma_k) \cdot \frac{1}{\delta_k} + (\sigma_{k+1} - \sigma_k) \cdot \frac{1}{\delta_k} \cdot \frac{1}{\delta_{k+1}} + \cdots,$$

[12] Shaku (尺) is a unit of length in old Japan and is about 30.3 cm.

[13] The number of digits of these values are not exaggerated; Takebe actually determined the value of π to over 40 digits.

where $k = 1, 2, \ldots, 7$, and got

$$\sigma_k^{(2)} = \sigma_k + \frac{\sigma_{k+1} - \sigma_k}{1 - 1/4} = \frac{1}{4-1}(4\sigma_{k+1} - \sigma_k), \quad k = 1, \ldots, 7.$$

These values are :

$$\begin{aligned}
\sigma_1^{(2)} &= 3.14143\ 77167\ 03830\ 32282\ 08505\ 70095\ 41082\ 84271\ 69012,\\
\sigma_2^{(2)} &= 3.14158\ 29366\ 41901\ 58989\ 48247\ 60704\ 60013\ 84608\ 09427,\\
\sigma_3^{(2)} &= 3.14159\ 20457\ 57690\ 79515\ 14053\ 50963\ 40715\ 36728\ 13131,\\
\sigma_4^{(2)} &= 3.14159\ 26155\ 92112\ 85331\ 03201\ 86273\ 72250\ 04583\ 16605,\\
\sigma_5^{(2)} &= 3.14159\ 26512\ 14810\ 47908\ 40134\ 89013\ 02494\ 11205\ 30666,\\
\sigma_6^{(2)} &= 3.14159\ 26534\ 41354\ 82007\ 15617\ 93875\ 40780\ 33997\ 45787,\\
\sigma_7^{(2)} &= 3.14159\ 26535\ 80515\ 80612\ 65389\ 80178\ 97117\ 60211\ 61879.
\end{aligned}$$

He calculated ratios of differences of this sequence $\{\sigma_n^{(2)}\}$ $(n = 1, \ldots 7)$ once again :

15.77019 25921 41822 63102 98558 96589 02108 75824 90883,

15.94226 50267 94338 75049 26503 15588 87737 19533 28524,

15.98554 84972 36192 63376 78777 25229 40940 35534 23891,

15.99638 60133 34177 35818 54333 92360 73694 71019 22183,

15.99909 64338 81847 93127 87854 93761 92951 03117 17904,

15.99977 41041 29484 56059 89557 53853 99682 27728 18987.

He realized that this sequence tends to 16 and that 16 is 4^2. Similar calculations ultimately yielded the value of π to over 40 digits.

This inductive inference with a great number of numerical calculations was not mere guesswork to Takebe; it was a method of proof. He never intended to prove the result of the above inference, an attitude shared by his contemporaries.

Occasionally, such inferences were incorrect and were disputed by later scholars. For instance, Seki introduced a method of calculating the determinant $|A|$ of a matrix A in 1683. His method was the same as the rule of Sarrus if the degree was 3, but Seki reasoned inductively (and incorrectly) that the rule also held in higher degrees. It took some ten years to correct the error[14]. While Japanese mathematics was not rigorous by modern standards, its inference techniques sufficed for the purposes of its practitioners.

2.5. Is Japanese mathematics just a poor imitation of Chinese mathematics?

It is certain that Japanese mathematics had its origins in Chinese mathematics. Beginning in the Nara period (8th century), the Japanese have introduced numerous aspects of Chinese culture, including mathematics, into their own culture. In the Edo period, in particular, mathematics developed through the assimilation of Chinese mathematical textbooks such as *Suanfa Tongzong* and *Suanxue Qimeng* (算学啓蒙, Zhu Shijie in the Yuan Dynasty, 1299). To understand Japanese mathematics, then, it is critical to understand Chinese mathematics.[15] It is, however, a mistake to assume that Japanese mathematics was merely a poor imitation of Chinese mathematics. Japanese mathematicians in the Edo period set original problems, developed new methods of solving them, and got advanced results. Problems in plane geometry are typical examples which show the originality of Japanese mathematics.[16] The inversion with respect to a circle was used for solving some of them and yielded results about the Steiner porism.[17] Japanese mathematicians also made effective use of the expansion formula for the determinant $|A|$ of a matrix A to solve simultaneous equations and used infinite series for finding the lengths of various curves and the volumes of solids. These are all manifestations of the higher aspects of Japanese mathematics.[18]

2.6. Was Japanese mathematics useful in other fields?

External motivations for the study of mathematics in Japan related

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[14] See [Horiuchi 1994a] and [Mikami 1914-1919].

[15] Mikami Yoshio, for example, wrote a number of articles on Chinese mathematics with an eye to Japanese developments [Mikami 1909, 1910a, 1911a, 1911b, 1928].

[16] [Fukagawa and Pedoe 1989], [Hirayama 1936], [Rothman 1998], and [Shinomiya and Hayashi 1936] collect and discuss such problems.

[17] See Dan Pedoe, *Geometry, A Comprehensive Course* (New York : Dover, 1988), p. 98 for the Steiner porism and for a detailed description of the inversion.

[18] There are many articles or books on this subject : [Fujisawa 1900], [Hayashi 1903–1905], [Horiuchi 1994a], [Kikuchi 1895a, 1895b, 1895c, 1895d, 1896], [Mikami 1909–1910a, 1911–1912, 1912b, 1913a, 1913b, 1913c, 1913d, 1914–1919, 1930], [Nakamura 1994], [Smith and Mikami 1914], [Zu 1999].

to trade, home building, calendar-reasonings, and so on. Yoshida Mitsuyoshi's (吉田光由) *Jinkouki* (塵劫記, 1627) is a compilation of all the knowledge necessary for daily commerce, and Heinouchi Masaomi's (平内廷臣) *Kujutsu Shinsho* (矩術新書, the early 19th century) is a mathematical textbook for carpenters. There was, however, no remarkable growth in these fields. It was establishing the calendar that stimulated such outstanding mathematicians as Seki Takakazu and Takebe Katahiro to study mathematics. They thoroughly investigated a Chinese calendar, *Shoushi Li* (授時暦, 1280), and studied mathematical problems derived from it. Their analytical studies of curves may have had their origins in it. Similarly their calculations of the value of π were stimulated by the necessity to establish the calendar.[19]

There were, however, a great number of studies without application in daily life. The geometry inscribed on sangaku or the geometry of a fan in Iesaki Yoshiyuki's (家崎善之) *Gomei Sanpou* (五明算法, 1814) are typical examples.[20] These geometries had no application to other fields, that is, they had interaction neither with production techniques nor with the natural sciences. The figures in these geometries were considered as a kind of art. Fujita Sadasuke (藤田貞資, 1734–1807) remarked in his book, *Seiyou Sanpou* (精要算法, 1781), that mathematics involved something useful, something not immediately useful, and something totally useless. He criticized the custom of hanging sangaku as totally useless work.[21]

2.7. How was mathematics learned in Japan?

People in the Edo period enjoyed learning mathematics, and a sangaku was one of fruits of their learning. Since many amateurs learned mathematics solely for pleasure, there was also a certain need for teachers. Teachers and pupils were organized in the form of the *iemoto* (家元) system. An iemoto system is usually defined as the system of licensing the teaching mainly of a traditional Japanese art. Iemoto means literally the main branch or the head of a school. *Shoryuu Iemoto Kagami* (諸流家元鑑, the 18th or 19th century) listed all kinds of iemotoes : the teaching of Shintoism, Buddhism, Confucianism, medicine, fortune-telling, etiquette,

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19 [Horiuchi 1994a] discusses the calendar in Japan.

20 See [Ogura 1993 p. 61] for some figures in *Gomei Sanpou*. See remark 16 for sangaku.

21 See [Ogura 1993, p. 49–50] for Fujita's philosophyof mathematics.

flower arrangement, the tea ceremony, various kinds of musical instruments (like the flute, drums, *shamisen*, *koto*, and *shakuhachi*), painting, dance, calligraphy, *tanka*, *haiku*, the game of *go*, *shougi* (Japanese chess), and so on. It also included the teaching of mathematics and distinguished seven iemotoes in it. It is, however, controversial whether people who learned mathematics could be organized in the same manner as the other iemoto systems of arts. In the Seki school, the biggest school in Japanese mathematics, the iemoto certainly had the right of fixing the contents of his art and licensing his pupil, but the iemoto of the Seki school was not hereditary. The heritability of an iemoto was essential for and characteristic of iemoto systems. The iemoto system of a mathematical school, therefore, differed significantly from that of other iemoto systems.[22]

3. A HUNDRED YEARS OF STUDY

The study of the history of Japanese mathematics started in the closing years of the 19th century, soon after the introduction of Western mathematics into Japan. The latter was a consequence of the industrial or technological policy of the Meiji government and brought about the decline of traditional Japanese mathematics.

The first article written in European language on the history of Japanese mathematics was by Kikuchi Dairoku in 1895[23]; Fujisawa Rikitarou read a paper on the topic at the International Congress of Mathematicians in Paris 1900.[24]

Research in the 100 years from 1800 to 1900 was done almost solely by Japanese scholars.[25] It was not until the 1920's that articles in English or German began to appear.[26] The leading authors in those days were Hayashi Tsuruichi, Kikuchi Dairoku, Mikami Yoshio, Shinomiya Asaji,

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[22] See Francis L.K. Hsu, 'Japanese Kinship and iemoto,' Chapter XI–XII of the Japanese Translation of *Clan, Caste, and Club* (Tokyo : Baifukan, 1970), for more details.

[23] [Kikuchi 1895a, 1895b, 1895c, 1895d].

[24] [Fujisawa 1900].

[25] One of the exceptional authors is Paul Harzer. He was one of the first foreign scholars who gave attention to the exact sciences in old Japan. See [Harzer 1905].

[26] See, for example [Smith 1911–1912], where two main problems for study are laid out : the influence of Western mathematics on Japanese mathematics and the nature of Japanese mathematics.

and Yanagihara Kichiji, all of whom (except Mikami) were also active as mathematical researchers.[27] They tended to analyze the mathematical contents of texts without studying the various contexts in which they developed. Mikami Yoshio, on the other hand, was an historian[28]. The controversy that took place around 1930 between Mikami and Hayashi on the origin of analytical computation in Japanese mathematics reflected the two different perspectives.[29]

Although the number of articles in Japanese has increased since the 1920's, the study of the history of Japanese mathematics remains a minor field both in Japan and internationally ; it has been almost entirely ignored in postwar Japan as a result of the country's technically oriented scientific policy.

In recent years, however, there has been a growing tendency to study the history of Japanese mathematics as part of a reevaluation of the history of Asian sciences. Sasaki, for example, emphasizes a precise reading of texts and adopts a social historical viewpoint in his approach to Asian mathematics.[30] Generally speaking, the translation into modern mathematical notation can be problematic because scholars can easily be led to make improper assumptions about earlier mathematical understanding.

4. AREAS IN THE STUDY OF THE HISTORY OF JAPANESE MATHEMATICS

We can distinguish six areas in the history of Japanese mathematics :

1) Clarification of the mathematical contents of texts.
2) Bibliographical study.
3) Biography.

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[27] Fujiwara Matsusaburou wrote a complete history, *Meijizen Nihon Suugakushi* (History of Japanese mathematics before the Meiji period, 明治前日本数学史, 1956) in 5 volumes in Japanese that is still frequently referred to, but his articles in European languages are all mathematical.

[28] His first article [Mikami 1905] was, however, on mathematics.

[29] For example, see [Mikami 1930] and Hayashi Tsuruichi, 'Seki Takakazu no Enri, I, II' (関孝和ノ円理, The Circle Principle due to Seki Takakazu, I, II), *Tokyo Butsuri Gakkou Zasshi* 469 (1930), 472 (1931) (in Japanese). One of the most important problems they concerned themselves with was whether Seki had gotten an infinite series using the circle principle.

[30] [Sasaki 1994].

4) Search for and deciphering of sangaku.
5) Social history of Japanese mathematics.
6) Mathematical study using historical material.

Understanding the mathematical contents of texts quite naturally forms the basis for any history of mathematics. There is a striking contrast between earlier Japanese mathematics and today's mathematics; the sort of problems, methods of proof and writing, manner of teaching or publishing all differ. While a mathematical approach to texts was practiced from the earliest stages of historical study, there remain many texts whose mathematical contents are obscure.[31]

Bibliographical studies are necessary because of the increasing number of books or manuscripts on Japanese mathematics. Identifying an author, dating a text, and searching for and comparing of editions – all these tasks are still important in the study of Japanese mathematics. For example, with respect to Seki Takakazu's *Hatsubi sanpou* (発微算法, 1674), the only book published during his lifetime, it was long believed that only one copy remained. Recently, however, two additional copies have been discovered, and it was proved in 1994 that there are at least two editions of the book.[32]

Biographical studies have also developed since the early 20th century. People who studied mathematics belonged to all social classes – warriors (*samurai*, 侍), farmers, craftsmen, and merchants – yet we have practically no knowledge of their lives regardless of their class.[33] For example, the birthday or birthplace of the most famous mathematician, Seki Takakazu, is unknown. Since biography is studied mainly within the framework of local history, it is primarily written in Japanese only.

The search for and deciphering of sangaku is flourishing. Most of the problems engraved in sangaku are, however, stereotyped, and there is

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[31] See bibliography for mathematical papers on Japanese mathematics, written around 1890–1920's by Fujisawa Rikitarou, Hayashi Tsuruichi, Kikuchi Dairoku, Mikami Yoshio, Shinomiya Asaji, and Yanagihara Kichiji. They may have intended to introduce Japanese mathematics to the West.

[32] Sato Ken'ichi, 'The Study of Seki Takakazu, *Hatsubi Sanpo*(1674)', *Journal of History of Science, Japan* 35 (1994), p. 179–187 (in Japanese). There are as yet no treatises on this subject written in a European language.

[33] There are some exceptions such as Takebe Katahiro. He served the Shogun or Arima Yoriyuki (1714–1783), who was a feudal lord.

no indication of a proof or a process of solution. Still, the study of sangaku represents an effort to understand the cultural history of Japanese mathematics, and there is abundant literature on it in Japan.[34]

There are, however, relatively few such social historical studies of Japanese mathematics, that is studies that aim to situate Japanese mathematics within Japanese culture as a factor in Japanese history. The analysis of the organization of pupils in a system close to iemoto, or of the custom of sangaku, constitute an attempt.[35]

Doing mathematics using historical material involves, for example, generalizations of problems in sangaku, books and manuscripts.[36] It is, in a sense, a continuation of wasan itself, and not a part of the historical study of mathematics.

5. CONCLUSION

Thus far, few, other than Japanese scholars, have devoted themselves to the history of Japanese mathematics. I would hope that some of that work might be translated into European languages. Japanese mathematics, even if ended a century ago, is nevertheless part of our common mathematical 'patrimony' and, as such, deserves to be more widely understood and appreciated.

[34] [Fukagawa and Pedoe 1989] deals with sangaku comprehensively.

[35] The principal paper to be mentioned here is 'Bunkashijou yori mitaru nihon no suugaku' (文化史上より見たる日本の数学, Japanese Mathematics Viewed in the Cultural History), *Tetsugaku Zasshi*, 37 (1922); revised edition, Tokyo : Sougensha (1947); enlarged and revised edition by Hirayama Akira, Ohya Shin'ichi and Shimodaira Kazuo, Tokyo : Kouseishakouseikaku, 1984; revised edition by Sasaki Chikara, Tokyo : Iwanami, 1999.

[36] [Yanagihara 1912, 1913] are among the earliest works.

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The following list contains treatises written in European languages, from the beginning of historical studies in 1895 to the present.[37] Relative to Hayashi and Mikami, treatises on the history of Chinese mathematics or on Dutch astronomy that might affect Japan are also included.[38]

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