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A STATISTICAL MODEL OF MESONS

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ABSTRUCT

A statistical model of mesons is discussed. The freedom of strings of mesons is interpreted as spatial equilibrium distributions of a system of interacting "particles". The classification of the meson-family is reduced to statistical and dynamical aspects of string distributions. To get the equilibrium distributions the Vlasov-McKean limit is employed and finally the problem is reduced to an eigenvalue problem. The eigenvalue problem is, however, stated in §2 as one of postulates in order to reach numerical evaluations directly. The mass-spectrum of the meson-family is computed numerically in §3. Some empirical rules, such as the mass-spin dependency and the quark-capacity of string distributions, are discussed. The mathematicl basis of the statistical model is explained in the last section.

1. INTRODUCTION

The gauge theory of Weinberg and Salam succeeded in describing the elementary processes in which electromagnetic and weak interactions are involved [1,2]. Hence we can obtain satisfactory theoretical conclusions on dynamics and classification of elementary particles appearing in this elementary process. There is, however, no well established powerful theoretical framework for hadrons which take part in elementary processes governed by the strong interaction. Although the quantum chromodynamics (QCD) is widely recognized as being a gauge theory which might describe the strong interaction, it is still on the way of development [3].

On the other hand, if we treat a hadron as a composit quantum system of several basic particles which are called quarks, we can analyse mass, spin, strangeness, etc. of baryons and mesons by purely algebraic methods which have been familiar since long time in quantum theory [4]. This model has, however, a weak point that the theory itself does not contain dynamics explicitly. There are some trials to overcome this weak point and to introduce dynamics into the composite model; the string model for mesons and the bag model of baryons [5,6]. At the same time some attempts are made to deduce the string freedom of mesons and bag freedom of baryons asymptotically from the fundamental concept of QCD, but satisfactory results have not been achieved yet.

Nevertheless, reliable is the picture of hadrons as composite particles consisting of several quarks. It seems reasonable to adopt this picture independent of whether we start from the compositve models or from QCD . This means that a statistical treatment would be required to recover, for example, the mass-spectrum of the meson-family, even when a reasonable theory of the strong interaction would be successfully well settled. It is, therefore, considered reasonable in the present stage of development to try a statistical and semi-empirical theory which does not depend on explicit form of strong interaction and which allows us to deduce intrinsic properties (mass, spin, strangeness, etc.) of hadrons from the inner motion of constituents. The string model and the bag model should be considered as notable examples of such semi-empirical theories. However, a string (or a bag), which is a fundamental concept in the model, is an intuitive and geometrical object that is already in existence mechanically, and therefore it can carry only the mechanical freedom [6]. There is still no theoretical framework which can provide a deductive construction of the spatial, geometrical and mechanical freedom of a string (or a bag). In fact, the freedom of a string or a bag is not yet deduced from QCD . Therefore it is meaningful to try to find out a (statistical) framework which can provide those freedoms and enables us numerical computation that can be compared with experiments, say, the mass-spectrum of mesons which is the fundamental experimental data.

2. A MODEL AND POSTULATES

In this section we will state a composite model as three postulates without physical explanation. A possible physical interpretation of the model will be given in § 6. Our model consists of one eigenvalue problem and a set of parameters. First of all we need: The eigenvalue problem given by

$$\frac{1}{2}\{\sigma^2\frac{\partial^2\psi}{\partial x^2}+a(\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2})\} + \{\lambda - k|x| - \varepsilon(y^2+z^2) + m_a\}\psi = 0 \ , \tag{1}$$

where $m_a = \sqrt{2a\varepsilon}$, and σk and $a\varepsilon$ will be determined empirically in comparison with experimental data [7]. The eigenvalues of (1) are given by

$$\lambda_{n,j} = \lambda_n + j \cdot m_a \ , \quad n=1,2,3,\ldots, \ j=0,1,2,\ldots, \tag{2}$$

where λ_n are the eigenvalues of

$$\frac{1}{2}\sigma^2\frac{d^2u}{dx^2} + (\lambda-k|x|)u = 0 \ , \tag{3}$$

and $j \cdot m_a$ are the eigenvalues of

$$\frac{1}{2}a(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}) + (\lambda+m_a-\varepsilon(y^2+z^2))v = 0 \ . \tag{4}$$

We will denote the corresponding eigenfunctions by

$$\psi_{n,j} = u_n \cdot v_j \ :$$

We introduce, next, the following parameters

$$m_q = m_{\bar{q}}$$

where $q = u,d,s$, or c (possibly also b). We will assume

$$m_q = m_{\bar{q}} = 0 \text{ , for } q = u \text{ and } d \tag{5}$$

and hence there remain two parameters m_s and m_c , which will be determined through comparison with experimental data.

Then we state three postulates as follows:

(P.1) We call $(q,\varphi_{n,j},\overline{q'})$ a "meson" with the mass in the rest frame

$$M_{n,j}(q,q') = \lambda_{n,j} + m_q + m_{\overline{q'}} \tag{6}$$

where $n=1,2,3,...,$ and $j=0,1,2,...,$ and put

$$\varphi_{n,j} = |\psi_{n,j}|^2 \text{ .} \tag{7}$$

$\varphi_{n,j}$ can be understood as the string degree of freedom, and therefore $\lambda_{n,j}$ as its energy; and m_q as the mass of a quark q .

(P.2) The spin of a meson $(q,\varphi_{n,j},\overline{q'})$ is the sum of the spin of q and $\overline{q'}$ and of angular momentum j of the string degree of freedom $\varphi_{n,j}$.

(P.3) The string degree of freedom does not affect the strageness and charm of mesons, that is, they are the sum of q and $\overline{q'}$ in $(q,\varphi_{n,j},\overline{q'})$.

Among the composit mesons there is the one which has the smallest mass, that is $(q,\varphi_{1,0},\bar{q}')$, where $q,q' = u$ or d, with the mass $M_{1,0}(q,q) = \lambda_1$. Since $\pi^{\pm}$ are the mesons which have the smallest mass among mesons that have relatively longer life time, we identify

$$\pi^{\pm} \text{"="} (u \text{ or } d , \varphi_{1,0} , \bar{u} \text{ or } \bar{d}) . \tag{8}$$

Therefore,

$$\lambda_1 = m(\pi^{\pm}) = 139.5669 \text{ Mev} \tag{9}$$

where $m(\pi^{\pm})$ denotes the mass of $\pi^{\pm}$, and hence we have

$$\left\{\frac{(\sigma k)^2}{2}\right\}^{1/3} = m(\pi^{\pm})/\mu_1 = 136.99236 \text{ Mev} \tag{10}$$

where μ_n is the eigenvalue of the dimensionless version

$$\frac{d^2u}{dy^2} + (\mu - |y|)u = 0 \tag{11}$$

which is obtained from (3) putting $x = \alpha y$, $\mu = \lambda(\alpha k)^{-1}$ and $\alpha = (\sigma^2/2k)^{1/3}$. Thus $\sigma \cdot k$ is determined by (10).

The eigenvalues of the equation (11) are given by zeros of [8]

$$J_{1/3}(\tfrac{2}{3}\mu^{3/2}) + J_{-1/3}(\tfrac{2}{3}\mu^{3/2})$$

and

$$J_{2/3}(\tfrac{2}{3}\mu^{3/2}) - J_{-2/3}(\tfrac{2}{3}\mu^{3/2}) .$$

The eigenvalues μ_n of (11) are given in Table 1, and the eigenvalues λ_n of (3) with σ and k determined by (10) in Table 2. The eigenvalues μ_n (and therefore λ_n) is asymptotically proportional to $n^{2/3}$. An interesting implication of this fact will be discussed in § 4 . It is natural to interprete

$$\ell_n = 2\lambda_n k^{-1} \tag{12}$$

as the effective geometrical length of the string distribution $\varphi_{n,j}$ with the energy $\lambda_{n,j}$. However, since k can not be determined separately (see (10)), it is not possible to compute ℓ_n in our model.

There remain three parameters m_s, m_c, and m_a indetermined. The parameter m_s will be fixed through comparison of $M_{n,0}$(s,u or d) with the mass of $K^{\pm}$, which are the lightest mesons that contain one s (cf. Case 2 in § 3). We will take, then

$$m_s \cong 50 \text{ Mev.} \tag{13}$$

The parameter m_c will be determined through comparison of $M_{n,0}$(c,u or d) with the mass of $D^{\pm}$, D^o, which are the lightest mesons that contain one c (see Case 37 in § 3). We will take

$$m_c \cong 700 \text{ Mev.} \tag{14}$$

The parameter m_a will be determined by the difference of the masses of $\omega(1)$ and $\rho(1)$ (cf. Case 4 and 5 in § 3) which are the lightest mesons having spin 1. Through the comparison we will take

$$m_a \cong 15 \text{ Mev.} \tag{15}$$

All parameters are now fixed. We will identify the mass-spectrum of the meson-faimly with our $M_{n,j}(q,q')$ in the next section.

3. IDENTIFICATION OF MESONS

We will use the notation "=" for the identification and denote our composit model as $(q,\varphi_{n,j},\bar{q}')(M_{n,j}(q,q'))$ with its mass.

Case 1. $\left.\begin{array}{l}\pi^{\pm}(0)(139.5669)\\ \pi^{o}(0)(134.9626)\end{array}\right\}$ "=" $(u \text{ or } d,\varphi_{1,0},\bar{u} \text{ or } \bar{d})(139.5669)$.

This identification is one of our postulates. The difference $m(\pi^{\pm}) - m(\pi^{o}) = 4.6$ Mev is neglected.

Let us consider $(q,\varphi_{n,o},\bar{q}')$, where $n \geqq 2$ and $q,q' = u$ or d . Then the distribution $\varphi_{n,0}$ has zeros. If a pair $(q,\bar{q})$ (or pairs) is (are) created by some external disturbances at one (or some) of the zeros of the distribution, then it performs a fission and creates a pair of (or several) mesons which are characterized by $\varphi_{n',0},\varphi_{n'',0}$ etc., corresponding to smaller eigenvalues $\lambda_{n'},\lambda_{n''}$ etc. For example, because

$$\lambda_2 > 2\lambda_1$$

the following reaction

$$(u,\varphi_{2,0},\bar{u}) \rightarrow (u,\varphi_{1,0},\bar{u}) + (\bar{u},\varphi_{1,0},u)$$

takes place through a pair creation at the zero of $\varphi_{2,0}$. Therefore $(q,\varphi_{n,0},\bar{q}')$, $q,q' = u$ or d , $n \geqslant 2$, does not exist in principle. For an exceptional case, see Case 18.

Case 2. $\left.\begin{matrix} K^{\pm}(0)(493.67) \\ K^{o}(0)(497.67) \end{matrix}\right\}$ "=" $\left\{\begin{matrix} (u \text{ or } d,\varphi_{3,0},\bar{s})(495) \\ (\bar{u} \text{ or } \bar{d},\varphi_{3,0},s)(495) \end{matrix}\right.$.

This identification is one of our postulates by which m_s is determined. It should be remarked that we can not identify $K^{\pm}$, K^{o} with $(s,\varphi_{1,0},\bar{u}$ or $\bar{d})$, because, if do so, we must take m_s = 354 Mev (cf. (6)) and then identify $\eta(549)$ with $(s,\varphi_{1,0},\bar{s})(847)$. This is not possible. If we identify $K^{\pm}$, K^{o} with $(s,\varphi_{2,0},\bar{u}$ or $\bar{d})$ and take m_s = 175 Mev, then we must identify η with $(s,\varphi_{1,0},\bar{s})(490)$ or $(s,\varphi_{2,0},\bar{s})(670)$. This identification will not meet better results and is not accepted. $K^{\pm}$, K^{o} are relatively stable, because they are the smallest meson which contain one s .

Case 3. $\eta(0)(549)$ "=" $(s,\varphi_{3,0},\bar{s})(545)$.

η is relatively stable, because it is the smallest meson which contains s and $\bar{s}$, and $m_{\eta} < 2m_K$.

Case 4. $\rho(1)(769)$ "=" $(u$ or $d,\varphi_{6,0},\bar{u}$ or $\bar{d})(756)$.

By this identification

$$m_a = \omega(1) - \rho(1) \cong 15 \text{ MeV}$$

is chosen as the difference of the mass of $\omega(1)$ and $\rho(1)$. We interprete that the spin unity of $\rho(1)$ is from quarks and the spin unity of $\omega(1)$ from the angular momentum of the string distribution. These cases will be discussed again in § 4.

<u>Case 5.</u> $\omega(1)(783)$ "=" $(u,\varphi_{6,1},\bar{u}),(d,\varphi_{6,1},\bar{d})(771)$.

<u>Case 6.</u> $K^*(1)(892)$ "=" $(s,\varphi_{7,1},\bar{u}$ or $\bar{d}),(u$ or $d,\varphi_{7,1},\bar{s})(894)$.

We interprete that the spin unity of K^* is from the pair of quarks. This case will be discussed again in Section 4.

<u>Case 7.</u> $\eta'(0)(958)$ "=" $(s,\varphi_{7,0},\bar{s})(944)$.

$\eta'(0)$ is relatively stable, because $m_{\eta'} < 2m_K$ and the decay

$$(s,\varphi_{7,0},\bar{s}) \rightarrow (s,\varphi_{3,0},\bar{u} \text{ or } \bar{d}) + (u \text{ or } d,\varphi_{3,0},\bar{s})$$

thorugh a pair creation of $(u,\bar{u})$ or $(d,\bar{d})$ is not possible.

<u>Case 8.</u> $S^*(0)(975)$ and $\delta(0)(980)$ are not well identified with our composite model $(q,\varphi_{n,0},\bar{q}')$ [9].

<u>Case 9.</u> $\Phi(1)(1020)$ "=" $(s,\varphi_{8,0},\bar{s})(1030)$.

Since $m_\Phi > 2m_K$, the decay through a pair creation of $(u,\bar{u})$ or $(d,\bar{d})$

$$(s,\varphi_{8,0},\bar{s}) \rightarrow (s,\varphi_{3,0},\bar{u} \text{ or } \bar{d}) + (u \text{ or } d,\varphi_{3,0},\bar{s})$$

is possible. This is observed in the decay mode of Φ .

<u>Case 10.</u> $H(1)(1190)$ "=" $(u,\varphi_{11,1},\bar{u}),(d,\varphi_{11,1},\bar{d})(1178)$.

<u>Case 11.</u> $B(1)(1233)$ "=" $(u$ or $d,\varphi_{12,0},\bar{u}$ or $\bar{d})(1236)$.

We interprete that the spin unity of B is from quarks (cf. Case 18).

Case 12. $\rho'(1)(1250)$ is not a well-established resonance. It may be identified with $(u \text{ or } d, \varphi_{12,1}, \bar{u} \text{ or } \bar{d})(1251)$.

Case 13. $Q_1(1)(1270)$ "=" $(s, \varphi_{12,0}, \bar{u} \text{ or } \bar{d})$ or $(u \text{ or } d, \varphi_{12,0}, \bar{s})(1286)$.

Case 14. $f(2)(1273)$ "=" $(u, \varphi_{12,2}, \bar{u}), (d, \varphi_{12,2}, \bar{d})(1266)$.

This case will be discussed in § 4 in connection with its spin.

Case 15. $A_1(1)(1275)$ "=" $(u \text{ or } d, \varphi_{12,1}, \bar{u} \text{ or } \bar{d})(1251)$.

Case 16. $\eta(0)(1275)$ is not a well-established resonance. It may be identified with $(s, \varphi_{11,0}, \bar{s})(1263)$.

Case 17. $D(1)(1283)$ "=" $(s, \varphi_{11,1}, \bar{s})(1278)$.

Case 18. $\varepsilon(0)(1300)$ "=" $(u, \varphi_{13,0}, \bar{u}), (d, \varphi_{13,0}, \bar{d})(1306)$.

To adopt this (exceptional) identification, we remark the following interesting inequalities: There is a critical number $n = 13$ such that

$$\begin{aligned} \lambda_n < \lambda_{n+1} < \lambda_1 + \lambda_{n-1} , \quad \text{for} \quad n \geqq 13 , \\ \lambda_n < \lambda_1 + \lambda_{n-1} < \lambda_{n+1} , \quad \text{for} \quad n \leqq 12 . \end{aligned} \tag{16}$$

This implies that, if $n \geqq 13$, and if the energy of the composite meson fluctuates, the next string distribution φ_{n+1} can be attained before performing fission into $\varphi_1 + \varphi_{n-1}$. However, for $n \leqq 12$, the fission into $\varphi_1 + \varphi_{n-1}$ occurs first. This indicates that (u or d,$\varphi_{n,0}$,$\bar{u}$ or $\bar{d}$) , $n \geqq 13$, is more probable to be observed as a mesonic resonance than those of $n \leqq 12$. This arguments can be applicable for fissions which involve u,d , whose mass is neglisible.

<u>Case 19.</u> $\pi(0)(1300)$ is not a well-established resonance. It may be identified with (u or d,$\varphi_{13,0}$,$\bar{u}$ or $\bar{d}$)(1306) .

<u>Case 20.</u> $A_2(2)(1318)$ "=" (u or d,$\varphi_{13,1}$,$\bar{u}$ or $\bar{d}$)(1321) .

This case will be discussed in Section 4 in connection with its spin.

<u>Case 21.</u> $\kappa(0)(1350)$ "=" (s,$\varphi_{13,0}$,$\bar{u}$ or $\bar{d}$), (u or d,$\varphi_{13,0}$,$\bar{s}$)(1356) .

<u>Case 22.</u> K'(0)(1400) is not a well-established resonance, but may be identified as in the Case 23.

<u>Case 23.</u> $Q_2(1)(1414)$ "=" (s,$\varphi_{14,0}$,$\bar{u}$ or $\bar{d}$), (u or d,$\varphi_{14,0}$,$\bar{s}$)(1425) .

<u>Case 24.</u> E(1)(1418) "=" $\begin{cases} (s,\varphi_{13,0},\bar{s})(1406) & \text{or} \\ (s,\varphi_{13,1},\bar{s})(1421) . \end{cases}$

This ambiguity comes form the fact that spin unity can be contributed either by quarks or by angular momentum of the string distribution.

Case 25. $K^*(2)(1434)$ "=" $(s,\varphi_{14,2},\bar{u}$ or $\bar{d})$, (u or $d,\varphi_{14,2},\bar{s})(1440)$.

This case will be discussed again in Section 4 in connection with its spin 2 .

Case 26. $f'(2)(1520)$ "=" $(s,\varphi_{14,2},\bar{s})(1505)$.

This case with spin 2 will be discussed again in Section 4.

Case 27. $L(2)(1580)$ is not a well-established resonance, but may be identified with $(s,\varphi_{16,1},\bar{u}$ or $\bar{d})(1573)$.

Case 28. $\rho'(1)(1600)$ "=" (u or $d,\varphi_{17,1},\bar{u}$ or $\bar{d})(1587)$.

Case 29. $\theta(2)(1640)$ is not a well-established resonance. It may be identified with $(u,\varphi_{18,1},\bar{u}),(d,\varphi_{18,1},\bar{d})(1650)$ or $(s,\varphi_{16,2},\bar{s})(1638)$.

Case 30. $K^*(1)(1650)$ is not a well-established resonance, but may be identified with $(s,\varphi_{17,1},\bar{u}$ or $\bar{d})(1637)$.

Case 31. $\omega(3)(1668)$ "=" $(u,\varphi_{18,2},\bar{u})$, $(d,\varphi_{18,2},\bar{d})(1665)$.

This case will be discussed in Section 4 in connection with their spin 3 .

<u>Case 32.</u> $A_3(2)(1680)$ "=" $(u \text{ or } d, \varphi_{18,2}, \bar{u} \text{ or } \bar{d})(1665)$.

<u>Case 33.</u> $\phi'(1)(1684)$ "=" $(s, \varphi_{17,0}, \bar{s})(1672)$, or with $\varphi_{17,1}(1687)$.

<u>Case 34.</u> $g(3)(1691)$ "=" $(u \text{ or } d, \varphi_{18,3}, \bar{u} \text{ or } \bar{d})(1680)$.

This case will be discussed in § 4 in connection with its spin 3 .

<u>Case 35.</u> $L(2)(1770)$ "=" $(s, \varphi_{19,1}, \bar{u} \text{ or } \bar{d})$, $(u \text{ or } d, \varphi_{19,1}, \bar{s})(1760)$ or with $\varphi_{19,2}(1777)$.

<u>Case 36.</u> $K^*(3)(1775)$ "=" $(s, \varphi_{19,2}, \bar{u} \text{ or } \bar{d})$, $(u \text{ or } d, \varphi_{19,2}, \bar{s})(1777)$.

This case will be discussed in Section 4 in connection with its spin 3 .

<u>Case 37.</u> $\phi(1850)$ is not a well-established resonance. We postpone identifying it with our composite model.

<u>Case 38.</u> $X(2)(1850)$ is not a well-established resonance. It may be identified with $(u \text{ or } d, \varphi_{21,2}, \bar{u} \text{ or } \bar{d})(1847)$, or $(s, \varphi_{19,2}, \bar{s})(1827)$.

<u>Case 39.</u> $$\left.\begin{array}{l} D^0(0)(1864) \\ D\ (0)(1869) \end{array}\right\} \text{"="}\ (c, \varphi_{11,0}, \bar{u} \text{ or } \bar{d}), (u \text{ or } d, \varphi_{11,0}, \bar{c})(1863)\ .$$

This identification is one of our postulates through which $m_c = 700$ Mev is obtained.

<u>Case 40.</u> S(1935) is not a well-established resonance. We postpone identifying it with our composite model.

<u>Case 41.</u> $\left.\begin{array}{l} D^{*o}(1)(2007) \\ D^{*\pm}(1)(2010) \end{array}\right\}$ "=" $(c,\varphi_{13,0},\bar{u}$ or $\bar{d})$, $(u$ or $d,\varphi_{13,0},\bar{c})$ (2006) .

We assume that the spin unity of D^{*o} and $D^{*\pm}$ is of quarks.

<u>Case 42.</u> F(0)(2021) is not well-identified with our composite model. If we would adopt the mixture of quark states, then it could be identified with $(c,\varphi_{13,0},(\bar{s}+\bar{u}$ or $\bar{d})/2)$(2031) .

<u>Case 43.</u> $\delta(4)$(2030) is not a well-established resonance. It may be identified with $(u$ or $d,\varphi_{24,3},\bar{u}$ or $\bar{d})$(2035) .

<u>Case 44.</u> h(4)(2040) "=" $(u,\varphi_{24,3},\bar{u})$, $(d,\varphi_{24,3},\bar{d})$(2035) .

This case will be discussed in § 4 in connection with its spin 4 .

<u>Case 45.</u> $\pi(3)$(2050) is not a well-established resonance. It may be identified with $(u$ or $d,\varphi_{24,3},\bar{u}$ or $\bar{d})$(2035) .

<u>Case 46.</u> K*(4)(2060) is not a well-established resonance. It may be identified with $(s,\varphi_{23,4},\bar{u}$ or $\bar{d})$(2048) .

<u>Case 47.</u> $\pi(2)(2100)$ is not a well-established resonance. It may be identified with $(u \text{ or } d, \varphi_{26,1}, \bar{u} \text{ or } \bar{d})(2117)$.

<u>Case 48.</u> $F^*(2140)$ is not a well-established resonance. It may be identified with $(c, \varphi_{14,1}, \bar{s})(2140)$.

<u>Case 49.</u> $\rho(1)(2150)$ is not a well-established resonance. It may be identified with $(u \text{ or } d, \varphi_{27,0}, \bar{u} \text{ or } \bar{d})(2156)$.

<u>Case 50.</u> $\varepsilon(2)(2150)$ is not a well-established resonance. It may be identified with $(u, \varphi_{27,1}, \bar{u}), (d, \varphi_{27,1}, \bar{d})(2171)$ or $(s, \varphi_{25,1}, \bar{s})(2161)$.

<u>Case 51.</u> $K^*(2200)$ is not a well-established resonance. We postpone identifying it with our composite model.

<u>Case 52.</u> $\rho(3)(2250)$ is not a well-established resonance. It may be identified with $(u \text{ or } d, \varphi_{28,3}, \bar{u} \text{ or } \bar{d})(2255)$.

<u>Case 53.</u> $(4)(2300)$ is not a well-established resonance. It may be identified with $(u, \varphi_{29,3}, \bar{u}), (d, \varphi_{29,3}, \bar{d})(2308)$, or $(s, \varphi_{27,3}, \bar{s})(2301)$.

<u>Case 54.</u> $\rho(5)(2350)$ is not a well-established resonance. It may be identified with $(u \text{ or } d, \varphi_{29,5}, \bar{u} \text{ or } \bar{d})(2338)$.

<u>Case 55.</u> $\delta(6)(2450)$ is not a well-established resonance. It may be identified with $(u \text{ or } d, \varphi_{31,6}, \bar{u} \text{ or } \bar{d})(2458)$.

Case 56. e^+e^-(1100-2200), $\bar{N}N$(1400-3600) and X(1900-3600) are not possible to identify.

Case 57. η_c(2981) "=" $(c,\varphi_{17,0},\bar{c})$(2972) .

See § 4. We assume that its spin is zero.

Case 58. J/ψ(1)(3097) "=" $(c,\varphi_{19,0},\bar{c})$(3097) .

Case 59. χ(0)(3415) "=" $(c,\varphi_{24,0},\bar{c})$(3390) . See Section 4.

Case 60. P_c or χ(1)(3510) "=" $(c,\varphi_{26,0},\bar{c})$(3502) , or with $\varphi_{26,1}$(3517).

Case 61. χ(2)(3556) "=" $(c,\varphi_{27,1},\bar{c})$(3571) .

Case 62. η_c'(3590) is not a well-established resonance. It may be identified with $(c,\varphi_{28,0},\bar{c})$(3610) or $(c,\varphi_{27,2},\bar{c})$(3586) . See § 4.

Case 63. ψ(1)(3686) "=" $(c,\varphi_{29,1},\bar{c})$(3678) .

Case 64. ψ(1)(3770) "=" $(c,\varphi_{31,0},\bar{c})$(3768) .

Case 65. ψ(1)(4030) "=" $(c,\varphi_{36,0},\bar{c})$(4020) , or with $\varphi_{36,1}$(4035) .

Case 66. ψ(1)(4159) "=" $(c,\varphi_{39,0},\bar{c})$(4166) .

Case 67. ψ(1)(4415) "=" $(c,\varphi_{44,1},\bar{c})$(4415) .

<u>Case 68.</u> $\gamma(1)(9456)$, $\gamma(1)(10016)$, $\gamma(1)(10347)$, and $\gamma(1)(10569)$ will be discussed in § 5.

4. SPIN OF MESONS

4.1 A relation between spin and mass of mesons

Let us consider a set $SD = \{\rho(1), \omega(1), f(2), A_2(2), g(3), \omega(3), h(4)\}$. We write simply $(\cdot, \varphi_{n,j}, \cdot)$ for $(q, \varphi_{n,j}, \bar{q})$, $q = u$ or d . Then

$$
\begin{aligned}
\rho(1) \quad &\text{"="} \quad (\cdot, \varphi_{6,0}, \cdot) \\
\omega(1) \quad &\text{"="} \quad (\cdot, \varphi_{6,1}, \cdot) \\
f(2) \quad &\text{"="} \quad (\cdot, \varphi_{12,2}, \cdot) \\
A_2(2) \quad &\text{"="} \quad (\cdot, \varphi_{13,1}, \cdot) \\
\omega(3) \quad &\text{"="} \quad (\cdot, \varphi_{18,2}, \cdot) \\
g(3) \quad &\text{"="} \quad (\cdot, \varphi_{18,3}, \cdot) \\
h(4) \quad &\text{"="} \quad (\cdot, \varphi_{24,3}, \cdot)
\end{aligned}
\tag{17}
$$

The above identification of the set of mesons suggest us an empirical rule, namely, if we denote by n_J the suffix of the string distribution corresponding to mesons with total spin J , then we have

$$n_{J+1} - n_J \cong 6 \ , \quad J = 1,2,3 \ . \tag{18}$$

For the set $S = \{K^*(1), K^*(2), K^*(3)\}$ of strange mesons, we have

$$
\begin{aligned}
K^*(1) \quad &\text{"="} \quad (\cdot, \varphi_{7,0}, \cdot) \\
K^*(2) \quad &\text{"="} \quad (\cdot, \varphi_{14,1}, \cdot) \\
K^*(3) \quad &\text{"="} \quad (\cdot, \varphi_{19,2}, \cdot)
\end{aligned}
\tag{19}
$$

where an obvious abbreviation of notation is employed. Here we see the same rule (18) holds.

For $S\bar{S} = \{\phi(1), f'(2)\}$ of $(s,\bar{s})$-mesons, we have

$$\begin{aligned} \phi(1) \ \text{"="} \ & (s, \varphi_{8,0}, \bar{s}) \\ f'(2) \ \text{"="} \ & (s, \varphi_{14,2}, \bar{s}) \end{aligned} \tag{20}$$

where the same rule (18) holds.

Analysis of the above three sets of mesons indicates us an empirical rule: The suffix n of distribution $\varphi_{n,j}$ increases by the number six to get each additional spin one. If we denote by M_J the mass of a meson with the total spin J which is the sum of spin j of angular momentum of the string distribution and that of quarks, then we have a mass-spin formula

$$M_J = \lambda_{J_0+6(J-1)} + j \cdot m_a + m_q + m_{\bar{q}'} \tag{21}$$

where $J_0 = 6$ for the set UD , $J_0 = 7$ for the set S and $J_0 = 8$ for the set $S\bar{S}$. If we combine (21) with

$$\alpha_n \lambda_n = n^{2/3} \qquad (\lambda_n \ : \text{Gev}) \tag{22}$$

which is remarked in § 2, then it is straightforward to get

$$J = -(J_0/6-1) + \beta(M_J - j \cdot m_a - m_q - m_{\bar{q}'})^{3/2} \tag{23}$$

where $\beta = (1/6)(\alpha_n)^{3/2}$. For example, $\beta_6 = 1.520$, $\beta_8 = 1.487$, $\beta_{12} = 1.455$, $\beta_{18} = 1.435$, $\beta_{24} = 1.424$, $\beta_{100} = 1.402$, and $\beta_{200} = 1.398$. For the set UD , neglecting $j \cdot m_a$, we have

$$J = 1.43(M_J)^{3/2} \qquad (M_J : \text{Gev}) \tag{24}$$

as a good approximation. Our result differs from the conventional Regge rule, say, $J = 0.5 + 0.86(M_J)^2$.
(Notice that our potential in (1) is not rotation invariant in R^3 .) [10]

4.2 <u>On C-mesons and (c,c)-mesons</u>

The number of c-mesons is not so many as the other kinds of mesons. If the same arguments in § 4.1 should be applied to c-mesons, it would be natural to expect the existence of other c-mesons (and mesonic resonances). If we apply the formula (21) to c-mesons, then $J_o = 13$ and a c-meson of spin two with the mass

$$M_2(c) = \lambda_{13+6} + m_c + j \cdot m_a \qquad j = 1 \text{ or } 2 ,$$

may exist, i.e. a composite meson

$$(c, \varphi_{19,j}, \bar{u} \text{ or } \bar{d})(2397 + j \cdot m_a) \qquad j = 1 \text{ or } 2 .$$

Since a slight deviation from the formula (21) might occur, the following should also be considered:

$$(c,\varphi_{18,j},\bar{u} \text{ or } \bar{d})(2335 + j\cdot m_a) \qquad j = 1 \text{ or } 2$$

and

$$(c,\varphi_{20,j},\bar{u} \text{ or } \bar{d})(2457 + j\cdot m_a) \qquad j = 1 \text{ or } 2 \ .$$

As for $(c,\bar{s})$-meson, there is $F(0)(2021)$. Let us assume $F^*(2140)$ "=" $(c,\varphi_{14,1},c)$ and with spin unity, then the formula (21) implies, because $J_0 = 14$,

$$M_2(c,\bar{s}) = \lambda_{14+6} + m_c + m_s + jm_a \qquad j = 1 \text{ or } 2 \ ,$$

i.e. we expect the existence of a mesonic resonance

$$(c,\varphi_{20,j},\bar{s})(2507 + jm_a) \qquad j = 1 \text{ or } 2 \ .$$

Since a slight deviation from the formula (21) might occur, the following should be considered also;

$$(c,\varphi_{19,j},\bar{s})(2447 + jm_a) \qquad j = 1 \text{ or } 2 \ ,$$

and

$$(c,\varphi_{20,j},\bar{s})(2567 + jm_a) \qquad j = 1 \text{ or } 2 \ .$$

In the case 57 of § 3, we have identified $\eta_c(2981)$ with $(c,\varphi_{17,0},\bar{c})(2971)$ with spin zero. If this is the case, then $J_0 = 19$ and it might appear a composite particle of spin two with the mass

$$M_2(c,\bar{c}) = \lambda_{19+6} + 2m_c + jm_a \qquad j = 1 \text{ or } 2 .$$

In the case 62, it is probable that

$$\eta_c'(3590) \quad "=" \quad (c,\varphi_{27,2},\bar{c})(3586)$$

and spin of η_c' is two. Moreover, in Case 59, $(c,\varphi_{24,1},\bar{c})$ (or with $\varphi_{24,2}$) is expected in place of $(c,\varphi_{24,0},\bar{c})$.

5. QUARK-CAPACITY OF STRING DISTRIBUTIONS

When we identified our composite models with the family of mesons in Section 3, we observed that not all of "possible" composite particles were found as real mesons and mesonic resoncances. It seems that there are some prohibited states. We will attempt to find out some empirical rules in connection with this in the following.

Each of quarks which have different masses seems to have a proper accessible range of inner energy which is carried by a string distribution. As explained in Case 18 in Section 3, the distribution density $\varphi_{13,0}$ represents a threshold value in the sense that distributions which are smaller than $\varphi_{13,0}$ (except φ_1) are unstable if they have no angular momentum. However, those distributions become relatively stable, if they are coupled with s-quark. This indicates, in other words, that there exists an intrinsic capacity of string distributions against different kind of quarks. Our synthesis in Section 3 tells us that inner energy carried by a string distribution should be large enough to accept a heavier quark (or quarks). The u,d-quarks which have negligible mass may be coupled with all distributions. However, s-quark can be coupled with only those distributions larger than $\varphi_{3,0}$. The c-quark can not be coupled with a distribution unless it is larger than $\varphi_{11,0}$. Moreover, when two of heavy quarks are coupled with a string distribution, it has higher inner energy than in the case of with one quark of the same kind.

Actually, if we observe the mass-spectrum of mesons, we find that above 1600 Mev the frequency of mesonic resonances which contain only u,d or s-quarks decreases, that is, string distributions $\varphi_{n,j}$, $n \geqq 17$ ($\varphi_{17,0}$ has the string mass of 1572 Mev) compose (u,d,s)-mesons only with angular momentum. As if compensating this, the string distributions $\varphi_{n,j}$, $n \geqq 17$, obtain the capacity for $(c,\bar{c})$-quarks and are relaized rather as $(c,\bar{c})$-mesons $(c,\varphi_{17,0},\bar{c})$ "=" η_c , $(c,\varphi_{19,0},\bar{c})$ "=" J/ψ , $(c,\varphi_{24,0},\bar{c})$ "=" χ , etc. Therefore, in the mass-spectrum of the family of mesons there appears a wide gap of the order of $2m_c = 1400$ Mev caused by the mass of c-quarks. This implies that mesonic resonances with larger angular momentum might be observed in the range of 1600 - 3000 Mev (except c-mesons), and $(c,\bar{c})$-mesons appear above this level.

The identification in Section 3 seems to suggest us an interesting rule: let us denote a quark ($\neq$ u,d) by q . Let $(q,\varphi_{n(2),0},\bar{q})$ be the one identified with the smallest meson of this type, and let $(q,\varphi_{n(1),0},\bar{u}$ or $\bar{d})$ or (u or $d,\varphi_{n(1),0},\bar{q})$ be the one identified with the smallest meson of these types. Then

$$n(1) \geqq [\frac{n(2)}{2}] + 2 \text{ (or 3)} , \qquad (25)$$

where [a] denotes the largest integer smaller than a .

According to this empirical rule, the distributions $\varphi_{1,0}$

and $\varphi_{2,0}$ can not be coupled with an s-quark, i.e., they have no s-capacity. In fact, the smallest $(s,\varphi_{3,0},\bar{s})$ is η and $n(2) = 3$. Therefore we have $n(1) \geqq [3/2] + 2 = 3$, and hence $(s,\varphi_{3,0},\bar{u}$ or $\bar{d})$ and $(u$ or $d,\varphi_{3,0},\bar{s})$ that are identified with $K^{\pm}$ and K^{o} are the smallest strange mesons.

Applying the same rule to c-quarks, we find the smallest $(c,\varphi_{17,0},\bar{c})$, which is identified with $\eta_c(2981)$, that is, $n(2) = 17$, and hence we have $n(1) \geqq 11$. This means that string distributions larger than $\varphi_{11,0}$ have c-capacity. In fact $(c,\varphi_{11,0},\bar{u}$ or $\bar{d})$ and $(u$ or $d,\varphi_{11,0},\bar{c})$ which are identified with $D^{\pm}$ and D^{o} are the smallest charm mesons.

Finally using the rule (25) let us predict possible masses of b-mesons. $\gamma(9456)$, $\gamma(10016)$, $\gamma(10347)$ and $\gamma(10569)$ are mesons which contain b-quarks. Assuming that all of the four mesonic resonances are composite particles of the form $(b,\varphi_{n,0},\bar{b})$, we first determine the mass of b-quark and suffix n . Some possible values are shown in Table 3. The best among them is the case that we take

$$m_b = 3650 \text{ Mev} , \tag{26}$$

and

$$\begin{aligned} \gamma(\ 9456) \quad &"=" \quad (b,\varphi_{27,0},\bar{b})(9456) \\ \gamma(10016) \quad &"=" \quad (b,\varphi_{38,0},\bar{b})(10018) \\ \gamma(10347) \quad &"=" \quad (b,\varphi_{45,0},\bar{b})(10346) \\ \gamma(10569) \quad &"=" \quad (b,\varphi_{50,0},\bar{b})(10570) \ . \end{aligned} \tag{27}$$

The other probable combinations are, though not the best,

$$m_b = 2840 \text{ Mev} \qquad (\text{resp. } m_b = 2214) \tag{28}$$

and

$$\begin{aligned}
&\gamma(\ 9456)\ \text{"="}\ (\cdot,\varphi_{62,0},\cdot)(9459) \quad ((\cdot,\varphi_{95,0},\cdot)(9461)) \\
&\gamma(10016)\ \text{"="}\ (\cdot,\varphi_{76,0},\cdot)(10013) \quad ((\cdot,\varphi_{111,0},\cdot)(10014)) \\
&\gamma(10347)\ \text{"="}\ (\cdot,\varphi_{85,0},\cdot)(10351) \quad ((\cdot,\varphi_{121,0},\cdot)(10346)) \\
&\gamma(10569)\ \text{"="}\ (\cdot,\varphi_{91,0},\cdot)(10570) \quad ((\cdot,\varphi_{128,0},\cdot)(10573)) \ .
\end{aligned} \tag{29}$$

If we assume these combinations and apply the rule (25), we can predict a lower bound for the mass of a meson which contains only one b-quark. Since $[27/2] + 2 = 15$, if we assume $m_b = 3650$ Mev, it would be [11]

$$(b,\varphi_{15,0},\bar{u} \text{ or } \bar{d})(5092) \ . \tag{30}$$

If $m_b = 2840$ Mev (resp. 2214 Mev) is assumed, the smallest b-meson would be

$$(b,\varphi_{33,0},\bar{u} \text{ or } \bar{d})(5310) \quad (\text{resp. } (b,\varphi_{49,0},\bar{u} \text{ or } \bar{d})(5440)) \ . \tag{31}$$

This analysis implies that mesonic resonances which contain only one b-quark can appear above 5000-5500 Mev .

6. A STATISTICAL MODEL OF SYSTEMS OF INTERACTING PARTICLES [12]

6.1 A statistical model

We consider in the following particles moving in one-dimensional space without boundary to make the statement simple and clear, although the model can be formulated for those in a Riemannian space of higher dimensions with boundary [13]. A system of interacting particles $X^1, X^2, \ldots, X^N$ which we consider is given by the following system of stochastic differential equations [14]:

$$dX^i = \sigma dw^i + \{U(X^i) + \frac{1}{N}\sum_{j=1}^{N} h(X^i - X^j)\}dt \ , \ i=1,2,\ldots,N \tag{32}$$

where $\{w^i;\ i=1,2,\ldots,N\}$ are independent one-dimensional Wiener processes (Brownian motion), $\sigma > 0$, $U(x)$ is a microscopic drift field, and $h(x-y)$ is a pair-interaction between particles. If we define the empirical distributions of particles $X^1, X^2, \ldots, X^N$ by

$$\mu_N(dy) = \frac{1}{N}\sum_{j=1}^{N} \delta_{X^j}(dy) \tag{33}$$

where δ_X denotes the one-point distribution at X ,then the equation (32) can be written as

$$dX^i = \sigma dw^i + \{U(X^i) + \int h(X^i - y)\mu_N(dy)\}dt, \ i=1,2,\ldots,N \ . \tag{34}$$

Under some regularity conditions on $U(x)$ and $h(x)$, McKean proved a "law of large numbers", that is, there exists the limit

$$\mu_N \to \mu \quad (N \to \infty) \ . \tag{35}$$

We will call it "Vlasov-McKean limit", and assume the existence of the limit in the following. Then we obtain from (34), as the limit $N \to \infty$, the following stochastic differential equation which describes the movement of a single particle:

$$dX = \sigma dw + \{U(X) + \int h(x-y)\mu(dy)\}dt \ . \tag{36}$$

We will call this single particle which is governed by (36) "typical particle" of the observing system, because the distribution of the single particle X and the spatial distribution of the system of particles coincide in the limit $N \to \infty$.

It is easy to see, applying Itô's formula of stochastic calculus [14], that the distribution μ of the typical particle X satisfies the following Vlasow-McKean equation

$$\frac{\partial\langle f,\mu\rangle}{\partial t} = \langle \frac{1}{2}\sigma^2 \frac{d^2 f}{dx^2} + \{U + \int h(\cdot - y)\mu(dy)\} \frac{df}{dx} \ , \ \mu \rangle \tag{37}$$

where f is a test function and $\langle f,\mu\rangle = \int f d\mu$. Although the equation (37) is in general a non-linear evolution equation of μ , we assume that the distribution μ turns out to be stationary, and that the distribution has the densitiy φ , i.e. $\mu(dx) = \varphi(x)dx$. Then the typical particle X is not anymore a "non-linear diffusion process", but a one-dimensional diffusion process with the drift coefficient

$$b(x) = U(x) + \int h(x-y)\varphi(y)dy \tag{38}$$

and with the stationary distribution density φ ; in other words, the transition probability density p of the diffusion process is the fundamental solution of the diffusion equation

$$\frac{\partial p}{\partial t} = \frac{1}{2}\sigma^2 \frac{\partial^2 p}{\partial x^2} + b(x)\frac{\partial p}{\partial x^2} \tag{39}$$

and the stationary distribution density φ of the process satisfies the Fokker-Planck equation

$$\frac{1}{2}\sigma^2 \frac{d^2\varphi}{dx^2} - \frac{d}{dx}(b\varphi) = 0 \tag{40}$$

from which we obtain the relation between the drift coefficient b and the distribution density φ ,

$$b = \frac{1}{2}\sigma^2\varphi^{-1}\frac{d\varphi}{dx} \tag{41}$$

which is called "Kolmogoroff's relation" in the contexts of time reversibility [16]. Combining (38) and (41) we obtain an important relation which connects a macroscopic quantity (the <u>spatial equilibrium density</u> φ of the system) and microscopic quantities (the <u>microscopic drift field</u> $U(x)$ and <u>pair-interaction</u> $h(x)$, see (32)):

$$\frac{1}{2}\sigma^2 \frac{1}{\varphi}\frac{d\varphi}{dx}(x) = U(x) + \int h(x-y)\varphi(y)dy \, . \tag{42}$$

If we assume that a distribution density φ (and σ) is given, then the equation (42) is an integral equation of U and h (conversely if U and h are given in advance, one can obtain

φ solving (42)). We will assume, in this paper, the distribution φ is given. To obtain U and h we must specify microscopic models further on. We will discuss this point later.

Let us assume now that the distribution density φ is given by

$$\varphi = |\psi|^2 \tag{43}$$

where ψ is an eigenfunction of

$$\frac{1}{2}\sigma^2 \frac{d^2\psi}{dx^2} + (\lambda - V(x))\psi = 0 \ . \tag{44}$$

Thus, corresponding to an eigenfunction ψ of (44), we have a diffusion process (= typical particle) with the drift coefficient $b(x)$ determined by (41) and a system of interacting particles given by (32) and (42). It should be noticed that we need not interprete (44) as the Schrödinger equation in quantum mechanics. As we mentioned above we interprete the equation (44) as an equation which describes through (43) and (42) the <u>macroscopic behaviour</u> of a system of interacting particles and at the same time that of <u>a typical particle</u> of the system. If we interprete the equation (44) as the Schrödinger equation of quantum mechanics, then the statistical model which we described above throws fresh light to the Schrödinger equation [17].

Let us assume that (44) has descrete eigenvalues

$$0 < \lambda_1 < \lambda_2 < \lambda_3 < \dots ,$$

and denotes the corresponding eigenfunctions by $\psi_1, \psi_2, \ldots$. For the ground state with λ_1 , the eigenfunction ψ_1 has no zero, and the corresponding drift coefficient given by (41) has no singularity. Therefore we can construct a diffusion process which has $\varphi_1 = |\psi_1|^2$ as the stationary distribution density. Since ψ_k $(k \geq 2)$ has zeros, the corresponding drift coefficient diverges at the nodal points of ψ_k . Nevertheless we can construct a diffusion process with this singular drift coefficient and prove that the diffusion process will never move across over the nodal points [18].

In order to go into the microscopic picture, we need $U(x)$ and $h(x)$ determined by (42). Let us assume $U \equiv 0$ (the first model). Then the equation (42) turns out to be

$$\frac{1}{2}\sigma^2 \frac{1}{\varphi}\frac{d\varphi}{dx} = \int_{-\infty}^{\infty} h(x-y)\varphi(y)dy \ . \tag{45}$$

For the ground state the equation (45) is soluble, because the left hand side of (45) has no singularity. In the excited states the equation (45) is, however, not soluble because of the singularity which appears in the left hand side. To overcome this difficulty we modify the microscopic model as follows [19]. For simplicity let ψ have only one node at the origin. The singularity of the left hand side of (45) is, then, of $1/x$. We decompose the drift coefficient $b(x)$ into two parts, say, $b_1(x)$ and $b_2(x)$, where $b_1(x)$ has no singularity and $b_2(x)$ has the singularity of $1/x$. Then put

$$b_1(x) = \int_{-\infty}^{\infty} h_1(x-y)\varphi(y)dy \tag{46}$$

$$b_2(x) = \begin{cases} \int_{-\infty}^{0} h_2(x-y)\varphi(y)dy\ , & \text{if} \quad x > 0 \\ -\int_{0}^{\infty} h_2(x-y)\varphi(y)dy\ , & \text{if} \quad x < 0\ . \end{cases} \tag{47}$$

After this modification we can solve the equation (46) and (47) [20]. This means that particles in the system have an inner degree of freedom, say, colour, and "red" particles are distributed on the left of "blue" particles. The $h_2(x)$ which is a repulsive interaction works only between particles with different colours, while the interaction $h_1(x)$ works among all particles.Thus the particles in the system are segregated into two groups. If ψ has n-nodes, the particles are segregated into (n+1)-groups and distributed as reds, blues, reds, ...

Let us assume $U(x) = b_2(x)$ (the second model). The function $b_1(x)$ which has no singularity determins a pair-interaction $h_1(x)$, and the microscopic drift field $U(x)$, which has singularities at the nodal points of ψ , segregates particles. In this model we do not need "colours" of particles.

If we assume $h \equiv 0$ (the third model), then $U(x) = b(x)$, that is, the microscopic drift field $U(x)$ and the drift $b(x)$ of a typical particle coincide. In this model every particle in the system moves independently and is itself "typical".

6.2 A composite model of Mesons

We apply the statistical model explained in § 6.1 to the composite model which is discussed in § 2. That is, for each eigenfunction $\psi_{n,j}$ of the eigenvalue problem (1) there corresponds a typical (diffusion) particle in the three dimensional space, which has the stationary distribution density $\varphi_{n,j} = |\psi_{n,j}|^2$. Behind this single particle there is a system of (interacting) particles, which has the spatial equilibrium distribution density $\varphi_{n,j}$. We consider the distributions of the system of particles as "string distributions" (or string degree of freedom). One may interprete the string distributions, borrowing QCD-picture, as spatial distributions of virtual gluons in confined phases of QCD vacuum.

We will give here a possible physical interpretation of the typical particle. If we follow a "virtual gluon", it moves with infinite speed and interacts with other "virtual gluons" (and "virtual quarks") infinitely often in a finite time interval. When the "virtual gluon" which we are following disappears, we pick up one of "virtual gluons" in the neighborhood and follow it (revival of Markov processes [21]). Thus we obtain a continuous and everywhere non-differentiable "ideal gluon path". This is the typical particle of the model.

Let us assume the microscopic drift field $U \equiv 0$ (the first model). Then particles in the system must be coloured. Since they are in the three dimensional space and distributed by the eigenfunctions of (1), we need two colours in x-direction and four colours in (y,z)-plane. Thus we need eight colours for "virtual gluons".

In the model discussed in § 2 we neglected the interaction between "virtual gluons" and "quarks", and the quark mass m_q is simply added to the energy of string distributions $\lambda_{n,j}$ to obtain the mass of a meson (cf. (6)). We should, perhaps, discuss a simple statistical model which contains "quarks" in a more natural way. If we assume, for example, that "quarks" interact with "virtual gluons" in the string distribution in a similar way as "coloured gluons" i.e. "quarks" are attracted by "virtual gluons" in the string distribution and repelled by "colour" if they come too close to gluons, then we get such a statistical model of mesons.

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7. One of σ and k (respectively a' and k') is a redundant parameter.

8. E.C. Tichmarsh, Eigenfunction Expansions Associated with Second-order Differential Equations, I (Oxford, 1962).

9. We do not adopt the notion of "mixture of quark states" in this paper. It seems, however, plausible that the decay mode of $S^*(975) \to \pi\pi(78 \pm 3\ \%)$, $K\bar{K}(22 \pm 3\ \%)$ indicates the necessity of introducing this notion. If we do so, $((s+d)/2, \varphi_{8,0}, (\bar{s}+\bar{d})/2)(980)$ is a candidate for $S^*(975)$.

10. Compare two functions: $J = 1.43x^{3/4}$ and $J = 0,5 + 0.86x$, where $x = M^2$, $0 < x \leqq 5$. It is difficult to judge which function approximates the given experimental data better.

11. We are assuming that $(b, \varphi_{74}, \bar{b})$ is the smallest $(b,\bar{b})$-meson, since $\Upsilon(1)(9458)$ is the smallest one ever observed by now, although a smaller $(b,\bar{b})$-meson with spin zero is expected by our composite model.

12. For details and other applications of the model, see Nagasawa, M., Segregation of a population in an environment. J. Math. Biology (1980), 9, 213-235; An application of the segregation model for septation of Escherichia coli. J. Theor. Biol. (1981), 90, 445-455; A statistical model of systems of interacting diffusion-particles (in preparation). Albeverio, S., Blanchard, Ph., & Høegh-Krohn, R., A stochastic model for the orbits of planets and satellites: An interpretation of Titius-Bode law (preprint).

13. In higher dimensions we need duality arguments, which will not come across in one-dimension. See Nagasawa (1980).

14. For stochastic differential equations see, e.g. K. Itô and S. Watanabe, Introduction to stochastic differential equations, Proc. of Intern. Symp. SDE Kyoto, 1976 (Ed. by K. Itô) i-xxx, Kinokuniya Book-Store, Co. LTD, Tokyo.

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18. See Theorem 6.1 of Nagasawa (1980)(in the proof, (6.11) should be read as $|\psi| \, \|\nabla\beta\|^2 = O(1)$), and also Nelson, E., Critical diffusions, Journees de Probabilités,1983, Bern.

19. The following arguments are based on discussions with H. Föllmer.

20. For example take $\varphi = cx^2 e^{-x^2}$, then

$$\frac{1}{2}\frac{1}{\varphi}\varphi' = -x + \frac{1}{x} .$$

Hence, $b_1(x) = -x$ and $b_2(x) = \frac{1}{x}$. The solution of (46) for this $b_1(x)$ is $h(x) = -x$. The solution $h_2(x)$ of (47) for the $b_2(x)$ has a singularity of x^{-4}.

21. This is the so called "piecing together (or revival) technique" of the theory of Markov processes. Cf. Theorem 1 and 2 of Nagasawa,M., Basic models of Branching Processes, Proc. of 41st Session of ISI, New Delhi, 1977, XLVII (2), 423-445.

FIGURE CAPTION

Figure 1 : Calculated mass-spectrum (continuous lines) and experimentally obtained values (dotted lines). Notice that almost of all experimental data have some finite ranges of indeterminancy, although they are not drawn here.

TABLE CAPTION

Table 1: The eigenvalues μ of $\frac{d^2u}{dy^2} + (\mu - |y|)u = 0.$

Table 2: The eigenvalues λ (MeV) of $\frac{1}{2}\sigma^2 \frac{d^2u}{dx^2} + (\lambda - k|x|)u = 0,$

with $\{\frac{(\sigma k)^2}{2}\}^{1/3} = 136.99236$ MeV.

Table 1.

μ_1	1.018 793 3	μ_{41}	20.881 923	μ_{81}	33.011 829
2	2.338 107 7	42	21.224 830	82	33.284 885
3	3.248 197 6	43	21.563 888	83	33.556 376
4	4.087 949 6	44	21.901 367	84	33.827 215
5	4.820 099 2	45	22.235 232	85	34.096 539
6	5.520 560 0	46	22.567 613	86	34.365 232
7	6.163 307 2	47	22.896 589	87	34.632 457
8	6.786 708 1	48	23.224 165	88	34.899 070
9	7.372 177 4	49	23.548 526	89	35.164 260
10	7.944 133 7	50	23.871 564	90	35.428 856
11	8.488 486 7	51	24.191 560	91	35.692 071
12	9.022 650 7	52	24.510 301	92	35.954 710
13	9.535 449 2	53	24.826 156	93	36.216 008
14	10.040 174	54	25.140 821	94	36.476 747
15	10.527 660	55	25.452 742	95	36.736 182
16	11.008 524	56	25.763 531	96	36.995 074
17	11.475 056	57	26.071 708	97	37.252 699
18	11.936 016	58	26.378 805	98	37.509 795
19	12.384 788	59	26.683 410	99	37.765 659
20	12.828 777	60	26.986 985	100	38.021 009
21	13.262 219	61	27.288 179	101	38.275 159
22	13.691 489	62	27.588 388	102	38.528 808
23	14.111 502	63	27.886 318	103	38.781 290
24	14.527 830	64	28.183 305	104	39.033 283
25	14.935 937	65	28.478 110	105	39.284 139
26	15.340 755	66	28.772 009	106	39.534 519
27	15.738 201	67	29.063 814	107	39.783 790
28	16.132 685	68	29.354 751	108	40.032 597
29	16.520 504	69	29.643 675	109	40.280 323
30	16.905 634	70	29.931 764	110	40.527 597
31	17.284 695	71	30.217 918	111	40.773 814
32	17.661 300	72	30.503 269	112	41.019 591
33	18.032 345	73	30.786 756	113	41.264 337
34	18.401 133	74	31.069 468	114	41.508 652
35	18.764 798	75	31.350 385	115	41.751 961
36	19.126 381	76	31.630 556	116	41.994 849
37	19.483 222	77	31,908 993	117	42.236 754
38	19.838 130	78	32.186 710	118	42.478 248
39	20.188 631	79	32.462 753	119	42.718 780
40	20.537 333	80	32.738 099	120	42.958 911

μ_{121}	43.198 102	μ_{161}	52.294 619
122	43.436 900	162	52.511 701
123	43.674 780	163	52.728 154
124	43.912 274	164	52.944 343
125	44.148 870	165	53.159 914
126	44.385 089	166	53.375 225
127	44.620 428	167	53.589 928
128	44.855 398	168	53.804 375
129	45.089 507	169	54.018 223
130	45.323 255	170	54.231 819
131	45.556 158	171	54.444 827
132	45.788 709	172	54.657 586
133	46.020 432	173	54.869 766
134	46.251 809	174	55.081 701
135	46.482 376	175	55.293 065
136	46.712 603	176	55.504 190
137	46.942 035	177	55.714 751
138	47.171 134	178	55.925 076
139	47.399 455	179	56.134 847
140	47.627 448	180	56.344 385
141	47.854 678	181	56.553 377
142	48.081 587	182	56.762 139
143	48.307 746	183	56.970 364
144	48.533 590	184	57.178 362
145	48.758 699	185	57.385 830
146	48.983 499	186	57.593 075
147	49.207 576	187	57.799 798
148	49.431 351	188	58.006 301
149	49.654 416	189	58.212 288
150	49.877 183	190	58.418 059
151	50.099 253	191	58.623 322
152	50.321 031	192	58.828 372
153	50.542 125	193	59.032 920
154	50.762 931	194	59.237 259
155	50.983 064	195	59.441 103
156	51.202 915	196	59.644 739
157	51.422 105	197	59.847 888
158	51.641 017	198	60.050 832
159	51.859 280	199	60.253 296
160	52.077 269	200	60.455 557

Table 2.

λ_1	139.566 90	λ_{41}	2860.663 9	λ_{81}	4522.368 3
2	320.302 89	42	2907.639 5	82	4559.774 9
3	444.978 25	43	2954.087 9	83	4596.967 1
4	560.017 87	44	3000.320 0	84	4634.070 0
5	660.316 77	45	3046.056 9	85	4670.965 3
6	756.274 54	46	3091.590 5	86	4707.774 2
7	844.326 00	47	3136.657 7	87	4744.382 0
8	929.727 16	48	3181.533 2	88	4780.906 0
9	1009.932 0	49	3225.968 2	89	4817.235 0
10	1088.285 6	50	3270.221 9	90	4853.482 6
11	1162.857 8	51	3314.058 9	91	4889.541 1
12	1236.034 2	52	3357.724 0	92	4925.520 6
13	1306.283 7	53	3400.993 7	93	4961.316 4
14	1375.427 2	54	3444.100 4	94	4997.035 6
15	1442.209 0	55	3486.831 3	95	5032.576 3
16	1508.083 7	56	3529.407 0	96	5068.042 5
17	1571.995 1	57	3571.624 8	97	5103.335 1
18	1635.142 9	58	3613.694 8	98	5138.555 3
19	1696.621 4	59	3655.423 3	99	5173.606 8
20	1757.444 4	60	3697.010 8	100	5208.587 7
21	1816.822 7	61	3738.272 1	101	5243.404 3
22	1875.629 4	62	3779.398 4	102	5278.152 4
23	1933.167 9	63	3820.212 6	103	5312.740 4
24	1990.201 7	64	3860.897 5	104	5347.261 6
25	2046.109 3	65	3901.283 5	105	5381.626 9
26	2101.566 2	66	3941.545 4	106	5415.927 1
27	2156.013 4	67	3981.520 5	107	5450.075 3
28	2210.054 6	68	4021.376 5	108	5484.160 0
29	2263.182 8	69	4060.957 0	109	5518.096 5
30	2315.942 7	70	4100.423 0	110	5551.971 1
31	2367.871 2	71	4139.623 9	111	5585.701 0
32	2419.463 2	72	4178.714 8	112	5619.370 5
33	2470.293 5	73	4217.550 3	113	5652.898 9
34	2520.814 6	74	4256.279 8	114	5686.368 2
35	2570.634 0	75	4294.763 3	115	5719.699 7
36	2620.168 0	76	4333.144 5	116	5752.973 5
37	2669.052 5	77	4371.288 2	117	5786.112 6
38	2717.672 2	78	4409.333 3	118	5819.195 4
39	2765.688 3	79	4447.149 1	119	5852.146 5
40	2813.457 7	80	4484.869 5	120	5885.042 6

λ_{121}	5917.810 0	λ_{161}	7163.963 3
122	5950.523 5	162	7193.701 8
123	5983.111 1	163	7223.354 3
124	6015.646 1	164	7252.970 5
125	6048.057 8	165	7282.502 1
126	6080.418 0	166	7311.998 0
127	6112.657 7	167	7341.410 6
128	6144.846 8	168	7370.788 2
129	6176.917 9	169	7400.083 8
130	6208.939 6	170	7429.344 9
131	6240.845 6	171	7458.525 3
132	6272.703 3	172	7487.671 7
133	6304.447 6	173	7516.738 7
134	6336.144 5	174	7545.772 3
135	6367.730 3	175	7574.727 5
136	6399.269 7	176	7603.649 9
137	6430.700 1	177	7632.495 3
138	6462.085 0	178	7661.308 2
139	6493.363 1	179	7690.045 2
140	6524.596 5	180	7718.750 3
141	6555.725 2	181	7747.380 6
142	6586.810 0	182	7775.979 4
143	6617.792 1	183	7804.504 6
144	6648.731 1	184	7832.998 8
145	6679.569 2	185	7861.420 3
146	6710.365 1	186	7889.811 3
147	6741.062 0	187	7918.130 7
148	6771.717 4	188	7946.420 0
149	6802.275 6	189	7974.638 7
150	6832.793 0	190	8002.827 8
151	6863.214 9	191	8030.947 3
152	6893.596 8	192	8059.037 5
153	6923.884 9	193	8087.059 1
154	6954.133 7	194	8115.051 9
155	6984.290 3	195	8142.976 9
156	7014.408 2	196	8170.873 6
157	7044.435 5	197	8198.703 4
158	7074.424 8	198	8226.505 2
159	7104.325 1	199	8254.241 2
160	7134.188 0	200	8281.949 4

Table 3. (v takes local minimum at n = 27, 62, 95, 105, and 118)

$\gamma(9456\pm10)$, $\gamma(10016\pm10)$, $\gamma(10347\pm10)$, $\gamma(10569\pm10)$.

b = 3650, 2b = 7300

$\lambda_{27}(2156.0) + 2b = 9\,456.0$ (0.0) v = 2.7
$\lambda_{38}(2717.7) + 2b = 10\,017.7$ (+1.7)
$\lambda_{45}(3046.0) + 2b = 10\,346.0$ (−1.0)
$\lambda_{50}(3270.2) + 2b = 10\,570.2$ (+0.3)

b = 2840, 2b = 5680

$\lambda_{62}(3779.4) + 2b = 9\,459.4$ (+3.4) v = 6.9
$\lambda_{76}(4333.1) + 2b = 10\,013.1$ (−2.9)
$\lambda_{85}(4671.0) + 2b = 10\,351.0$ (+4.0)
$\lambda_{91}(4889.5) + 2b = 10\,569.5$ (+0.5)

b = 2214, 2b = 4428

$\lambda_{95}(5032.6) + 2b = 9\,460.6$ (+4.6) v = 6.9
$\lambda_{111}(5585.7) + 2b = 10\,013.7$ (−2.3)
$\lambda_{121}(5917.8) + 2b = 10\,345.8$ (−1.2)
$\lambda_{128}(6144.8) + 2b = 10\,572.8$ (+3.8)

b = 2035, 2b = 4070

$\lambda_{105}(5381.6) + 2b = 9\,451.6$ (−4.4) v = 10.1
$\lambda_{122}(5950.5) + 2b = 10\,020.5$ (+4.5)
$\lambda_{132}(6272.7) + 2b = 10\,342.7$ (−4.3)
$\lambda_{139}(6493.4) + 2b = 10\,563.4$ (−5.6)

b = 1822, 2b = 3644

$\lambda_{118}(5819.2) + 2b = 9\,463.2$ (+7.2) v = 11.7
$\lambda_{135}(6367.7) + 2b = 10\,011.7$ (−4.3)
$\lambda_{146}(6710.4) + 2b = 10\,354.4$ (+7.4)
$\lambda_{153}(6923.9) + 2b = 10\,567.9$ (−1.1)

b = 1838, 2b = 3676 v = 12.5

$\lambda_{117}(5786.1) + 2b = 9\,462.1$ (+6.1)
$\lambda_{134}(6336.1) + 2b = 10\,012.1$ (−3.9)
$\lambda_{145}(6679.6) + 2b = 10\,355.6$ (+8.6)
$\lambda_{152}(6893.6) + 2b = 10\,569.6$ (+0.6)

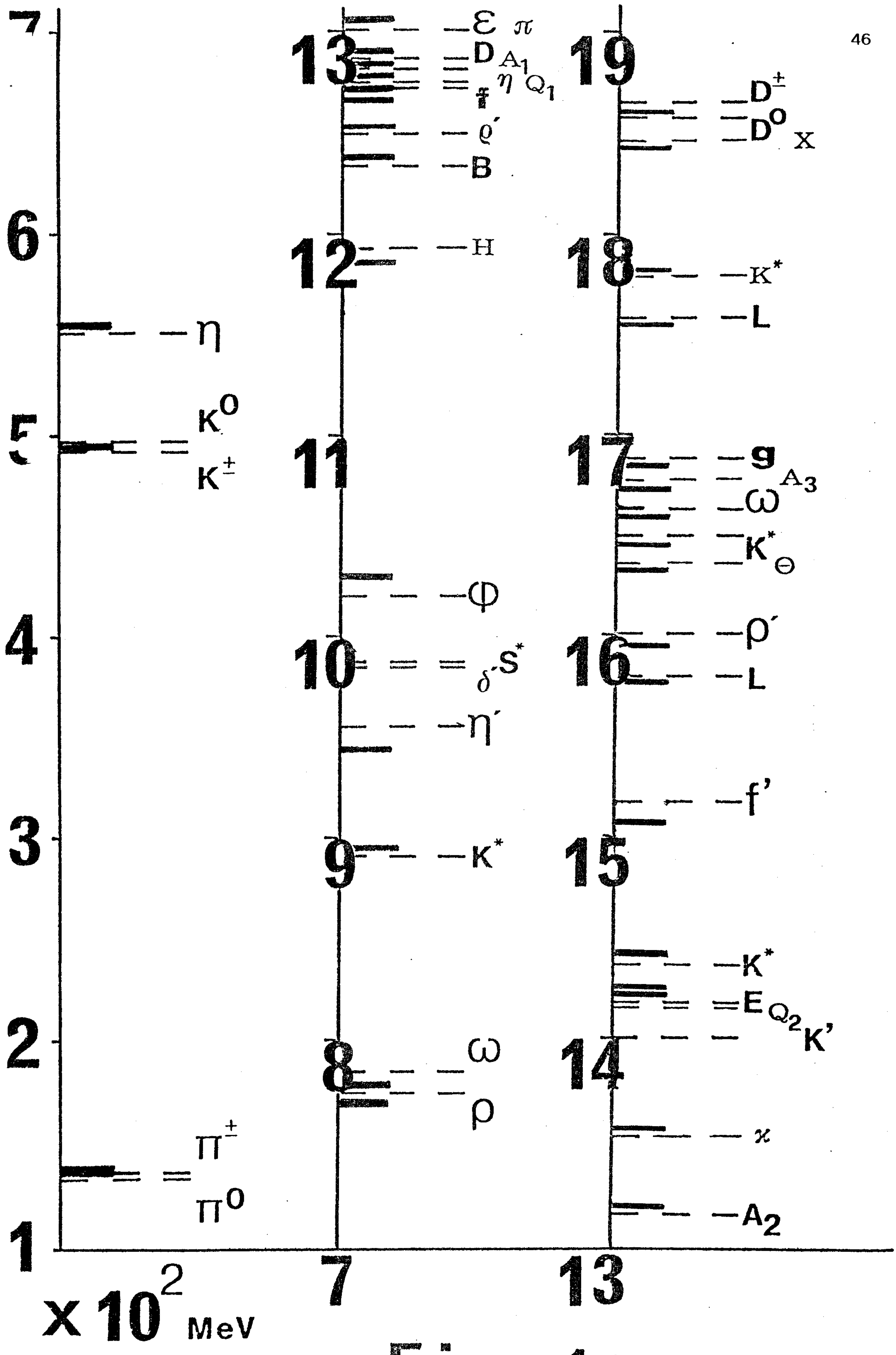

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Fig. 1a

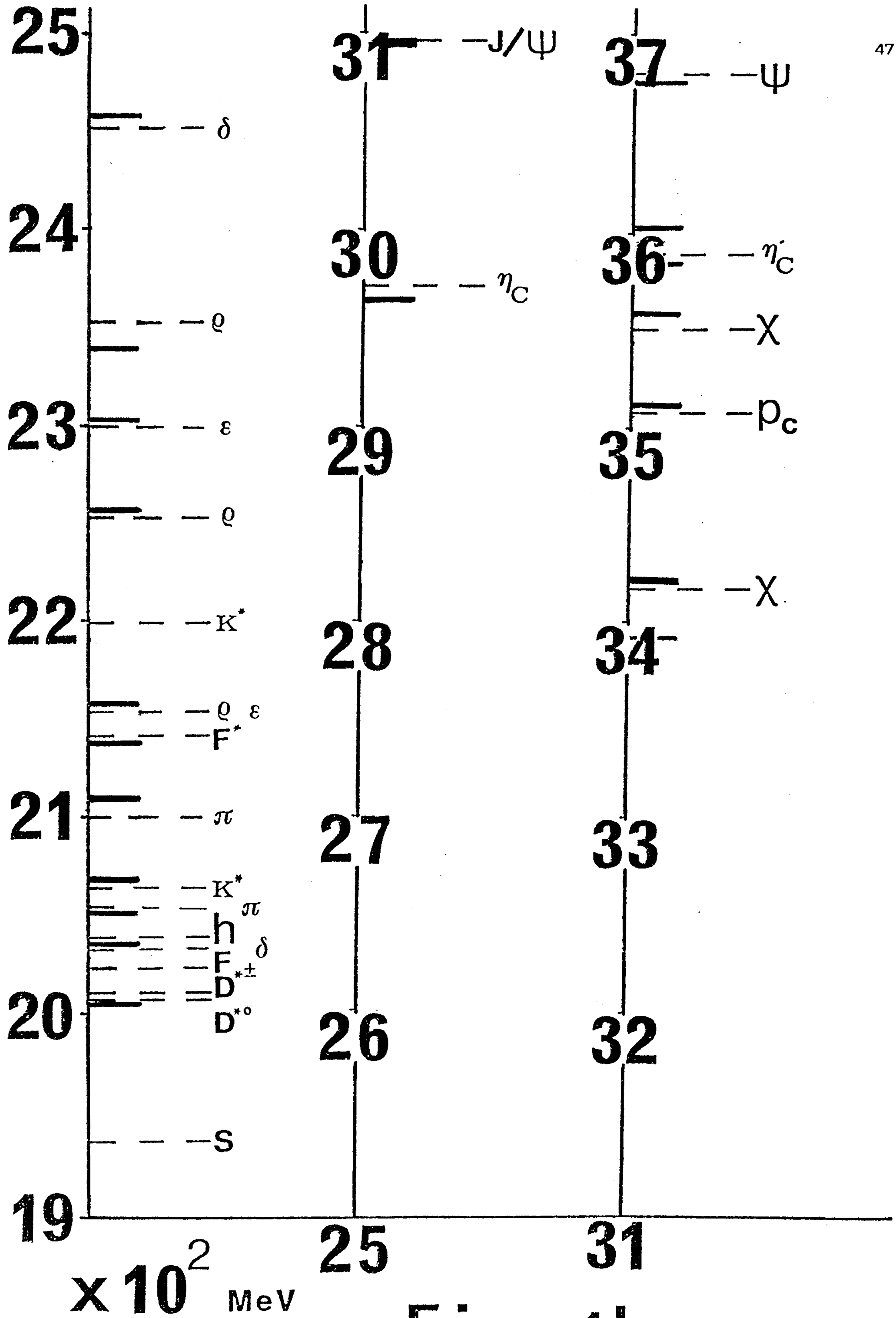
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Fig. 1b

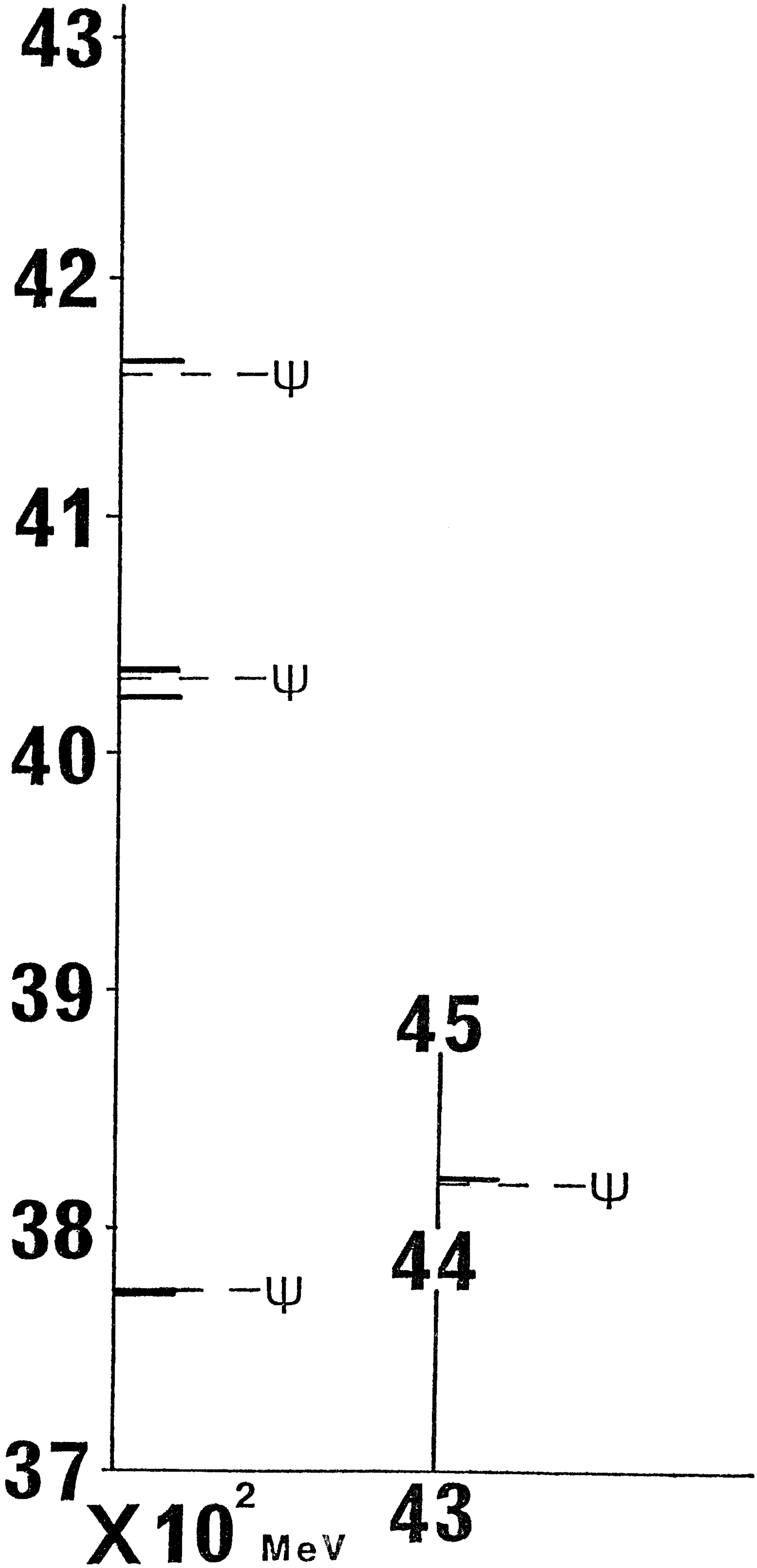

Fig. 1c