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TOPOLOGICAL ENTROPY AND HOMOLOGY

Anthony MANNING

Let X be a compact metric space and $f: X \rightarrow X$ a continuous map.

Definition.- (Bowen)

A (k, δ) spanning set for f is a finite set $Y \subset X$ s.t.

$\forall x \in X \exists y \in Y$ s.t. $d(f^j x, f^j y) < \delta$ for $0 \leq j < k$.

$$h(f, \delta) = \limsup_{k \rightarrow \infty} \frac{1}{k} \log \text{card}(\text{minimal } (k, \delta) \text{ spanning set})$$

The topological entropy is defined as :

$$h(f) = \sup_{\delta > 0} h(f, \delta)$$

$h(f)$ thus describes the exponential growth rate of the number of types of f -orbits, or roughly speaking the amount f mixes up the points of X .

$h(f) = \sup_{\mu} h_{\mu}(f)$ where μ ranges over all normalized Borel invariant measures (Dinaburg).

Conjecture.- (Shub)

Let $f: M \rightarrow M$ be a diffeomorphism of a smooth compact manifold M . Then $h(f) \geq \log \text{sp } f_*$ where $\text{sp } f_*$ is the largest modulus of any eigenvalue of the induced map of homology $f_*: H_*(M; \mathbb{R}) \rightarrow H_*(M; \mathbb{R})$.

$H_i(M; \mathbb{R})$ is a finite dimensional real vector space describing the i dimensional structure of M , f describes the action of f on this structure and so the conjecture says that a diffeomorphism that mixes the i dimensional structure of M for some i must also have at least the corresponding amount of entropy I.e. the homology of f detects some of the dynamics of f .

The conjecture has been proved for diffeomorphisms satisfying Axiom A and the no cycle property (Shub and Williams). Hence it holds for a C^0 open and

dense set of diffeomorphisms of M . For C^0 maps the inequality fails even on the manifold S^2 . However, any C^0 map of M satisfies $h(f) \geq \log |\lambda|$ for each eigenvalue λ of f_* in $H_1(M; \mathbb{R})$ the first homology group (Manning).

For eigenvalues in higher groups differentiable hypotheses would usually be required. In this direction Misiurewicz and Przytycki showed recently that any C^1 map $f: S^2 \rightarrow S^2$ satisfies $\exp h(f) \geq |\deg(f)|$.

REFERENCE

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