

DAVID RUELLE

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HENRI POINCARÉ'S "SCIENCE ET MÉTHODE"*

by DAVID RUELLE

Henri and Raymond Poincaré were a gifted pair of cousins. Raymond, born in 1860, would become President of the French Republic. Henri, born in 1854, was to be a famous mathematician, and he is the one whom scientists simply call Poincaré. Young Henri had varied interests, and spent his spare moments writing a novel: the story of a heartbroken beautiful young woman, in an aristocratic setting. The unfinished manuscript of the novel now seems to be lost and, judging by a few extant fragments, this may be just as well. The masterpiece of Henri Poincaré would come later: the three volumes of *Les méthodes nouvelles de la mécanique céleste* (1892-1899) were a different kind of literature. In this opus, and a deluge of articles and books, Poincaré opened or revolutionized vast areas of mathematics: topology, the theory of algebraic curves, dynamical systems, and so on. He was a most powerful mathematician, with universal interests extending also to physics. His intellectual vigor was unaltered when he died in 1912 from complications of a prostate operation.

To be confronted with an intelligence of this magnitude is a bit disturbing. Should one exclaim that such genius is beyond our comprehension? Or rather look for weak points, as French mathematicians of a later generation have done, finding the methods too intuitive and the interests outmoded? Or should one perhaps approach the general problem of how a mathematician's brain works? This problem, characteristically, was one on which Poincaré himself pondered. He knew, as every mathematician does, that if you have to solve a difficult problem, you first spend time looking at it from different angles. A number of ideas present themselves, which you pursue conscientiously, but you fail to solve your problem. How then do you proceed? Here is what may happen: "One evening I took black coffee, contrary to my custom. I could not go to sleep. Ideas came up in swarms, I sensed them clashing until a pair would hook together, so to say, to form a stable combination. By morning . . . I had just to write the results, which only took me a few hours."

Poincaré's hooking together of ideas may take place unconsciously. He describes how he had at some point to abandon his mathematical preoccupations to go on a geological trip and ". . . we took a bus for some excursion or other; the instant I set my foot on the step

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the idea came to me, with nothing apparently in my previous thoughts having prepared me for it, that the transformations I had used to define Fuchsian functions were identical with those of non-Euclidean geometry. I did not make the verification; I should not have had the time, . . . but I instantly felt a complete certainty”.

Reflecting on a number of such experiences, Poincaré expresses his belief that what appears to be a sudden illumination is in fact the result of a previous long subconscious work. Some card-carrying rationalists might frown at the role attributed by Poincaré to unconscious thought in a higher intellectual process. Poincaré was actually a very rational man: here he presents some interesting facts of observation, and boldly discusses their implications. But, characteristically, he stops when he reaches the end of what he can seriously argue.

The quotations made above are excerpted from Chapter 3 of *Science et méthode* (1908). This is one of several books in which Poincaré presented his ideas on the philosophy of science to the general public. Young Henri had failed in his early attempt at writing a heart-stirring novel, but there is no doubt that he had literary gift. The style of the philosophical writings is fluid and unhurried, and a delight to read. The clarity of exposition is such that you forget about the style and concentrate on the ideas. Poincaré’s books on the philosophy of science consist of short chapters which are as many little essays. Some chapters on physics are outdated, but other chapters have aged well, and sometimes show uncanny premonition of future scientific developments.

Chapter 4 of *Science and method* is about chance, or randomness, which the layman as well as the physicist encounters every day. Where does it come from? Poincaré finds not one but several sources of randomness (the list would be longer now with the advent of quantum mechanics). He explains for instance that randomness in the game of roulette arises from our lack of muscular control in spinning the wheel. Another source of randomness is what we now call chaos: “A very small cause, which escapes us, determines a considerable effect, which we cannot ignore, and we then say that this effect is due to chance.” As an example he discusses meteorology: “Why do meteorologists have such difficulty to predict the weather with any certainty? Why do rainstorms . . . seem to happen at random, so that many people find it natural to pray for rain or fair weather, when they would judge ridiculous to ask for an eclipse?” The answer he gives, sensitivity to initial condition, would be rediscovered (and justified) much later. The elegant simplicity of Poincaré’s presentation hides the vast amount of mathematical expertise and thinking underlying his philosophical discussions. And all this thinking did not always reach a satisfactory conclusion. It is probable for instance that he spent time thinking about hydrodynamic turbulence. He wrote a book related to the topic (*Théorie des tourbillons*, 1893), a study of vortices and their stability, but does not obtain any dramatic result.

One can guess that Poincaré’s intelligence has a significant moral aspect. Because he was basically modest, and his work had received considerable recognition, he showed little of the bitterness and need to assert his superiority which are so common among great (and less great) scientists. He was not a militant person: he said of the rather more ideological

Kronecker that the latter was able to do so much fine mathematics because he frequently forgot his own mathematical philosophy. Poincaré was thus not driven by strong ambition or ideological preconceptions, but he had extreme curiosity for the true nature of things. And so we see him pushing his investigations often far ahead of his time, unafraid to venture into physics and psychology, but stopping short of poetical guesswork, which seems to have held no attraction for him.

David RUELLE
IHÉS, 35, route de Chartres,
91440 Bures-sur-Yvette, France