PUBLICATIONS MATHÉMATIQUES DE L'I.H.É.S.

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Erratum : Carleman estimates for the Laplace-Beltrami equation on complex manifolds

Publications mathématiques de l'I.H.É.S., tome 27 (1965), p. 153-155
<http://www.numdam.org/item?id=PMIHES_1965__27__153_0>

ERRATUM TO :

CARLEMAN ESTIMATES FOR THE LAPLACE-BELTRAMI EQUATION ON COMPLEX MANIFOLDS

by ALDO ANDREOTTI *and* EDOARDO VESENTINI

(Publications Mathématiques de l'Institut des Hautes Études Scientifiques, n° 25)

1. We draw the attention to a mistake which appears in the above quoted paper due to an error in computation. This has to be corrected as indicated below; the rest of the paper remains unchanged.

In the formula (29) p. 102 the curvature tensor $s^{a}_{b\bar{\beta}_i\beta}$ of the metric on the bundle E should be replaced by the expression of the " covariant curvature tensor "

$$\widetilde{s}^{a}_{b\bar{\beta}_i\beta}=\nabla_{\bar{\beta}_i}l^{a}_{b\beta},$$

and this is because we use the Riemannian connection on the bundle $\overline{\Theta}^{*}$.

Accordingly in formula (31) p. 102 the curvature forms s and L should respectively be substituted by the " covariant " curvature forms $\widetilde{s}$ and $\widetilde{\mathrm{L}}$.

2. In the following § 5 the use of covariant curvature would however be cumbersome and it is therefore better to abandon the Riemannian connection and make use exclusively of the ∂-connections and $\bar{\partial}$-connections (see 2 *b)* and 3 *a)*) to establish the inequality we need following the same lines. The argument we shall presently give has to be added as section 15 *c)* of the paper.

First we note that, in terms of covariant derivatives with respect to the considered connection, the operator $\bar{\partial}$ has the expression

$$\bar{\partial}=\hat{\partial}+\mathrm{S},$$

where for $\varphi\in \mathrm{C}^{pq}(\mathrm{X},\mathrm{E})$:

$$(\hat{\partial}\varphi)^{a}_{\mathrm{A}\bar{\beta}_1\ldots\bar{\beta}_{q+1}}=(-1)^{p}\sum_{r=1}^{q+1}(-1)^{r-1}\nabla_{\bar{\beta}_r}\varphi^{a}_{\mathrm{A}\bar{\beta}_1\ldots\hat{\bar{\beta}}_r\ldots\bar{\beta}_{q+1}},$$

$$(\mathrm{S}\varphi)^{a}_{\mathrm{A}\bar{\beta}_1\ldots\beta_{q+1}}=(-1)^{p}\sum_{i,r=1}^{q+1}(-1)^{r-1}\overline{\mathrm{S}^{\alpha}_{\beta_i\beta_r}}\varphi^{a}_{\mathrm{A}\bar{\beta}_1\ldots(\bar{\alpha})_i\ldots\hat{\bar{\beta}}_r\ldots\bar{\beta}_{q+1}},$$

$\mathrm{S}^{\alpha}_{\beta\gamma}$ being the torsion tensor as defined in n° 3 *a)*.

Analogously we see that

$$\theta = \hat{\theta} + T$$

where $T = -*\#^{-1} S \# *$, and for $\varphi \in C^{pq}(X, E)$

$$(\hat{\theta}\varphi)^{a}_{A\bar{\beta}_1 \ldots \bar{\beta}_{q-1}} = (-1)^{p-1} \Sigma \nabla_{\alpha} \varphi^{a\alpha}_{A\,\bar{\beta}_1 \ldots \bar{\beta}_{q-1}}.$$

We now consider the vectors ξ and η introduced in 14 *b)* (the covariant derivatives being intended to be with respect to the ∂- and $\bar{\partial}$- connections). We have

$$\operatorname{div} \xi - \operatorname{div} \eta = \Sigma \nabla_{\beta} \xi^{\beta} - \Sigma \nabla_{\bar{\gamma}} \eta^{\bar{\gamma}} - 2\Sigma (S_{\tau} \xi^{\tau} - \bar{S}_{\tau} \eta^{\bar{\tau}}),$$

where $S_{\tau} = \Sigma S^{\beta}_{\tau\beta}$. A calculation of the same type as the one given in 14 *b)* gives:

$$(33') \qquad \Sigma \nabla_{\beta} \xi^{\beta} - \Sigma \nabla_{\bar{\gamma}} \eta^{\bar{\gamma}} = (q-1)! \{ A(\bar{\nabla}\varphi, \bar{\nabla}\varphi) + A(\hat{\mathscr{K}}\varphi, \varphi) - A(\hat{\partial}\varphi, \hat{\partial}\varphi) - A(\hat{\theta}\varphi, \hat{\theta}\varphi) \},$$

where now

$$(\hat{\mathscr{K}}\varphi)^{a}_{A\bar{B}} = \sum_{i=1}^{q} (-1)^{i} \{ \Sigma s^{a}_{b\bar{\beta}_i;\beta} \varphi^{b\beta}_{A\,\bar{B}'_i} + \Sigma L^{\sigma}{}_{\beta\bar{\beta}_i;\sigma} \varphi^{a\beta}_{A\,\bar{B}'_i} + \Sigma (-1)^{k} \Sigma L^{\alpha}{}_{\alpha_k \bar{\beta}_i;\beta} \varphi^{a}{}_{\alpha A'_k}{}^{\beta}{}_{\bar{B}'_i} \}.$$

For $\varphi \in \mathscr{D}^{pq}(X, E)$, by Stokes formula we get

$$(*) \qquad ||\bar{\nabla}\varphi||^2 + (\hat{\mathscr{K}}\varphi, \varphi) = ||\hat{\partial}\varphi||^2 + ||\hat{\theta}\varphi||^2 + 2\int_X \Sigma (S_{\tau}\xi^{\tau} - \bar{S}_{\tau}\eta^{\bar{\tau}}) dX.$$

We have the following estimates

$$||\hat{\partial}\varphi||^2 \leq 2(||\bar{\partial}\varphi||^2 + ||S\varphi||^2) \leq 2||\bar{\partial}\varphi||^2 + c\int_X |S|^2 A(\varphi, \varphi) dX,$$

$$||\hat{\theta}\varphi||^2 \leq 2(||\theta\varphi||^2 + ||T\varphi||^2) \leq 2||\theta\varphi||^2 + c\int_X |S|^2 A(\varphi, \varphi) dX,$$

$$\left| \int_X (\Sigma S_{\tau}\xi^{\tau}) dX \right| \leq c\{ \varepsilon \int_X |S|^2 A(\varphi, \varphi) dX + \frac{1}{\varepsilon} \int_X A(\bar{\nabla}\varphi, \bar{\nabla}\varphi) dX \},$$

$$\left| \int_X (\Sigma \bar{S}_{\tau}\eta^{\bar{\tau}}) dX \right| \leq c\{ \varepsilon \int_X |S|^2 A(\varphi, \varphi) dX + \frac{1}{\varepsilon} \int_X A(\hat{\theta}\varphi, \hat{\theta}\varphi) dX \},$$

where c is a universal positive constant, ε any positive number and $|S|^2$ is the length of the torsion tensor.

We substitute these four estimates, with $\varepsilon = 4c$, in $(*)$. If we set

$$\mathscr{K} = \hat{\mathscr{K}} - (16c^2 + \frac{5}{2}c)|S|^2$$

we obtain an inequality of the type

$$||\bar{\nabla}\varphi||^2 + c_1(\mathscr{K}\varphi, \varphi) \leq c_2\{ ||\bar{\partial}\varphi||^2 + ||\theta\varphi||^2 \}$$

where c_1, c_2 are positive universal constants. Proposition 15 and its corollaries can thus be stated using this expression of $\mathscr{K}$.

3. The use of identity (33′) instead of identity (33) simplifies the argument of section 17 *d)* in that that no covariant Levi form has to be introduced. What follows replaces thus 17 *d)* from line 7 from below of p. 112 onwards. From (33′) we get (with the notations of 17 *d)*)

$$\frac{1}{(s-1)!}\int_{\partial Y}(\xi-\eta)_n dS = ||\overline{\nabla}\varphi||_Y^2 + (\hat{\mathscr{K}}\varphi, \varphi)_Y - ||\hat{\partial}\varphi||_Y^2 - ||\hat{\theta}\varphi||_Y^2 - 2\int_Y \Sigma (S_\tau \xi^\tau - \overline{S}_\tau \eta^{\bar\tau}) dY;$$

hence

$$||\overline{\nabla}\varphi||_Y^2 + (\hat{\mathscr{K}}\varphi, \varphi)_Y + \frac{1}{(s-1)!}\int_{\partial Y}\frac{1}{|\partial f|}\mathscr{L}(f)\{\varphi, \varphi\} dS = ||\hat{\partial}\varphi||^2 + ||\hat{\theta}\varphi||^2,$$

and thus as in section 15 *c)* we derive the inequality

$$||\overline{\nabla}\varphi||_Y^2 + c_1(\mathscr{K}\varphi, \varphi)_Y + \frac{1}{(s-1)!}\int_{\partial Y}\frac{1}{|\partial f|}\mathscr{L}(f)\{\varphi, \varphi\} dS \leq c_2\{||\overline{\partial}\varphi||^2 + ||\theta\varphi||^2\}.$$

If $\mathscr{L}(f)$ has $n-q+1$ positive eigenvalues at each point of ∂Y we can choose a hermitian metric so that $l_{\mathscr{L}(f)} \geq c_0$ at each point of ∂Y with $c_0 > 0$. Also we can replace the metric h on the fibers of E by $e^{\tau|f|}h$. We will have

$$\frac{1}{(s-1)!}\int_{\partial Y}\frac{1}{|\partial f|}\mathscr{L}(f)\{\varphi, \varphi\} dS \geq c_3\int_{\partial Y} A(\varphi, \varphi) dS,$$

$$A_\tau(\mathscr{K}\varphi, \varphi) \geq c_4 \tau A_\tau(\varphi, \varphi),$$

with positive constants, c_3 and c_4, for any $\tau \geq \tau_0$ with a convenient $\tau_0 \geq 0$, and for any $\varphi \in B^{r,s}(Y, E)$ with $\text{supp}(\varphi) \subset U$, provided $s \geq q$. From this we obtain by the same argument as the one given there (p. 113 from line 11 from below onwards) the las inequality of 17 *d)* [1].

[1] In the expression of $||\varphi||_{\tau,\eta}^{(n)}$ there is a misprint: $e^{\tau|f|}$ should replace $e^{\tau f}$.