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Abstract. Contemporary intuitionistic mathematicians and logicians, who have put forward theories of constructions, have considered the proofs of mathematical and logical theorems via the model of geometrical problems. In his book, *Mathematical Intuition* [Tieszen 1989], Richard Tieszen connected that kind of approach with Husserl's phenomenological studies. The present paper seeks to bring the comparison between ancient geometry and Husserl's phenomenology into a somewhat different, even if not opposite, direction, and thereby to throw some new light on Husserl's concept of intuition. The main conclusion of the paper is that the model of geometrical problem-solving manages to illuminate Husserl's distinction between sensuous and categorial intuition presented in the *Logische Untersuchungen*.

1. On Contemporary Theories of Constructions

In Euclid's geometry, problems were tasks of constructing figures. Each problem gave a description of the kind of construction that was desired. The general method of finding the solution was that of analysis. The geometer started with a model-figure, which represented the imagined result of constructing activities. He moved backwards from the imagined end-state to something that was already admitted at the starting-point, in order to find out how the desired construction can be brought about [See Euclid, Book XIII, prop. 1] [Pappus 1965, *Vol II*, 634-635]. It has been argued both by Thomas Heath, who comments on the *Elements*, and by Wilbur Knorr, who has worked on ancient geometry, that for Greek geometers the method of analysis was more important in solving problems than in proving theorems. In solving problems, analysis offered heuristic power to the geometers [See Heath 1926, 140] [Knorr 1986, 356]. Knorr argues that the questions of construction were primarily applicable to problems and then transferred to theorems [Knorr 1986, 360 and 368]. However, from the point of view of Plato's philosophy, theorems were the proper objects of geometry and constructing geometrical figures turned out to be lower activity typical of those who are close to sensory impulses [Knorr 1986, 351] [Plato, *Republic* 510d and 527; *Euthydemus* 290c].

Following the ancient tradition, contemporary intuitionistic mathematicians and logicians, who have put forward theories of constructions, have taken problems to be conceptually primary and considered the proofs of logical and mathematical theorems via the model of geometrical problems. Göran Sundholm ascribes to Heyting the view that "the meaning of a proposition is explained in terms of what constructions have to be carried out in order to prove the proposition" [Sundholm 1983, 160]. In his book *Mathematical Intuition* [Tieszen 1989], Richard Tieszen has presented this kind of
approach by means of Husserl’s conceptual tools, namely, by means of the distinction between the meaning-intentions and the meaning-fulfilments of expressions.

In the contemporary theories of constructions, propositions are taken to be tasks of carrying out proofs. As Sundholm puts it, the proposition itself is not a theorem, but every theorem is of the form: “A proposition has been proved” [ibid, 161]. Expressed in terms of geometrical problem-solving, to carry out a proof is to construct a theorem. Moreover, expressed in Husserl’s terms, a logical or a mathematical proposition expresses a meaning-intention, that is, an intention towards a proof construction, and to fulfil the intention is to produce an intuition or a construction, which is a proof for that proposition.

The present paper has several aims which are related to the above theme. First, it seeks to outline a new interpretation of Husserl’s distinction between noesis and noema which relies on the similarities between geometry and phenomenology. I will argue that even if Husserl takes phenomenology to be a science which only describes or analyzes what is given in experience, hence, a science which does not prove anything, phenomenological studies themselves can be considered via the model of geometrical problem-solving. I have also put forward this interpretation in my earlier articles. It is Husserl himself who proposes the model of geometry, as he compares phenomenology with geometry in the Ideen I [Husserl 1913]. There are also a few interesting details in Husserl’s views of geometry which are expressed in his writings in the 1890’s. Second, in the present paper I will discuss two of the ways in which Husserl uses the concept of intuition in the Logische Untersuchungen, which have not received much attention in the literature. Third, I will discuss Husserl’s theory of judgements, on which the model of geometry throws some new light. In general, my intention is to show that geometry played an important role for Husserl when he developed his own philosophical project, including his search for the origins of logical concepts.

2. Geometry and Husserl’s Phenomenology

My interpretation of Husserl’s phenomenological enterprise relies on the analogy between geometry and phenomenology which Husserl discusses in the Ideen I [Husserl 1950, 160-168]. For Husserl, geometry and phenomenology are material, hence, non-formal sciences of essences. Both a geometer and a phenomenologist

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1 See [Haaparanta 1990 and 1994]. I will rely on those articles, when I present the model of geometry. Also see [Haaparanta 1995].
use imaginary examples. A geometer studies such concepts as triangle and parallelogram by considering figures which exemplify those geometrical concepts. Likewise, a phenomenologist considers intentional acts, such as perceptual and emotional acts, when he or she wishes to study pure consciousness. The phenomenologist's examples satisfy the condition of intentionality, which characterizes conscious acts. By analyzing examples, the phenomenologist seeks to find new essential features of those acts and thereby features of pure consciousness. It is typical both of geometrical and of phenomenological studies that the results do not become more reliable even if the number of examples is increased. However, geometry and phenomenology have to do with systematic free variation of examples in imagination, which amounts to saying that the essences are seen by means of thought-experiments [Spiegelberg 1975, 10-11 and 62-63]. Hence, in Husserl's view, as the geometer resorts to imagery when he or she studies geometrical concepts, likewise the phenomenologist makes use of concrete observable acts and phantasy in his or her search for the essences.

On the basis of the analogy between geometry and phenomenology, we can develop an interpretation for Husserl's concepts of noesis and noema. Husserl states in the *Ideen* that intentional experience (*intentionales Erlebnis*) consists of morphe, form, and hyle, matter, and that when the morphe of intentional experience is stressed, we may speak about noesis [Husserl 1950, 207-212]. Noesis is a form-giving act, for example, a perceptual, volitional or emotional act, hence, it shapes the hyletic material and gives it the logical form [ibid., 209, 237-239 and 290]. The hyletic material may be a visual or an auditory sensation or an emotional impulse. We cannot consider it as detached from the form-giving acts of our consciousness. However, what Husserl suggests is that the conceptual distinction between morphe and hyle is made by the one who carries out the phenomenological analysis of experience.

The objects to which the noetic acts are directed are parenthesized. However, there are curious objects which interest the phenomenologist and which Husserl calls noemas. His examples of noemas are such as the perceived object as perceived (*das Wahrgenommene als solches*), the perceived tree as perceived (*das Baumwahrgenommene als solches*), the remembered object as remembered (*das Erinnerte als solches*), and the loved object as loved (*das Geliebte als solches*) [ibid., 218-222]. Husserl thinks that the noema somehow belongs to the intentional experience, even if it is not its component in the same way as noesis and hyle. His noesis and noema also depend on each other in a curious way; Husserl states that there cannot be any noetic moment without noematic moment and that *infimae species* on the noematic side refers backwards to the
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*infimae species* on the noetic side [ibid., 244-215 and 314]. The components of the noema are quality (*Qualität*), which comes from the act, and matter (*Materie*), which comes from the parenthesized object [ibid., 316-317 and 324]. On the basis of these clues, it seems that hyle and the material component of the noema have the very same origin. Such qualities as the judgement-quality, the wish-quality and the perception-quality are ‘posited’ characteristics in the noemas.

How could we make sense of this strange connection between noesis and noema? If we rely on the model of geometry, we may suggest that noesis is mental constructing, which has hyle as its material, and noema is the construction which is brought about. If we have a geometrical problem, for example, if we have to draw a triangle the sides of which are known, we have three segments of a line, which are our material, and we also have our geometrical knowledge or capacity. By means of these two we are able to draw the figure. When we have the triangle, we can analyze it and find in it the material and the traces of the process of constructing, that is, the matter and the ‘posited’ characteristics, which appear as qualities of the construction. The deep philosophical idea which Husserl expresses is that our world is not given to us as it is in itself; the phenomenologist is the one who shows that it is our own construction, and he or she also shows how it is constructed. The phenomenologist analyzes the given objects of experience, just as the geometer analyzes model-figures. He or she goes backwards both to the material, that is, to sensations and to emotional and volitional impulses, and to the cognitive perceptual, emotional and volitional acts by means of which form has been given to the material. After this analysis, the phenomenologist is as it were able to draw the desired figure, that is, to show how the objects of experience, which he or she now calls noemas, are constructed.

It is important to note that among Husserl’s noeses and noemas, there is the noesis called judging (*urteilen*) and the noema called the judged as judged (*das Geurteilte als solches* or *das Urteil*). On the model of geometry, which I developed above, we may say that judging is constructing, which requires both the very act of judging together with logical forms, and hyle, which is the empirical material. Moreover, this model tells us that the judgement is a construction, which has both the component which comes from the cognitive act and the material component. From judgements we can go backwards to the process of constructing, in which we manipulate empirical material by means of logical forms and by means of the very act of judging.

It seems as if the distinction between noesis and noema,
constructing and construction, and judging and judgement opened up for Husserl a new way of looking at logic. For example, in the preparatory considerations of his *Formale und transzendentale Logik* [Husserl 1929 or FTL] he writes as follows:

A sharp distinction must be made between the mental process of meaning and the meaning intended, the sense \(\text{zwischen meinen und Meinung}\): between judging and judgement \(\text{Urteilen und Urteil}\), wishing and wish, and so forth; so that, in fact, there turns out to be a triality to which the distinction between thinking and what is thought (the thought) has already pointed. [Husserl 1929, 22 or 1969, 25]

Earlier, in the *Ideen*, Husserl writes about judgements as follows:

The noema of the judging, i.e., of the concrete judgmental process, is the ‘judged as judged’; that, however, is nothing else, or at least with respect to its main core, it is nothing else than what we usually call simply the judgment. [Husserl 1950, 233; Kersten, 227]

He also concludes:

The difference between the *making of a judgment* and the *judgment made*, a difference already recognized in ordinary speech, can serve to point out the correct view, namely that to the judgmental mental process there belongs correlativevly as noema the judgment simpliciter. The latter, then, should be understood as the ‘judgment’ or *proposition in the sense of the word in pure logic* [Husserl 1950, 234-235; Kersten, 229]

3. On the Concept of Intuition in the *Logische Untersuchungen*

What I presented above is a philosophical reconstruction of Husserl’s phenomenology and two of its basic concepts, that is, noesis and noema, in terms of the model of geometry. I find it difficult to argue that the interpretation is historically true, meaning, it is what Edmund Husserl, the historical person, really meant in 1913. I find that claim problematic mainly because the concept of historical truth is philosophically problematic. However, we can proceed in the standard way and take up a few documents which speak in favour of the suggested interpretation.

When Husserl discusses geometry in the 1890’s, he remarks that for centuries geometry has been the methodological example for metaphysics [Husserl 1983, 268]. Even if Husserl himself does not work on metaphysics in the traditional sense, in this context he seems to mean that some features of geometry could be methodological examples for philosophy in general. Moreover, he states that the symbol-system of geometry, which consists of hieroglyphs is of great
importance in view of discovery. Husserl remarks that when a geometer wishes to explain the general, he also has a concrete, intuitive picture, which improves understanding [Husserl 1983, 294-295]. In Husserl’s view, a geometer who works on intuitions, works on symbols; the symbols of geometry are figures, and they symbolize geometrical concepts for us. It may also be noted that the very word ‘intuition’ (Anschauung) carries various meanings for Husserl, and he also distinguishes between some of those meanings in his writings in the 1890’s. However, I will not go into those distinctions in the present paper. Instead, I wish to show that there is an interesting connection between Husserl’s way of using geometry in the Ideen I and his early statements about geometry, especially in the second volume of the Logische Untersuchungen [Husserl 1901].

Husserl’s doctrine of noesis and noema can be traced back to his distinction between meaning-intention and meaning-fulfilment, which is introduced in the Logische Untersuchungen. However, it is not quite clear how these connections ought to be seen. At least it seems that the concept of intuition adopts various roles in Husserl’s early phenomenology. In what follows, I will try to show (1) that in some passages of the Logische Untersuchungen intuitions are understood both in terms of model-figures and as constructions made, and (2) that understood as constructions, they anticipate the noemas of the Ideen. This is somewhat surprising, as the meaning-intentions are usually regarded as the predecessors of the noemas.

In the first investigation of the second volume of the Logische Untersuchungen Husserl starts his discussion of meaningful signs. He states that what is essential for an expression is that it means something and thereby relates to what is objective. Husserl remarks that this objective can either be actually present via intuitions or it may appear in representation, for example, in a mental image; if this happens, the relation to an object is realized. If the expression functions significantly but lacks an intuition which gives its object, its relation to an object is unrealized. According to Husserl’s terminology, it has a meaning-intention but no meaning-fulfilment. For example, a name may name its object and thus have a meaning-intention, but if the object is not intuitively before the one who gives the expression, the meaning-intention is empty [LU II, A 37-38/B 37-38]. It is noteworthy that the meaning-intentions are not directed only to individual objects: rather, Husserl wants to speak about objective correlates (Gegenständlichkeiten), which include objects, properties, states of affairs, etc. [LU II, A 38/B 38].

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2 In what follows, I will further develop what I outlined in [Haaparanta 1994, 224-225].
In the *Logische Untersuchungen*, Husserl also compares his own studies of expressions and meanings with geometrical studies. On the basis of what Husserl tells us it seems that geometrical illustrations correspond to meaning-fulfilsments in intuition and representation and that geometrical concepts correspond to the meaning-intentions of expressions [LU II, A 65/B₁ 65]. Even if he does not lay much stress on the role of illustrations in geometry, he admits that sensuous pictures function in a phenomenologically graspable and describable manner; they are aids of understanding [ibid.]. Husserl thinks that both expressions and geometrical concepts can function without illustrative intuition [LU II, A 66/B₁ 66]. Still, he asks why we employ corresponding intuitions at all in order to know conceptual truths, which are truths known through an analysis of meanings [LU II, A 71/B₁ 71]. He answers that we construct corresponding intuitions in order to see what the expressions really mean [ibid.]. In this context, intuitions are considered via the model of geometry, as they turn out to be like geometrical drawings which can also serve as objects of analysis.

Hence, Husserl has a number of uses of the concept of intuition, but some of them are close to the concept of construction used in ancient geometry. First, in the *Logische Untersuchungen*, Husserl states that intuitions are representatives of concepts. This idea is related to Euclid's *ekthesis*. Second, he assumes that intuitions are needed if we wish to analyze meanings. That view can be understood in terms of geometrical problem-solving in two ways. Husserl both stresses the rôle of model-figures in phenomenological studies of meaning, and he thinks that we construct intuitions in which meanings reside as objective. The latter idea suggests that there is a connection between Husserl’s concept of meaning-fulfilment and his later concept of noema.

### 4. Sensuous and Categorial Acts in the *Logische Untersuchungen*

From the point of view of Husserl’s philosophy of logic, it is especially important that Husserl takes judgements to be noemas. It is equally noteworthy that in his *Formale und transzendentale Logik* [Husserl 1929 (FTL)] Husserl seeks to reveal the origins of logical concepts by analyzing judgements [FTL, 184 and 190]. The model of geometry can also be found behind Husserl’s view of judgements presented in the sixth investigation of the *Logische Untersuchungen*. In the first chapter of the second section of the sixth investigation entitled *Sinnliche und kategoriale Anschauungen*, Husserl discusses categorial meaning-forms and their fulfilment. This problem has to do with logical concepts; Husserl remarks that he is interested in “what may and can furnish fulfilment for those aspects of meaning which make up the propositional form as such, the aspects of
**categorial form** to which, e.g., the copula belongs” [LU II, A 601/B₂ 129; LI II, 773]. He writes as follows:

In our meanings [...] parts of very different kinds are to be found, and among these we may here pay special attention to those expressed by formal words such as the, a, some, many, few, two, is, not, which, and, and further expressed by the substantival and adjectival, singular and plural inflection of our words etc. [LU II, A 601/B₂ 129; LI II, 774.]

In the beginning, Husserl does not seem to have any intuitions which would be the fulfilments of the meaning-intentions of logical words. He remarks that only certain antecedently specifiable parts of our statements can have something which corresponds to them in intuition, while to other parts of the statements nothing intuitive can correspond [LU II, A 607/B₂ 135]. He then concludes that in such statements as “A is P”, “This S is P” and “All S are P” we may put meanings that are fulfilled in perception only at the places indicated by letters, that is, variables [ibid.]. He states:

The a and the the, the and the or, the if and the then, the all and the none, the something and the nothing, the forms of quantity and the determinations of number etc. — all these are meaningful propositional elements, but we should look in vain for their objective correlates (if such may be ascribed to them at all) in the sphere of real objects, which is in fact no other than the sphere of objects of possible sense-perception. [LU II, A 611/B₂ 139; LI II, 782.]

Husserl argues that the true source of logical forms lies in the fulfilments of judgements themselves [LU II, A 613/B₂ 141]. In order to develop this idea, he widens the concepts of perception and intuition. He states:

If we are asked what it means to say that categorically structured meanings find fulfilment, confirm themselves in perception, we can but reply: it means only that they relate to the object itself in its categorial structure. The object with these categorial forms is not merely referred to, as in the case where meanings function purely symbolically, but it is set before our very eyes in just these forms. In other words: it is not merely thought of, but intuited or perceived. When we wish, accordingly, to set forth what this talk of ‘fulfilment’ is getting at, what structured meanings and their structural elements express, what unitary or unifying factor corresponds to them objectively, we unavoidably come on ‘intuition’ (or on ‘perception’ and ‘object’). We cannot manage without these words, whose widened sense is of course evident. [LU II, A 615/B₂ 143; LI II, 785.]

After introducing the widened concepts of perception and intuition, Husserl wants to give a phenomenological analysis of the
distinction between sensuous and categorial perception. In his view, in sensuous perception the directly grasped object is real, whereas in categorial perception it is ideal. He characterizes sensuous or real objects as objects of the lowest level of possible intuition and categorial or ideal objects as objects of higher level [LU II, A 617/B2 146]. Husserl claims that sensuous objects are present in perception at a single act-level, that is, they need not be constituted in such acts as are founded on other acts which bring other objects to perception. Acts which are founded on other acts he calls acts of higher level [LU II, A 618/B2 146]. Those acts include acts of conjunction, disjunction, definite and indefinite individual apprehension (that - something), and generalization. The new objects which arise are based on the older ones; in Husserl's view, they are related to what appears in the basic acts [LU II, B2 146]. Hence, what Husserl argues is that logic needs objects in the sense that we can perceive logical forms only if we have objects of sense-perception.

Husserl characterizes sense-perception as straightforward perception and categorial acts as founded acts. He states that in the straightforward act the parts which constitute an object do not become our explicit objects. However, he claims that the same object can be grasped by us in explicating fashion [LU II, A 623 - 625/B2 151-153]. He then considers the relationships "A is or has a" and "a is in A" and states that in founded acts these states of affairs become constituted as data [LU II, A 625/B2 153]. Hence, we may conclude that Husserl takes such relationships as the relation between a whole and its parts to be of categorial and ideal nature. Husserl remarks that the part lies hidden in the whole before all division into members and we subsidiarily apprehend it when we grasp the whole. However, Husserl points out that "this fact, that it thus lies hidden in the whole, is at first merely the ideal possibility of bringing the part, and the fact that it is a part, to perception in correspondingly articulated and founded acts" [LU II, A 627/B2 155; LI II, 794]. Husserl thinks that there is an important distinction between sensuous or real forms of combination and categorial or ideal forms of combination; when we constitute the latter forms, we bring new objects into being. Those objects belong to the class of "states of affairs", and that class, Husserl argues, includes none but "objects of higher order" [LU II, A 628/B2 156].

Consequently, Husserl's wordings suggest that categorial intuition is an act which constructs, that is, gives being to certain new objects, which Husserl calls ideal. In this context, intuition is an act which constructs; on the other hand, it is the very construction which is produced by the act. Husserl explains his analysis of categorial acts even further. He writes that he considers founded acts to be intuitions and intuitions of the new types of object that they bring to light.
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[LU II, A 637/B₂ 165]. He gives grounds for his terminological choice as follows:

We call the new acts *intuitions* in that, with a mere surrender of a 'straightforward' relation to their object — the peculiar sort of immediacy defined by us as 'straightforwardness' — they yet have all the essential peculiarities of intuitions: we find in their case the same essential divisions, and they show themselves capable of achieving the same fully performed *fulfilments*. This last mentioned capacity is particularly important for our purposes, for it was with a view to such performances that this whole investigation has been conducted. Knowledge as the unity of fulfilment is not achieved on a mere basis of straightforward acts, but in general, on a basis of categorial acts: when, accordingly, we oppose *intuition* to *thought* (as meaning), we cannot mean by 'intuition' merely sensuous intuition. [LU II, A 637-638/B₂ 165-166; LI II, 803]

Husserl also assumes that, once established, categorial objects may become the objects of new "*connecting, relating or ideating acts*" [LU II, A 653/B₂ 181]. As we have seen, Husserl distinguishes between acts of straightforward intuition, which he calls sensuous, and founded acts, which he calls categorial and which lead back to sense. He stresses that everything categorial ultimately rests upon sensuous intuition, "*that a 'categorial intuition', an intellectual insight, a case of thought in the highest sense, without any foundation of sense, is a piece of nonsense*" [LU II, A 655/B₂ 183; LI, 818].

Husserl also has a number of interesting remarks in the *Logische Untersuchungen* in which he rejects the idea that logical forms are added to primary objects. He states that, in forming sensible objects, categorial functions leave their real essence untouched [LU II, A 657/B₂ 185]. He continues:

The new objects they [categorial forms] create are not objects in the primary, original sense. Categorial forms do not glue, tie or put parts together, so that a real sensuously perceivable whole emerges. They do not form in the sense in which the potter forms. Otherwise the original datum of sense-perception would be modified in its own objectivity: relational and connective thought and knowledge would not be what it is, but would be a falsifying transformation into something else. Categorial forms leave primary objects untouched: they can do nothing to them, cannot change them in their own being, since the result would otherwise be a new object in the primary, real sense. [LU II, A 658/B₂ 186; LI II, 820.]

Husserl thus seems to think that the one who seeks the origins of logical concepts in the sensuous acts finds them
unarticulated in primary objects and makes them explicit as objects of founded acts.

Hence, what is the procedure like which brings ideal objects such as logical forms to light? There are important lessons to be learned from Husserl’s statements, if we consider them via the model of geometrical problem-solving. In his *Formale und transzendente Logik* in 1929 Husserl stresses that logic needs a theory of experience and the theory of experience must be characterized as the first and the most fundamental judgement theory [FTL, 188]. Unlike that later study, the *Logische Untersuchungen* of 1901 is not meant to preach for constitutive phenomenology and transcendental philosophy. Still, in its sixth investigation Husserl already relies on the idea that sensuous acts give us objects which the one who studies the phenomenological foundations of logic can take as his or her starting-points. By analyzing the straightforwardly given objects, the phenomenologist tries to find the logical forms which reside in those objects. It turns out, however, that logical forms have their being in judgements, which the phenomenologist is able to construct as ideal objects. On the model of geometrical problem-solving, we may say that straightforwardly given sensuous objects serve as model-figures; by analyzing those figures, the phenomenologist is able to find the structure of sensible objects. He or she shows that structure by constructing judgements, which are constituted by logical forms. Categorial or founded acts turn out to be acts of constructing, and categorial forms are intuitions, hence, analogous to those constructions which are solutions to geometrical problems.

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