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BOUNDS FOR TWISTED CONVOLUTION OPERATORS

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Twisted convolution on a locally compact group G is defined by means of a T-valued 2-cocycle c : For appropriate functions (or distributions, if G is Lie) f and g on G the twisted product is given by

$$f *_c g\,(x) = \int f(xy)g(y^{-1})c(xy,y^{-1})dy \,.$$

If G is a connected Lie group, T a distribution with compact support and $g \in \mathcal{D}(G)$, then $T_c : g \mapsto T *_c g$ is a linear operator on $\mathcal{D}(G)$. T_c is densely defined on $L^p(G)$. Let $\| T_c \|_p$ be its (possibly infinite) L_p-operator-norm and let e be the trivial cocycle on G, so that T_e is the ordinary convolution operator. Theorem : $\| T_c \|_p$ is finite if and only if $\| T_e \|_p$ is finite. -- Ine the special case $G = \mathbb{C}^n$, c the Weyl cocycle, this was proved by M. Cowling (Rend. Circ. Mat. Palermo Ser. II, num. 1, 1981). Problem : How do the norms $\| T_c \|_p$ depend on the compact support of T ?

Let $c_k(u,v) = \exp(2\pi ik \operatorname{Im} (u|v))$ be the k^{th} power of the Weyl cocycle on $\mathbb{C}^n$, with $(u|v)$ the Hilbert-inner-product on $\mathbb{C}^n$ on define

$$\mu_k^p(K) = \inf(\| T_c \|_p / \| T_e \|_p)$$

the infimum taken over all T with support in the compact set K. Sharp estimates are given for these functions μ_k^p , in particular for $B(t) = \{z \in \mathbb{C}^n \;;\; |z| \leqslant t\}$ we show $\mu_k^p(G(t)) \sim (|k|t)^{rn}$ with $r = \min\{\frac{1}{p}, 1 - \frac{1}{p}\}$.

(Joined work with Detlef Müller).

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