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ON A METHOD OF MOMENTS

by

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By a linear system we shall understand a system of three Banach spaces X , Y and two continuous linear operators A, B

$$\begin{matrix} (X \rightarrow & \rightarrow Y) \\ A & B \end{matrix}$$

In our considerations the space Y will not play any role, because we shall not consider constraints on trajectories. For this reason we write briefly $C = BA$.

Let $y_0 \in Y$. If we are looking for a minimal norm solution of equation $Cu = y_0$, we said that we consider a minimum norm problem.

The first step for the solving of minimum norm problem is to find

$$(1) \quad a = \inf \{ \|u\| : Cu = y_0 \}$$

In the case when Y is one dimensional it can be easily reduced to a well known problem of calculating of the norm of the functional C .

A case when Y is finite dimensional was reduced to the one dimensional case by M.G. KREIN [1] by formula

$$(2) \quad \inf \{ \|u\| : Cu = y_0 \} = \sup_{C \in Y} \inf \{ \|u\| : C(Cu) = C(y_0) \}$$

In the theory of control formula (2) was used first time by N.N. KRASSOWSKI [3]. A.G. BUTKOWSKI [2] has proved formula (2) for $X = L^p$ and $Y = \ell^p$.

It was shown in [4] that formula (2) holds in general provided that the image Γ of the closed unit ball $K = \{u : \|u\| \leq 1\}$ is closed.

If Γ is not closed formula (2) may not hold. It follows from the following example [5]. Let $X = \ell$ and let Y be an arbitrary infinite dimensional Banach space. Let $y_0, y_1, \dots, y_n, \dots$ be a sequence of strongly linearly independent elements (i.e. such that if a series $\sum_{n=0}^{\infty} t_n y_n$ is convergent to 0, then $t_n = 0, n = 0, 1, \dots$) convergent to y_0 . Such sequences exist in each Banach spaces of infinite dimension. In fact as follows from a Banach theorem each infinite dimensional Banach contains an infinite dimensional subspace with a basis $\{e_n\} n = 0, 1, 2, \dots$. Let us put $y_0 = e_0$

$y_n = e_0 + \frac{1}{n} e_n$. It is easy to verify that the sequence y_n has desired properties.

Let

$$C(\{\ell_n\}) = \frac{1}{2} t_0 y_0 + \sum_{n=1}^{\infty} t_n y_n$$

The operator C is one to one. It is easy to verify and that

$$\inf \{ \|u\| : Cu = y_0 \} = 2$$

and that on the other hand for each $C \in Y^*$

$$\inf \{ \|u\| : C(Cu) = C(y_0) \} = 1$$

as follows from the fact that $y_n \rightarrow y_0$ and C is continuous.

Similar example was done by I. SINGER [6].

We say that maximum principle of Pontrjagin holds if there is

$$C_0 \in Y^* \text{ such that}$$

$$(3) \inf \{ \|u\| : Cu = y_0 \} = \inf \{ \|u\| : C_0(Cu) = C_0(y_0) \}.$$

The set CX is closed if and only if Γ has interior in CX . In this case using Hahn-Banach theorem one may easily to show that the principle of maximum holds for all $y_0 \in CX$. Thus the principle of maximum holds if either Y or X are finite dimensional.

If the set CX is not closed, there is such $y_0 \in CX$ that the principle of maximum does not hold. It means that for all $C \in Y^*$

$$(4) \inf \{ \|u\| : Cu = y_0 \} > \inf \{ \|u\| : C(Cu) = C(y_0) \}$$

It is a consequence of a following theorem of WOJTASZCZYK [7].

Let Y be a Banach space. Let Γ be a closed set in Y such that 0 is an algebraically internal point in Γ and let $\overline{\text{lin } \Gamma} = Y$. If for each point y_0 of the algebraic boundary of Γ , there is a continuous linear functional f such that $f(y_0) \geq f(x)$ for all $x \in \Gamma$, then Γ has interior in the norm sense.

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