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THE NOTION OF RANDOMNESS FROM ARISTOTLE TO POINCARÉ [1]

O.B. SHEYNIN [2]

RÉSUMÉ - La notion de hasard d'Aristote à Poincaré.
Aristote et même des philosohes et scientifiques plus anciens ont essayé de définir le concept de hasard. L'auteur décrit, depuis Aristote jusqu'à Poincaré, diverses tentatives de formalisation du hasard dans le domaine des mathématiques. Il insiste sur les interprétations du hasard qui ont été proposées dans les sciences de la nature et en philosophie, puis sur les relations entre le hasard et la nécessité.

SUMMARY - *Aristotle and even earlier scientist and philosophers attempted to define, or at least to through light upon randomness. The author sketches the attempts to direct concept of randomness into the realm of mathematical science from Aristotle up to Poincaré. He dwells on the various interpretations of randomness that were pronounced in natural science and philosophy, and on the interrelation between necessity and randomness.*

1. INTRODUCTION

Aristotle and even earlier scientists and philosophers attempted to define, or at least to through light upon randomness and in jurisprudence, about two thousand years ago, it was indirectly recognized in an ancient Indian book of instructions [1, §108], which determined the behavior of man both at home and in social life.

In §2 I sketch the attempts to direct the concept of randomness into the realm of mathematical science. In §§3-10 I dwell on various interpretations of randomness which were pronounced in natural science and philosophy. My §11 is devoted to the interrelation between necessity and randomness and, finally, in §12, I formulate my conclusions.

The history of the notion of randomness is especially interesting since the new approach to its understanding which had recently took shape in physics and mechanics has affected the fundamentals of these sciences [2].

The title of my paper should be understood as "... up to, but not including Poincaré". This great savant paid much attention to randomness and I describe his work on this topic and on probability in general in a special article to be published in the *Archive for history of exact science*. Here, I shall just mention that Poincaré directly linked chance to instability of motion [of the solution of differential equations]. There is a case for examining the attitude of ancient scientists preceding Aristotle towards randomness. However, my own experience [3, §§2.1 and 2.3] is that this topic is extremely difficult since their thoughts may be interpreted in different ways. Finally, I restrict myself with the fields of mathematics and natural science.

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[1] Russian version, Moscow, 1988.

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There is no general literature on my subject ; one author [4], however, has discussed randomness from a different point of view, some other [5--11] busied themselves with its particular problems. I shall mention contributions [5--7] in the sequel. I myself have touched the same topic in many articles published in the *Archive for history of exact sciences*. My excuse for doing so, and for returning to this subject in an *ad hoc* paper is that it is patently impossible to compile a contribution such as this one all at once.

Aristotle described, Darwin used, and Maxwell indicated various aspects of randomness and for this reason I repeatedly mention each of these great scholars.

2. MATHEMATICS AND THE CONCEPT OF RANDOMNESS

Lambert [12, p.238-239 ; 13, p.246 ; 3, p.136-137] made an endeavor to formalize randomness. His interest in this problem may be explained by the fact that he was the first follower of Leibniz in attempting to create a doctrine of probability pertaining to a general science of logic. Lambert's efforts, founded on an intuitive notion of normal numbers, proved to be ahead of its time. True, Cournot [14, p.57-58] and Chuprov [15, p.188] had noted Lambert's efforts, but nobody became interested in their accounts.

Poisson [16, p.140-141] hesitatingly offered a definition of a random variable as one that assumed several different values with corresponding probabilities. No one referred to this definition. Poisson also attempted to state the nature of chance (1.c., p.80). Randomness, he argued, was an *ensemble* of causes that produced an event without altering its (the event's) chances of happening or failing. This idea seems unsuccessful, but at least Poisson thus maintained that random events possessing stable probabilities of the two possible outcomes do occur.

While attempting to construct the theory of probability anew, von Mises [17, p.62] introduced the celebrated concept of *Kollektiv* and demanded that the order in which its elements followed each other be random (*mit zufallsartiger Zuordnung etc.*). Later on [18, 1928, 1936] he began to use an equivalent term, irregularity, and, finally [18, 1939, p.32], he equated chance with *complete 'lawlessness'*, cf. §6, and (1.c., p.133) noted its *fundamental importance* for the theory of probability. His endeavors bore fruit. As a result of his work, mathematicians [19] became interested in defining the *Kollektiv* (the infinite random sequence) and attempts of such kind are now continued in the modern theory of algorithms. Three approaches are now recognized [20, p.199--214]. The *frequency* approach had originated with von Mises (and even with Lambert) and Kolmogorov modified it in 1963. It is based on demanding that the various elements of a random sequence and of its *legitimate* subsequences should appear with stable frequencies. According to the approach founded on *complexity* (Kolmogorov, 1963), the entropy of the initial part of a random sequence should be sufficiently large. The main idea of the *quantitative* approach (Martin-Löf, 1966) is that a random sequence may only have a small number of regularities and, consequently, that it should pass certain tests. It is easy to see that these approaches are not independent. In 1963 Kolmogorov additionally outlined the concept of a finite random sequence ; according to his opinion, a finite sequence is the *more random* the more complex is the law that describes it. Quite recently there appeaared another Russian paper [21] on the same subject with no reference given to the previous one. As in the case of ref [20], Uspensky was its coauthor, but this time the second, or, rather, the first coauthor was Kolmogorov himself.

3. RANDOMNESS DOES NOT EXIST

Such was the standpoint of the most eminent thinkers and scholars who believed that its semblance resulted from ignorance of relevant causes. Sambursky [7, p.40-41] described the utterances of ancient Greek authors on this subject and Kendall [6, p.11] studied similar ideas due to St Augustine, Thomas Aquinas, Spinoza and d'Alembert. In turn, I discuss the thoughts of several scientists without dwelling on the writings of Bentley [22, p.316--318] who somewhat verbosely explicated Newton's point of view or Lamarck [23, p.74 and 97 ; 24, p.329]. Here are the statements of Kepler [25], Laplace [26, p.145], and Darwin [27, p.128], in that order :

1. Chance is an idol, an abuse of God Almighty.
2. Chance is only ignorance of the connections between phenomena.
3. That chance occasions variations between individuals is wrong, but this expression *serves to acknowledge ... our ignorance* of the relevant causes.

Kepler, however, was unable to deny that the eccentricities of the planetary orbits were random (§5). Newton left two utterances [28, Query 31 ; 29, p.49] which testify that he attached certain importance to chance and to which I shall return in §§7 and 8 :

> 1. ... *blind chance could never make all the planets move one and the same way in orbs concentrick, some inconsiderable irregularities excepted, which may have risen from the mutual actions of comets and planets upon one another, and which will be apt to increase, till this system wants a [*divine*] reformation. Such a wonderful uniformity in the planetary system must be allowed the effect of choice. And so must the uniformity in the bodies of animals.*
> 2. *Did blind chance know that there was light and what was its refraction, and fit the eyes of all the creatures after the most curious manner to make use of it ?*

Lamarck [30, p.450] thought that variations between individuals came into existence because of random causes ; and the Darwinian theory in its entirety hinged on the action of these same causes (I mention Laplace in §5). It is extremely strange that in spite of his own statistical explanation of the second law of thermodynamics, Boltzmann failed to recognize either the latter fact or the importance of randomness in nature [31, §4.3].

4. A POSSIBILITY

Randomness is a possibility. This definition goes back to Aristotle [32, 1064b-1065a] who moreover apparently believed that a chance event had a logical or subjective probability less than 1/2. Similarly, Thomas Aquinas [33, Vol.19, p.297] supposed that random events *proceed from their causes in the minority of cases...*

The followers of the Indian teaching of Syadvada which existed as early as in the sixth century B.C. studied the concepts of the possible, the indeterminate, *etc.* Mahalanobis [34] maintained that this doctrine was interesting for the history of statistics. He did not mention randomness but I believe that the Syadvada indirectly recognized it as a possibility.

Darwin [35, Vol.1, p.449], drawing on stochastic calculations made at his request by Stokes, decided that a particular deformity in man was passed from parent to child and did not occur by chance [was not merely possible]. W. Herschel [36, p.577] and Struve [37, note 72] left room for randomness of this kind. In their models of the stellar system, they only restricted the distances of the stars of a given magnitude without indicating their actual position. Maxwell [38, p.274] remarked that neither the form and dimensions of the planetary orbits, nor the size of the earth were determined by any law of nature [that the relevant quantities might have been different]. He did not mention randomness. His remark has to do with yet another interpretation of chance (§6).

Hegel [39, p.383] in addition to understanding randomness as a possibility, formulated the converse proposition : *Das Zufällige ist ein Wirkliches, das zugleich nur als möglich bestimmt* ... and *was möglich ist, ist selbst ein Zufälliges*. It is easy to illustrate this proposition. If a random variable X assumes values x_i with probabilities p_i (i=1,2,...,n) then any possible x_i is random in the sense that it occurs with probability p_i.

Note that Aristotle did not connect any definite probabilities with the possible values of x_i (i = 1, 2).

5. DEVIATION FROM LAWS OF NATURE

Randomness occurs when the purpose of nature is not attained, when hindering causes corrupt the operations of nature. This explanation is due to Aristotle [40, 199b] who thought that nature's accidental mistakes brought about the appearance of monsters and that the birth of female animals was the first departure from the *type* and, at the same time, a *natural necessity* [41, 767b]. His statements were the first to confront necessity and randomness. Indeed, the random occurrence of monsters accompanies the necessary acts of regular births whereas the birth of a female according to Aristotle's opinion is both necessary and random. Of course, from a modern point of view the second example is wrong ; in addition, it hardly corresponds to his own belief (§4) that the probability of a chance event is less than 1/2.

Referring to the Philosopher, Thomas Aquinas [33, Vol.19, p.489] pointed out that the birth of a girl was a random event.

Kepler [42, p.244 ; 43, p.932] suggested that only *zufällig* perturbations had forced the planets to deviate from circular motion. True, he also stated that the eccentricities regulated the planets' motions [44, p.317] but of course he was unable to say why the eccentricity of a given orbit had a particular value rather than any other one. Kant [45, p.337] repeated Kepler's pronouncement on the elliptic paths of the planets.

Lamarck [24, p.133] maintained that there existed deviations from the divine lay-out of the tree of animal life and he explained them by the action of a *cause accidentelle et par conséquent variable*.

The pronouncements described above pertained to determinate laws of nature. However, many natural scientists, while making similar statements, actually thought about mean states. Adanson [46, p.48] regarded intraspecific variations as digressions from the divine order and believed them necessary *pour l'équilibre des choses*. Lamarck [47, p.76] argued that *plusieurs causes*, some of them *variables, inconstantes et irrégulières dans leur action* corrupted [determined !] the [mean] state of the atmosphere. Humboldt [48, p.68] conditioned the study of all natural phenomena by discovering the appropriate mean values (mean states). As early as in 1817 he isolated climatology from meteorology [49]. His point of view was not, however, quite consistent in that he did not link his definition of climate [50, p.404] with mean states but at least subsequent scientists had improved on him [51, p.296].

A. de Moivre [52, p.253] declared that the value of the parameter of the binomial distribution of male and female births was of divine origin. Quite logically, he regarded as random only the deviation of the number of male (say) births from the corresponding number determined by the binomial law. Random, in modern notation, for de Moivre was not X itself, but rather (X - EX). A. de Moivre (l.c., p.251) also argued that *in process of Time, Irregularities* [produced by chance] *will bear no proportion to the recurrency of that Order which naturally results from Original Design.*

Being greatly influenced by Newton and having devoted to him the first edition of his book [53], de Moivre nevertheless did not repeat Newton's inference on the need for divine reformation (§3). Finally, de Moivre [53, p.329] effectively declared that the aim of the theory of probability was to isolate chance from divine design [from purpose], and he thus came close to another understanding of randomness (§6).

Similarly, for Laplace the theory of probability pertained to natural science rather than to mathematics, and its goal was not the study of mathematical objects (for example, of densities), but the discovery of the laws of nature. He therefore stood in need of analysing observations, of eliminating randomness from them, of separating chance from law.

6. LACK OF PURPOSE

Randomness is the lack of divine law or goal ; it occurs when independent chains of events intersect each other. Again, randomness is lack of purpose and, perhaps, "uniformity" (§7) as well. It was in this sense that chance was understood in ancient India, about two thousands years ago, although not in natural science, but in civil life [1, §108] : if shortly after giving evidence at a trial, a misfortune befell a certain witness or his family, it was believed that he was punished by God [that the evil did not happen without purpose, i.e., by chance].

6.1. Lack of Law or Goal

According to Aristotle, an unexpected meeting of two friends [32, 1025a] or a discovery of a buried treasure [40, 196b] are chance events. Each of these events could have been aimed at. Junkersfeld [5, p.22] who considered numerous examples contained in the great scientist's works inferred that he would not have thought that coming across a stranger or finding a rusty nail were random.

The ancient Indian Yadrichchha or Chance Theory contained an interesting illustration of randomness [54, p.458] :

> *The crow had no idea that its perch would cause the palm-branch to break, and the palm-branch had no idea that it would be broken by the crow's perch : but it all happened by pure chance.*

These examples show that the interpretation of chance as an intersection of chains of events was known even in antiquity. And in this connection Cournot [14, p.56] had quoted Boethius and Bru [55, p.306] noticed that Cioffari [56, p.77-84] had discussed/reproduced appropriate passages from several ancient scholars.

Hobbes [57, p.259] maintained that a traveller *meets with a shower* by chance since *the journey caused not the rain, nor the rain the journey*. Much the same was the opinion of many modern scientists [3, p.133] and of course in each reasoning of this kind the interpretation mentioned above simply suggests itself.

Darwin [58, p.395] argued that he had used the word *chance* only in relation to purpose [to lack of purpose] in the origination of species. He continued : *the mind refuses to look at* [the universe] *as the outcome of chance--that is, without design or purpose.*

The d'Alembert-Laplace problem merits special attention. The word *Constantinople* is composed of separate letters ; is it possible that the choice and arrangement of the letters were random ? D'Alembert [59, p.245-255] who questioned the fundamentals of the theory of probability maintained that all arrangements of the letters were equally probable only from the mathematical point of view but not in reality. Laplace [26, p.152 ; 60, p.XV] came to a different conclusion : since the word had a certain meaning [answered a particular purpose] the composition was not likely at all to have been accidental [aimless].

This reasoning helps to understand properly a number of earlier opinions. Aristotle [61, 289b] believed that it was impossible for the stars to move independently one from another [to move at random] and yet to remain fixed. They possessed common motion, he inferred. A similar idea can be traced in the theory of errors. A large deviation of an observation from the arithmetical mean had rather been assigned to a special reason (though not to a goal, or a law, but to a blunder) than attributed to an unlikely combination of admissible and mutually independent [accidental] errors.

Kepler [62, p.397] thought that a possible (a chance, see §4) appearance of a new star in a definite place and on a particular date was so unlikely that it had to be occasioned on purpose. By implication, he believed that each place (and each date) was equally probable. Thus, Kepler understood randomness not only as lack of purpose but as something [aimlessly] possible (§4) and, at the same time, as uniform (§7).

6.2. Intersections of Chains of Events

Randomness is an intersection of such chains. This interpretation is due to La Placette [63, last page of Preface] who contended that *le Hasard renferme... un concours de deux, ou de plusieurs événements contingents*. Each event had its own cause, the author continued, but we did not know the reason why they coincided. La Placette did not explain randomness ; his definition was no better than saying that the cause of any chance event was unknown (cf. §3). He devoted his book to proving that games of chance were not contrary to Christian ethics.

Cournot [55, §40 ; 14, p.52] took up La Placette's idea and in one instance [14, p.57] he referred to the latter. Cournot [55] initially mentioned chains of determinate events thus improving on La Placette :

> *Les événements amenés par la combinaison ou la rencontre de phénomènes qui appartiennent à des séries indépendantes, dans l'ordre de la causalité, sont ce qu'on nomme des événements fortuits...*

In his later work [14] Cournot regrettably omitted the phrase *dans l'ordre de la causalité*.

Cournot [55, §§41--48] apparently thought of using his definition of randomness to present the theory of probability as a science of chance events. He could not have succeeded. What was really needed was a systematic use of the notions of a random variable (cf. §2) and of its expectation and variance.

7. UNIFORMITY

Randomness is something *uniformly possible*, it can occur in one out of several equally possible ways.

7.1. *Uniform Randomness*

In §6.1. I stated that Kepler had equated chance with *uniform randomness*. This attitude was characteristic of natural scientists for about two centuries. Arbuthnot [64], in attempting to explain the prevalence of boys among the newly-born, contrasted uniform randomness and design without thinking of other possible laws of randomness. The same kind of comparison is implied in both of Newton's pronouncements (§3).

Jakob and Niklaus Bernoulli and de Moivre are known to have introduced the binomial distribution into the theory of probability. In spite of this, however, the former understanding of randomness persisted. Boyle [65, p.43], indicating that a chance composition of a long sensible text was impossible, declared that the world could not have been created randomly. The first part of his statement is also contained in the *Logique de Port-Royal* [66, chapter 16]. Kant [45, p.230] and Voltaire [67, p.316] maintained that a uniformly random origin of organic life was even less possible than a similar origin of the system of the world. Daniel Bernoulli [68] and Laplace [69], likely following Newton, calculated the probability that the regularities observed in the Solar system were due to randomness and they only contrasted blind chance and a determinate cause.

Maupertuis [70, p.120-121] indicated that the seminal liquid *de chaque individu* most often contained *parties* similar to those of their parents. He also mentioned rare cases when a child resembled one of his remote ancestors (p.109) as well as mutations [a subsequent term] (p.121). It could have been inferred that Maupertuis recognized randomness with a multinomial distribution. He was not, however, consistent : while discussing the origin of eyes and ears in animals [71, p.146], he restricted himself to comparing *une attraction uniforme & aveugle* and *quelque principe d'intelligence* (and came out in favor of design).

In the 19th century many scientists, imagining that randomness was only uniform, refused to recognize the evolution of species. While illustrating this idea, both the astronomer J. Herschel [72, p.63] and the biologist Baer [73, p.6] mentioned the philosopher depicted in the *Gulliver's Travels*. Hoping to get to know all the truths, this good-for-nothing inventor put on record each sensible chain of works which happened to appear among their *uniformly* random arrangements.

Also in the 19th century, Boole [74, p.256] argued that the distribution of stars was random if, owing to the ignorance of the relevant law, *it would appear to us as likely that a star should occupy one spot of the sky as another* (cf. §3). And he continued : *Let us term any other principle of distribution an indicative one.* Even in 1904 Newcomb [75, p.13] called the uniform distribution of stars *purely accidental.* Recalling the definition of a finite random sequence as outlined by Kolmogorov (§2) and bearing in mind that the number of stars of the first few magnitudes is finite, I note, however, that Boole's and Newcomb's inferences were quite modern.

The following examples which have to do with finite populations of stars or atoms are similar. Nevertheless, in these instances natural scientists reasonably believed that uniform randomness represented a statistical law of nature. Thus, Forbes [76, 1849] contended that *An equable spacing of stars...* [was] *far more inconsistent with a total absence of Law or Principle, than the existence of* [regions of condensation and paucity] *of stars.* He also asked [76, 1850, p.420] what distributions might be called random [as not representing any law, cf. §6.1].

In 1906 Kapteyn [77, p.400] declared that *The peculiar motions of the stars are directed at random, that is, they show no preference for any particular direction.* Struve [78, p.132-133] pronounced a similar weaker statement even in 1842. Boltzmann [79, p.237 ; 80, 321] held that gas molecules move with equal probability in whichever direction, but he did not mention randomness.

Sometimes chance might have been connected with the state of chaos, i.e., with the absence of any law of distribution. Since this possibility was hardly discussed before the 19th century, I believe that either nobody considered it, or, in any case, that it gradually gave way, perhaps unjustly, to uniform randomness. In those times, apparently only de Moivre [52, p.251-252] mentioned chaos and even he dismissed it out of hand. *Absurdity follows*, he declared, if a certain event happened not *according to any Law* [de Moivre meant one or another value of the parameter of the binomial distribution], *but in a manner altogether desultory and uncertain ; for then the Events would converge to no fix Ratio at all.*

While introducing his definition of probability as the limit of statistical frequency, von Mises [17, p.60] effectively excluded chaos.

Against the background of the abovementioned examples it is interesting to name two philosophers of the 18th century who expressly indicated that *non-uniform* randomness was indeed possible. Hume [81, Vol.1, p.425], while discussing chance events, illustrated his ideas by considering an imaginary die having *four sides marked with a certain number of spots, and only two with another.*He did not, however, refer to any law of nature. Holbach [82, pt 2, p.138-139] maintained that the *molecules* of various bodies greatly differed one from another and combined with each other in diverse ways and he compared them with *dice pipées ... d'une infinité de façons différentes* [with irregular dice].

7.2. Specifying Particular Problems

During the 19th century, it gradually became clear that the concept of uniform randomness *in general* was not sufficiently intelligible. The problem of determining the distance between two random points (*A* and *B*) on a sphere is highly relevant since Laplace [83, p.261] and Cournot [55, §148] understood it in differing senses. Laplace believed that *B* was with equal probability any point of the great circle *AB* whereas Cournot's solution implied that all possible situations of *B* on the given sphere were equally probable. Similarly, Daniel Bernoulli had calculated the probability that the planes of the planetary orbits were close to each other due to uniform randomness (cf. §6.1.) but Todhunter [84, §396] remarked that it would have been more natural to consider uniform randomness in respect to the closeness of the poles of the orbits.

Darwin [85, p.52-55] attempted to ascertain whether earth worms carrying small objects into their burrows seize *indifferently by chance* any part of the find. He considered four versions of such randomness in regard to the manner of capturing paper triangles which he had strewn about on the ground. After calculating the corresponding frequencies, Darwin decided that earth worms carry the triangles in a non-random manner, i.e., to a certain extent sensibly. Considering non-randomness on a par with reason Darwin therefore recognized chance as lack of purpose ; in §6.1. I have mentioned him exactly in this connection.

Bertrand [86, p.6-7] took up the problem of calculating the distance between *random* points on a sphere. Without mentioning Laplace or Cournot, he repeated their solutions and concluded that both were correct. In addition, Bertrand maintained that not only small distances but other geometric features as well might be used to characterize an unlikely scatter of the stars over the sky. He hardly knew about Darwin's experiment, but he provided a few more examples including his celebrated problem on the length of a *random* chord of a given circle. He thus proved that *uniform randomness* was not definite enough and he justly insisted that in particular instances this concept be specified.

8. INSTABILITY OF MOTION

Randomness is instability of motion, it involves slight causes leading to considerable consequences. Galen [87, p.202], without mentioning randomness, asserted that *in old men even the slightest causes produce the greatest change.* According to Newton (§3), the accumulation of irregularities in the planetary system may be interpreted as an action of slight causes giving rise to considerable effects.

Maxwell [88, p.366] prophetically argued that physicists will study *singularities and instabilities* thus moving away from mere determinacy. Illustrating this idea, he referred to the unstable refraction of rays within biaxial crystals (p.364). Maxwell thus connected randomness with instability, though he did not say so directly. And he expressed similar thoughts elsewhere [89, p.295-296] :

> *There is a very general and very important problem in Dynamics... It is this--'Having found a particular solution of the equations of motion of any material system, to determine whether a slight disturbance of the motion indicated by the solution would cause a small periodic variation, or a total derangement of the motion...'.*

Von Kries [90, p.58], while discussing a game of chance, noted that *eine kleine Variirung der Bewegung hinreichend, um an Stelle des Erfolges Schwarz den Erfolg Weiss herbeizuführen...* This remark was noticed by von Plato [91, p.83]. (Cf. the discussion of the game of roulette in §7.1).

Pirogov [92, p.518] called an event random if its dependence on the relevant causes was complicated and *mit Hülfe von nur analytischen Functionen gar nicht ausgedrückt werden kann.* His utterance may be considered as another hint at the connection between chance and instability. As to complicated causes, see §9.

As stated in §1, I am not discussing the work of Poincaré, but I shall at least emphasize that he was the first to say expressly that randomness is instability of motion.

9. COMPLICATED CAUSES

Randomness occurs when complicated causes are involved. In a heuristic sense Leibniz [93, p.288] anticipated this explanation by declaring that the *zufällige Dingen* were those *deren vollkommener Beweis jeden endlichen Verstand überschreitet.*

While formulating his celebrated law of the velocities of gas molecules , Maxwell [94] reasonably supposed that the distribution sought sets in *after a great number of collisions among a great number of equal particles.* He did not mention randomness. Elsewhere [95, p. 436] Maxwell remarked that the motion of heat is *perfectly irregular* and that the velocity of a given molecule can not be predicted. Once more, he did not mention rendomness and he said nothing about complicated causes. I have adduced his pronouncement since it supplements his previous idea. Also note that Maxwell actually corrected Laplace's famous declaration on the possibility of calculating the future states of the universe [60, p.VI].

10. SLIGHT CAUSES LEADING TO SMALL EFFECTS

Randomness occurs when slight causes lead to small effects. Laplace [96, p.504] qualitatively explained the existence of trifling irregularities in the system of the world by the action of countless [small] differences between temperatures and between densities of the diverse parts of the planets. He did not mention randomness. Kepler and Kant (§5) referred in similar cases to deviations from purpose.

11. - NECESSITY AND RANDOMNESS

In discovering laws and regularities of nature and in studying its mean states, scientists determined necessity. Besides this, they often revealed, or even attemped to isolate, the unavoidable accompanying phenomena of the second order, i.e., randomness. And it was exactly in this manner that many natural scientists imagined the relation between necessity and chance. Recall in this connection Aristotle's opinion (§5) on the appearance of monsters, Keplers's reasoning on the eccentricities of the planetary orbits (§5), Newton's thoughts (§3, also see below) on the planetary system, Lamarck's utterance (§5) on the tree of animal life, de Moivre's reasoning (§5) on the ratio of male and female births as well as the isolation of climatology from meteorology achieved by Humboldt (§5) and W. Herschel's and Struve's models of the stellar system (§4).

Lamarck's pronouncement [24, p.169] merits special attention. He apparently believed that necessity and chance were the two main *moyens* of nature. Without proving anything or producing any example he declared that these *moyens puissans et généraux* were universal attraction and a repulsive molecular action *qui... varie sans cesse...* He also argued that the *equilibre entre ces deux forces opposées...naissent... les causes de tous les faits que nous observons, et particulièrement de ceux qui concernent l'existence des corps vivans.*

Lamarck likely supposed that the molecular action was random since elsewhere (see §5) he maintained that by definition accidental causes were variable.

Without dwelling on the statistics of marriages, suicides, crime, *etc.* that reveals laws in apparently free [random] behavior of man, I note that Kant [97, p.508] compared the chance birth of a man with the stability of the birth-rate : *der Zufall im Einzelnen nichts desto weniger einer Regel im Ganzen unterworfen ist....*

Only Hegel, after offering his definition of randomness (§4), formulated a proposition on the unity [on the interdependence] between necessity and chance. Exactly this unity, he declared [39, p.389], *ist die absolute Wirklichkeit zu nennen.* Engels [98, p.213] approvingly called this thesis utterly unheard of and urged scientists to study both necessity and chance. However, it was Poincaré [99, p.1] who offered the most important statement :

> *Dans chaque domaine, les lois précises ne décidaient pas de tout, elles traçaient seulement les limites entre lesquelles il était permis au hasard de se mouvoir. Dans cette conception, le mot hasard avait un sens précis, objectif...*

A few words about the theory of probability. At the end of §5 I have mentioned de Moivre and Laplace in connection with the aim of this scientific discipline. To formulate it now more precisely, they entrusted the theory with delimiting randomness from necessity. In these days, the same goal is being achieved by mathematical statistics created since then.

K. Pearson [100] remarked that the development of the theory of probability was much indebted to Newton. I shall show that he thought about the great scientist's idea on the relation between necessity and chance.

> *Newton's idea of an omnipresent activating deity, who maintains mean statistical values .* Pearson stated, *formed the foundation of statistical development through Derham, Süssmilch, Niewentyt, Price to Quetelet and Florence Nightingale.* And, further :
>
> *A. de Moivre expanded the Newtonian theology and directed statistics into the new channel down which it flowed for nearly a century. The causes wich led de Moivre to his "Approximatio"* [to the memoir [52] where the normal approximation to the binomial distribution was first discovered] *or Bayes to his theorem were more theological and sociological than purely mathematical, and until one recognizes that the post-Newtonian English mathematicians were more influenced by Newton's theology than by his mathematics, the history of science in the 18th century - in particular that of the scientists who were members of the Royal Society - must remain obscure.*

Since Newton never mentioned the maintaining of mean values, I believe that Pearson actually thought about divine reformation, necessary, according to Newton (§3) for neutralizing the propagation of chance corruptions in the Solar system, for preserving the mean states. Thus, Pearson suggested that Newton's theologically formulated idea concerning the relation between necessity and chance had served as the basis for the development of the theory of probability. Pearson's general statement about the science in the 18th century may be specified : first, he apparently bore in mind Laplace (end of §5 and above). Second, restricting myself to the theory of probability, I note that Pearson [101, §§ 10.1, 10.2] put forward plausible arguments in favor of the thesis on Newton's influence upon Bayes (and Price, who communicated, and inserted comments in his classical memoir).

12. CONCLUSIONS

The denial of randomness (§3) was only formal and nowadays this attitude seems to be deservedly forgotten. Possibility (§4) found its way into laws and empirical regularities, but it was Hegel who declared that randomness was a possibility and, moreover, that the possible was random. Chance as deviation from the laws of nature (§5) is recognized as a perturbation (a noise) and natural scientists admitted that it indeed corrupted the laws. As far as the deviations obey the preconditions of the central limit theorem, this randomness is normal. Randomness as lack of law or purpose (§6) may be interpreted as an intersection of independent chains of events. The definitions of §4 and §6, while reflecting different heuristic features of randomness, essentially coincide. Randomness is a [random variable] having a uniform distribution (§7), i.e., it is a special case of the possible (§4). Therefore, this *uniform* randomness characterizes lack of determinate law or purpose (§6) ; at the same time, in some instances it signifies the existence of a special statistical law of nature.

Randomness is occasioned by instability (§8) and/or complicated causes (§9). It can also occur in the context of slight causes leading to slight effects (§10). This case partly includes *deviations from the laws of nature* (§5), as in meteorology and astronomy. The joint action of a large number of such causes can lead to random variables with a normal distribution (above).

It is scarcely possible to comprehend rendomness without studying its interconnection with necessity. Hegel stated that these concepts were united. However, even after Hegel scientists had recognized randomness only as a phenomena of the second order accompanying the main phenomenon, necessity (§11).

The explanations and definitions of chance (§§4--10) are heuristically connected with the modern interpretations of randomness (§2). Thus, §9 is closely linked with the complexity approah and to a lesser degree a similar link seems also to apply to §8 ; §7 illustrates a particular instance of the frequentist approach and the rest of these sections at least do not contradict the quantitative approach. Finally, I note that §§4--6 and 10 are linked with §11.

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BIBLIOGRAPHIE

[1] *Laws of Manu*, Oxford, 1886, (Sacred book of the East, Vol.25).

[2] CHIRIKOV, B.V., "The nature of the statistical laws of classical mechanics", *in : Methodological and philosophical problems of physics*, Novosibirsk, 1982, 181-196. *In* Russian. [*Metodologich i filosofsk. problemy fiziki*].

[3] SHEYNIN, O.B., "On the prehistory of the theory of probability", *Arch. hist. ex. sci.*, Vol.12, n°2, 1974, 97-141.

[4] NALIMOV, V.V., *The language of stochastic concepts*, Moscow, 1976, *in* Russian. [*Yazyk veroiatnostnykh predstavlenii*]

[5] JUNKERSFELD, J., *The Aristotelian-Thomistic concept of chance*, Notre Dame, Indiana, 1945.

[6] KENDALL, M.G., "The beginnings of a probability calculus", *Biometrika*, Vol.43, n°1-2, 1956, 1-14.

[7] SAMBURSKY, S., "On the possible and probable in ancient Greece", *Osiris*, Vol.12, 1956, 35-48.

[8] DAVID, F.N., *Games, gods and gambling*, London, 1962.

[9] BYRNE, E.F., *Probability and opinion*, The Hague, 1968.

[10] RABINOVITCH, N.L., *Probability and statistical inference in ancient and medieval Jewish literature*, Toronto, 1973.

[11] GNEDENKO, B.V., *From the history of the science of the random*, Moscow, 1981, *in* Russian. [*Iz istorii nauki o sluchainom*].

[12] SHEYNIN, O.B., "Newton and the classical theory of probability", *Arch. hist. ex. sci.*, Vol.7, n°3, 1971, 217-243.

[13] SHEYNIN, O.B., J.H., "Lambert's work in probability", *Ibid.*, 244-256.

[14] COURNOT, A.A., *Essai sur les fondements de nos connaissances*, t. 1. Paris 1851.

[15] CHUPROV, A.A., *Essays on the theory of statistics* (1909), Moscow, 1959, *in* Russian [*Ocherki po teorii statistiki*].

[15] POISSON, S.D., *Recherches sur la probabilité des jugements etc.*, Paris, 1837.

[17] von MISES, R., "Grundlagen der Wahrscheinlichkeitsrechnung (1919)", *Sel. papers*, Vol.2, Providence, Rhode Island, 1964, 57-105.

[18] von MISES, R., *Wahrscheinlichkeit, Statistik und Wahrheit.* (1928, 1936, 1951), Engl. transl. : London, 1939 ; New York - London, 1964.

[19] CHURCH, A., "On the concept of a random sequence", *Bull. Amer. Math. Soc.*, Vol.46, n°2, 1940, 130-135.

[20] USPENSKY, V.A. , SEMENOV, A.L., *The theory of algorithms : main discoveries and applications*, Moscow, 1987, *in* Russian. [*Teoria algoritmov* etc.].

[21] KOLMOGOROV, A.N., USPENSKY, V.A., "Algorithms and randomness", *in* Russian [*Teoria veroiatnostei i ee prim*], Vol.32, n°3, 1987, 425-455.

[22] BENTLEY, R., "Sermons (1693)", *in Newton*, I., *Papers and letters on natural philosophy*, Cambridge, 1958.

[23] LAMARCK, J.B., "Aperçu analytique des connaissances humaines", (MS, 1810-1814), *in* Vachon, M. et al., *Inédits de Lamarck*, Paris, 1972, 69-141.

[24] LAMARCK, J.B., *Histoire naturelle des animaux sans vertèbres*, t.I, Paris, 1815.

[25] KEPLER, J., "De stella nova (1606)", French transl. of a passage *in* : Servien, P., *Science et hasard*, Paris, 1952, p. 132.

[26] LAPLACE, P.S., "Recherches sur l'intégration des équations différentielles (1776)", *Œuvr. compl.*, t.8, Paris, 1891, 69-197.

[27] DARWIN, C., *Origin of species* (1859), London-New York, 1958.

[28] NEWTON, I., Optics (1704), *Opera*, Vol.4. London, 1782, 1-264.

[29] NEWTON, I., *Theological manuscripts*, Liverpool, 1950.

[30] LAMARCK, J.B., "Espèce", *Nouv. dict. hist. natur.*, t.10, 1817, 441-451.

[31] SHEYNIN, O.B., "On the history of the statistical method in physics", *Arch. Hist. Ex. Sci.*, Vol.33, n°4, 1985, 351-382.

[32] ARISTOTLE, *Metaphysica.Works*, Vol.8, Oxford.
[33] AQUINAS, THOMAS, *Summa theologica*, Engl. transl. : *Great books of the Western world*, Vols.19-20. Chicago, 1952.
[34] MAHALANOBIS, P.C., "The foundations of statistics", *Sankhya, Ind. J. Stat.*, Vol.18; n°1-2, 1957, 183-194.
[35] DARWIN, C., *The variation of animals and plants under domestication*, Vols1-2, London, 1885
[36] HERSCHEL, W., "Astronomical observations and experiments etc. (1817)", *Scient. papers*, Vol.2, London, 1912, 575-591.
[37] STRUVE, F.G.W., *Études d'astronomie stellaire*, St Petersbourg, 1847.
[38] MAXWELL, J.C., "Discourse on molecules (MS, 1873)", *in* : Campbell, L. & Garnett, W., *Life of Maxwell* (1882), London, 1884, 272-274.
[39] HEGEL, G.W.F., *Wissenschaft der Logik*, T.1.1 (1812), Hamburg, 1978.
[40] ARISTOTLE, *Physica.Works*, Vol.2.
[41] ARISTOTLE, *De generatione animalum. Works*, Vol.5.
[42] KEPLER, J., *Neue Astronomie*(1609, in Latin), München-Berlin, 1929.
[43] KEPLER, J., *Epitome of Copernican astronomy*, book 4 (1620, in Latin), *in Great books of the Western World*, Vol.16, Chicago, 1952, 845-1004 (books 4 and 5).
[44] KEPLER, J., *Welt-Harmonik* (1619, in Latin), München-Berlin, 1939.
[45] KANT, I., *Allgemeine Naturgeschichte und Theorie des Himmels* (1755), *Werke*, Bd 1, Berlin, 1910, 215-368.
[46] ADANSON, M., "Examen de la question si les espèces changent parmi les plantes", *Hist. Acad. Roy*, Paris avec *Mém. math. et phys.*, 1769 (1772) ; *Mém.*, 31-48.
[47] LAMARCK, J.B., *Annuaire météorologique* [t.1], Paris, pour l'an 8 (1800).
[48] SHEYNIN, O.B., "On the history of the statistical method in meteorology", *Arch. Hist. Ex. Sci.*, Vol.31, n°1, 1984, 53-95.
[49] HUMBOLDT, A., "Des lignes isothermes", *Mém. Phys. Chim. Soc. d'Arcueil*, t.3, 1817, 462-602.
[50] HUMBOLDT, A., *Fragmens de géologie et de climatologie asiatiques*, t.2. Paris, 1831.
[51] KÖRBER, H.G., "Über Humboldts Arbeiten zur Meteorologie und Klimatologie", *in* : *Humboldt, Gedenkschrift*, Berlin, 1959, 289-335.
[52] MOIVRE (de), A., *A method of approximating the sum of the terms of the binomial* $(a+b)^n$ *etc.*(1733, *in Latin*), [53, 1756, 243-254].
[53] MOIVRE (de), A., *Doctrine of chances* (1718, 1738, 1756), New York, 1967.
[54] BELVALKAR, S.K., RANADE , R.D., *History of Indian philosophy*, Vol.2. Poona, 1927.
[55] COURNOT, A.A., *Exposition de la théorie des chances et des probabilités* (1843), Paris, 1984, ed. B.Bru.
[56] CIOFFARI, V., *Fortune and fate from Democritus to St Thomas Aquinas*, New York, 1935.
[57] HOBBES, T., *Of liberty and necessity* (1646), *Engl. works*, Vol.4, London, 1840, 229-278.
[58] DARWIN, C., *More letters*, Vol.1, London, 1903.
[59] ALEMBERT (d'), Le ROND, J., "Doutes et questions sur le calcul des probabilités", *in* author's *Mélanges de littérature, d'hist. et de philos.*, t.5, Amsterdam, 1768, 239-264.
[60] LAPLACE, P.S., *Essai philosophique sur les probabilités* (1814). *Œuvr. compl.*, t.7, n°1, Paris, 1886, separate paging.
[61] ARISTOTLE, *De Caelo, Works*, Vol.2.
[62] KEPLER, J. , *Bericht vom neuen Stern. Ges. Werke*, Bd 1., München, 1938, 391-399.
[63] LA PLACETTE, J., *Traité des jeux de hasardl*, La Haye, 1714.
[64] ARBUTHNOT, J., "An argument for divine Providence etc (1712)", *in* : *Studies in the history of statistics and prob.*, Vol.2, London, 1977, 30-34.
[65] BOYLE, R., "Some considerations touching the usefulness of experimental natural philosophy (1663-1671)", *Works*, Vol.2, London, 1772, 36-49.

[66] ARNAULT, A., NICOLE, P., *Logique de Port-Royal* (1662), Paris, 1877.
[67] VOLTAIRE, *Homélies*, Première homélie (1767), *Œuvr. compl.*, t.26, Paris, 1879, 315-354.
[68] BERNOULLI, D., "Recherches physiques et astronomiques etc", *in Pièces qui ont remporté le prix de l'Acad. Roy. Sci. en 1734*, Paris, 1735, 95-122.
[69] LAPLACE, P.S., "Mémoire sur l'inclinaison moyenne descomètes (1776)", *Œuvr. compl.*, t.8, 1891, 279-321.
[70] MAUPERTUIS, P.L.M., *Venus physique* (1745), *Oeuvr.*, t.2. Lyon, 1756, 1-133.
[71] MAUPERTUIS, P.L.M., *Système de la nature* (1751), *Ibid.*, 135-184.
[72] HERSCHEL, J.F., "Sun (lecture, 1861)", *in* author's *Familiar lectures on scient. subjects*, London-New York, 1866, 47-90.
[73] BAER, K., *Zum Streit über den Darwinismus*, Dorpat, 1873.
[74] BOOLE, G., "On the theory of probabilities (1851)", *in* author's *Studies in logic and probability*, London, 1952, 247-259.
[75] NEWCOMB, S., *On the position of the Galactic*, Carnegie Instn. of Washington, 1904, n°10.
[76] FORBES, J.D., "On the alleged evidence for a physical connection between stars", *Phil. Mag.*, Vol.35, 1849, 132-133 ; Vol.37, 1850, 401-427.
[77] KAPTEYN, J.C., "Statistical methods in stellar astronomy", [*Repts*] *Intern. congr. arts and sci. St Louis-Boston*, 1904, N.p., 1906, Vol.4, 396-425.
[78] STRUVE, F.G.W., "Review of Struve, O., Bestimmung der Constante der Praecession. Petersburg, 1842", *Bull. scient. Acad. Imp. Sci. St. Pétersb.*, t.10, n°9 (225), 1842, 129-139.
[79] BOLTZMANN, L., "Über das Wärmegleichgewicht (1871)", *Wiss. Abh.*, Bd. 1. Leipzig, 1909, 237-258.
[80] BOLTZMANN, L., "Weitere Studien über das Wärmegleichgewicht (1872)", *Ibid.*, 316-402.
[81] HUME, D., *Treatise on human nature*, Vols.1-2 (1739), London, 1874.
[82] HOLBACH (d'), P.H.T., *Système de la nature*, pt.2, Paris, 1781.
[83] LAPLACE, P.S., *Théorie analytique des probabilités* (1812), *Œuvr. compl.*, t.7, Paris, 1886.
[84] TODHUNTER, I., *A history of the mathematical theory of probability* (1865), New York, 1965.
[85] DARWIN, C., *The formation of vegetable mould* etc., (1881), London, 1945.
[86] BERTRAND, J., *Calcul des probabilités* (1888), New York, 1972.
[87] GALEN, C., *De sanitata tuenda*. Engl. transl. : *Hygiene*, Springfield, Illinois, 1951.
[88] MAXWELL, J.C., *Does the progress of physical science tend to give any advantage to the opinion of necessity* etc., (Read 1873), In Campbell, L. et al [38], 357-366.
[89] MAXWELL, J.C., *On the stability of the motion of Saturn's rings* (1859), *Scient. papers*, Vol.1 (1890), Paris, 1927, 288-376.
[90] von KRIES, J., *Die Principien der Wahrscheinlichkeitsrechnung*, Freiburg i.B., 1886 (reprint : Tübingen, 1927).
[91] von PLATO, J., "Instability and the accumulation of small effects", *in Probability and conceptual change in scient. thought*, eds, M. Heidelberger & L. Krüger, Bielefeld, 1982, 77-93.
[92] PIROGOV, N.N., "Uber das Gesetz Boltzmann's", *Repertorium Phys.*, Bd.27, 1891, 515-546.
[93] LEIBNIZ, G. W., *Allgemeine Untersuchungen über die Analyse der Begriffe und wahren Sätze* (MS, 1686), in author's *Fragmente zur Logik*, Berlin, 1960, 241-303.
[94] MAXWELL, J.C., "Illustrations of the dynamical theory of gases (1860)", *Scient. papers*, Vol.1, 377-410.
[95] MAXWELL, J.C., "On the dynamical evidence of the molecular constitution of bodies (1875)", *Scient. papers*, Vol.2 (1890), Paris, 1927, 418-438.
[96] LAPLACE, P.S., "Exposition du système du monde", *Œuvr. compl.*, t.6. Paris, 1884 (reprint of the 1835 ed.).
[97] KANT, I., *Kritik der reinen Vernunft* (1781), *Werke*, Bd.3, Berlin, 1911.

[98] ENGELS, F., *Dialektik der Natur*, written 1873-1882, first publ. 1925, Berlin, 1971.
[99] POINCARE, H., *Calcul des probabilités*, Paris, 1896 et 1912, The second ed. was reprinted in 1923. The Introduction to this edition is a reprint of the article "Le hasard", *La Rev. du Mois*, t.3, 1907, 257-276.
[100] PEARSON, K., A. de Moivre, *Nature*, Vol.117, n°2946, 1926, 551-552.
[101] PEARSON, K., *The history of statistics in the 17th and 18th centuries*, lectures 1921-1933, ed. E.S. Pearson, London a.o., 1978.