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NOTE ON THE CHI-SQUARE STATISTIC OF ASSOCIATION IN 2×2 CONTINGENCY TABLES AND THE CORRECTION FOR CONTINUITY

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Research workers frequently do not have to make a decision but merely want some idea of how likely or unlikely the observed sample result is when some null hypothesis is true. Recently some authors argued that in those situations we can better report the smallest value of α for which the hypothesis would have been rejected (i.e. the exceedance probability of the sample result under this specific hypothesis), rather than the sole fact of whether or not this hypothesis should be rejected at the traditional .05 or .01 levels of significance (see for example Hays and Winkler [1]).

Suppose we apply this recommendation to the chi-square statistic of association in 2×2 contingency tables withe cell counts A,B,C,C and grand total N. First, we would have to calculate the χ_P^2-statistic by means of the well-known formula :

$$\chi_P^2 = \frac{N\left(|AD - BC|\right) - \frac{N}{2}\;^{2}}{(A+B)\,(C+D)\,(A+C)\,(B+D)}$$

(see Siegel [2]), and consequently we would determine the probability that χ^2 exceeds the obtained value by means of the tabular values of a chi-square

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distribution with one degree of freedom. This procedure seems self-evident, but as we will see, it is not always the most appropriate.

It is well-known that the use of Formula 1 implies a reference to the conditional distribution of χ_P^2 given the marginal totals. Fisher [3] stated that, so far as tests of independence are concerned, this restriction remains valid in situations where one or both margins are not fixed. However, is this restriction still valid when we merely want to know the exceedance probability of a sample result under the assumption that the hypothesis of independence is true ?

In field studies, for example, psychologists do not know until the end of the classification what the marginal totals are. Moreover, most of the time they do not know in advance the exact size of the sample since this depends on the free cooperation of the subjects. If a psychologist wants to know the relationship between, for example, sex and smoking, he will examine a number of subjects (e.g. all first year students !) and he subsequently classifies every observation into the double dichotomy Men/Women and Smokers/Non-smokers. It is clear that such a psychologist is not so interested in knowing the probability in χ_P^2 values which are greater than the observed χ_P^2-value in samples *with the same marginal totals*, but rather he wants to know the probability of χ_P^2 values greater than the observed χ_P^2-value in samples obtained by repeated application of *the whole sampling process*. The same holds for many experimental and differential studies in which the χ_P^2 statistic is computed. Mostly only the margins related to the 'stimulus alternatives' are fixed in advance, whereas the margins related to the 'response alternatives' are only known at the end of the classification. Also the reader in those domains is mainly interested in the interpretation of the results against the background of repeated application of the whole experiment since the frequency concept appeals to the common user's understanding.

In the present note we want to state that if one wishes merely to tell how 'unusual' the sample result is as compared with the sampling distribution under the assumption that the hypothesis of independence is true, it is better to use a chi-square statistic which does not include a correction for continuity rather than a test statistic which does include such a correction (as Formula 1), at least if one wishes to relate the chi-square statistic to the sampling distribution observed by repeated application of the whole sampling process[2].

However, we are not against a correction for continuity as such. We are only against it when it is computed by Formula 1 (or a similar formula) in the above mentioned situation where the marginal totals are not fixed in advance of the sampling. In that case we object because Formula 1 assumes that the margins are fixed. If, in spite of that reason Formula 1 is used in a 2×2 table with random margins, the result can no longer be considered a 'corrected' χ_P^2 in the sense that Formula 1 leads to a better estimation of the exact exceedance probability of the chi-square statistic, than can a formula without a built-in correction for continuity. The explanation for this can easily be found in the relationship between Yates' correction and Formula 1.

The principle of the correction for continuity as presented originally by Yates [4] amounts to reading the χ^2 table, not at the point that corresponds to the value of the χ_P^2 actually to be evaluated, but at a point halfway between this value and the next lowest possible value of χ_P^2 in a sample of equal size. It can be proved that for 2×2 contingency tables with fixed marginal totals and with one degree of freedom, the required midpoint can be found without knowing the next lowest possible value of χ_P^2 : it suffices to reduce the absolute value of each difference between the observed and expec-

2. If one is interested in the answer to the question as to whether the hypothesis of independance should or should not be rejected, the situation is different. In that case only a method is required which helps to decide between alternative hypotheses and it is clear that there are many possible ways of making this kind of decision. The reader may be referred, concerning this issue, to Tocher [5], Lehman [6], Plackett [7], Harkness & Katz [8], Mantel & Greenhouse [9], and Clark [10].

ted frequencies *fo-fe* in Formula 2 by 0.5 before squaring.

$$\chi_P^2 = \sum \frac{(fo-fe)^2}{fe} \qquad (2)$$

This practical rule is based on the fact that in all four cells the successive values of the difference *fo-fe* differ by unity if the estimated expected frequencies remain constant over the universe of samples. Consequently, decreasing the absolute difference *fo-fe* by 0.5 as well as using Formula 1 only make sense if the contingency table has one degree of freedom and if the marginal totals are considered as fixed in the sense that the same marginal totals must appear in any repetition of the "experiment" on N new random elements of the population.

For very large samples, the above restriction concerning the use of Formula 1 does not hold because when $N \to \infty$, χ_P^2 has the same limiting distribution for fixed and random marginal totals. But this is not so in small samples where Yates' correction is traditionally recommended.

An example for illustration of the foregoing is provided by Table 1. For the sake of simplicity we demonstrate our argument for the case $N = 8$, but the conclusions holds also for larger samples, i.e. situations where psychologists use traditionally the chi-square statistic. In the first column are specified the different sets of cell frequencies which can be registered in a sample with size 8 and with random marginal totals ≥ 1.[3] For each observed contingency table the χ_P^2 value was calculated from Formula 2 and from Formula 1, these values being indicated by χ_P^2 and χ_c^2. In column (3) are listed the different χ_P^2 values that can be registered. The contingency tables leading to these χ_P^2-values are specified in the first two columns. So, there is one table (2222) with χ_P^2 value 0 ; four tables (1133, 3311, 1313 and 3131) with $\chi_P^2 = 0$; and four tables (1222, 2132, 3221 and

3. By requiring that the marginal totals ≥ 1, 32 of the 165 possible tables with $N = 8$ were excluded.

Table 1

Comparison of the Distribution of Uncorrected and Corrected Values of χ_P^2

$N = 8$ (fixed) ; Marginal Frequencies Random (≥ 1) ; $P_{11} = P_{12} = P_{21} = P_{22} = 0.25$

Cell Frequencies $a\ b\ c\ d$ ($a \mid b$ / $c \mid d$)	Number of Permutations with the Same Value of χ_P^2	Magnitude of Chi Square: Uncorrected χ_P^2	Magnitude of Chi Square: Corrected χ_c^2		Tabular p-Value: Uncorrected	Tabular p-Value: Corrected		Exact p-value: Uncorrected	Exact p-value: Corrected		
		Formula 2	Gen.prin.	Formula 1		Gen. prin.	Formula 1		Gen.prin.	Formula 1	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
2222	1	0	0	0.500	1.000	1.000	0.480	1.000	1.000	0.074	0.297
1133	4			0.667			0.414			0.122	0.223
1223	4	0.036	0.018	0.320	0.850	0.893	0.572	0.891	0.891	0.156	0.481
0116	4	0.163	0.100	1.469	0.686	0.752	0.226	0.787	0.787	0.004	0.067
1124	8	0.178	0.171	0.178	0.673	0.679	0.673	0.784	0.784	0.174	0.655
0215	8	0.381	0.280	0.381	0.537	0.597	0.537	0.680	0.680	0.028	0.325
1232	8	0.533	0.457	0	0.465	0.499	1.000	0.659	0.659	0.290	1.000
0314	8	0.686	0.610	0.076	0.408	0.435	0.783	0.451	0.451	0.056	0.710
0224	4	0.889	0.788	0	0.346	0.375	1.000	0.416	0.416	0.290	1.000
1115	4										
0413	8	1.143	1.016	0	0.285	0.313	1.000	0.369	0.369	0.290	1.000
0233	8	1.600	1.375	0.178	0.206	0.241	0.673	0.334	0.334	0.174	0.655
2114	4	1.742	1.671	0.320	0.187	0.196	0.572	0.265	0.265	0.156	0.481
1205	8	1.905	1.824	0.076	0.168	0.177	0.783	0.213	0.213	0.056	0.710
1331	2	2.000	1.953	0.500	0.157	0.162	0.480	0.192	0.192	0.074	0.297
0422	8	2.667	2.334	0.667	0.102	0.127	0.414	0.157	0.157	0.122	0.223
0332	4	2.880	2.774	0.889	0.090	0.096	0.346	0.105	0.105	0.035	0.102
1016	8	3.429	3.155	0.381	0.064	0.076	0.537	0.070	0.070	0.028	0.325
2015	8	4.444	3.937	1.600	0.035	0.047	0.206	0.063	0.063	0.021	0.063
3014	8	4.800	4.622	2.133	0.028	0.032	0.144	0.043	0.043	0.035	0.042
1007	4	8.000	6.400	1.469	0.005	0.011	0.226	0.008	0.008	0.004	0.067
2006	4			3.556			0.059			0.002	0.007
3005	4			4.302			0.038			0.003	0.006
4004	2			4.500			0.034			0.002	0.002

2312) with χ_P^2 = 0.036, etc.

In column (4) are listed the χ_P^2 values corrected according to the general principle : thus χ_P^2 value 0.036 becomes 0.018 after correction because (0 + 0.036)/2 = 0.018. In column (5) are listed the χ_c^2 values obtained through the traditional Formula 1.

When comparing columns (3), (4) and (5), one notes that differences in uncorrected values are not necessarily accompanied by differences in corrected values. Also the order of corrected values calculated from Formula 1 corresponds only partially with the order of the uncorrected values. Hence, the results obtained through Formula 1 do not conform to the principle of the correction for continuity if the row and column totals are random variables.

In columns (6), (7) and (8) the exceedance probabilities of the values from columns (3), (4) and (5) are estimated from the tabular χ^2 distribution. The accuracy of these estimates can be evaluated by comparing these probabilities with the respective exact upper tail probabilities calculated from the multinomial rule. These values are listed in columns (9), (10) and (12).[4] Since the χ_c^2 values in column (5) are not given in increasing order, we have also listed, for the sake of clarity, the exact probabilities of these χ_c^2 values in column (11). For example, the value 0.074 at the top of column (11) is the probability of observing a χ_c^2 value of 0.500. The value 0.074 was obtained by adding the exact probabilities of all tables for which χ_c^2 = 0.500. In this case, the calculation was made from the tables 2222 (× 1) and 1331 (× 2). The exact probabilities for these tables are :

$$P(2222) = \frac{8!}{2!2!2!2!} (0.25)^2(0.25)^2(0.25)^2(0.25)^2 \frac{1}{0.984436} = 0.039$$

$$P(1331) = \frac{8!}{1!3!3!1!} (0.25)^1(0.25)^3(0.25)^3(0.25)^1 \frac{2}{0.984435} = 0.035$$

4. It was taken into account that 32 of the 165 possible tables were not counted. Therefore, the probabilities obtained through the multinomial rule were always divided by 0.984, i.e. 1 minus the sum of the probabilities of the 32 non-calculated tables.

and hence $p(\chi_c^2 = 0.500) = 0.039 + 0.035 = 0.074$. From these ordered χ_c^2 values, the exact exceedance probability of the different χ_c^2 values can easily be calculated.

Table 1 shows that the exact exceedance probabilities of the uncorrected χ_c^2 values (i.e. the values in column (9)) are better approximated by the tabular exceedance probabilities of the uncorrected χ_P^2 values (i.e. the values in column (6)) than by the tabular upper tail probabilities of the χ_c^2 values obtained through Formula 1 listed in column (8). Hence it may be contended that if the marginal totals are random variables, the use of Formula 1 cannot be considered a device for a more precise evaluation of the exceedance probability of an observed (uncorrected) chi-square statistic. This claim is in agreement with the ideas proposed by Pearson [11], Grizzle [12], and Conover [13, 14, 15].

However, when the correction for continuity is performed according to the general principle, the correction still improves the estimation of the exceedance probability of the uncorrected statistic. Table 1 shows that the values in column (7), i.e. the tabular upper tail probabilities of the chi-square values corrected according to the general principle, usually correspond better with the exact exceedance probabilities of the uncorrected chi-square values (see column (9)) than do the values from column (6) (i.e. the tabular exceedance probabilities of the uncorrected values) correspond with the exact exceedance probabilities of the uncorrected chi-square values in column (9). This result shows that Conover's [13] statement "that the Yates continuity correction should not be used in 2×2 contingency tables unless row and column totals are nonrandom" is rather misleading.[5] The proper con-

5. In the same journal issue Conover's article as attended by three comments. (Starmer, Grizzle and Sen [16] ; Mantel [17] ; Miettinen [18]) which are essentially not a discussion of Conover's article. Two years later Mantel [19] protested again from an other point of view against improper justification for use of non-corrected statistics.

clusion is, that *the use of formula* (1) which incorporates a correction for continuity, provides poor estimates of the desired exceedance probabilities ; but *the application of the principle of the correction for continuity* as presented originally by Yates, still improves the estimation of the unknown probabilities. Formula (1) agrees only with the general principle of Yates' correction for continuity if row and column totals are fixed.

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