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VOLATILITY MODEL RISK MEASUREMENT AND AGAINST WORST CASE VOLATILITIES

RISKLAB PROJECT IN MODEL RISK*

MODEL RISK : OUR APPROACH

Equilibrium or (absence of) arbitrage models, but also portfolio management applications and risk management procedures developed in financial institutions, are based on a range of hypotheses aimed at describing the market setting, the agents risk appetites and the investment opportunity set. When it comes to develop or implement a model, one always has to make a trade-off between realism and tractability. Thus, practical applications are based on mathematical models and generally involve simplifying assumptions which may cause the models to diverge from reality. Financial modelling thus inevitably carries its own risks that are distinct from traditional risk factors such as interest rate, exchange rate, credit or liquidity risks.

For instance, suppose that a French trader is interested in hedging a Swiss franc denominated interest rate book of derivatives. Should he/she rely on an arbitrage or an equilibrium asset pricing model to hedge this book? Let us assume that he/she chooses to rely on an arbitrage-free model, he/she then needs to specify the number of factors that drive the Swiss term structure of interest rates, then choose the modelling stochastic process, and finally estimate the parameters required to use the model.

In order to characterize the random evolution of the term structure of interest rates, models with one-factor, generally chosen as the short term rate, have been developped because they are easy to implement (see, *e.g.*, Merton [21] , Vasicek [25], Cox, Ingersoll and Ross [10], Hull and White [17], etc.), even if most empirical studies using a principal component analysis have decomposed the motion of the interest rate term structure into three independent and non-correlated factors, which respectively capture the level shift in the term structure, the twist in opposite direction of short and long term rates, and the butterfly factor that captures the fact that the intermediate rate moves

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in the opposite direction of the short and the long term rates. The multi-factor models do significantly better than single-factor models in explaining the dynamics and the shape of the entire term structure, but the latter provide analytical expressions for the prices of simple interest rates contingent claims, whereas a multi-factor model generally leads to numerically or quasi analytically solve partial differential equations in a higher dimension to obtain prices and hedge ratios for the interest rate-contingent claims.

The factor(s) dynamics specification is another source of Model Risk. The dynamics specifications cover a large spectrum of distributional assumptions such as pure diffusion processes or mixed jump-diffusion processes. The stochastic processes can have time-varying or constant drift and/or volatility parameters and they can rely on a linear or a non-linear specification of the drift (see Ait- Sahalia [3])

Once the model is fixed, one has to estimate its parameters. This step does not really provide help to verify the adequacy of the model on past data, since the theory of parameter estimation generally assumes that the true model belongs to a parametrized family of models. Moreover, a mis-specified model does not necessarily provide a bad fit to the data.

A study by Jacquier and Jarrow [18] proposes to incorporate model error and parameter uncertainty into a new method of contingent claims models' implementation. Using Markov Chains, Monte Carlo estimators, their conclusion, for a single stock option case study, suggests that the pricing performance of the «extended» Black and Scholes model dominates the simple Black and Scholes model within but not out-of-sample. Usually, the model parameters must be estimated by fitting a given set of market data. However, in finance it has been proved that natural estimators such as maximum likelihood and generalized method of moments estimators applied to time-series of interest rates may require a very large observation period to converge towards the true parameter values, yet it seems highly unrealistic to assume constant parameters over such a long period, see, Fournié & Talay [13]. Another important problem arises from the time discretization when we numerically compute the statistical estimates of the parameters of a stochastic model.

All the sources of Model Risk we have just listed, have strong financial repercussions (losses incurred by a bank or a financial institution due to Model Risk are fairly common) on the pricing, hedging, risk management and the definition of regulatory capital adequacy rules.

As far as derivatives pricing/hedging is concerned, a large part of the literature relies on the assumption of absence of arbitrage opportunities since the seminal articles of Black and Scholes [8] and Merton [21]. The principle is simple : given a distribution on a primitive asset price (for a stock, a bond, or a commodity), if we make the simplified hypothesis that there are no frictions and that the risk structure is not too complex, one can show that the derivative cash-flows can be replicated by a dynamic trading strategy involving the primitive assets. The Absence of Arbitrage hypothesis stipulates that the price of the derivative should be the same as the price of the replicating portfolio or there would be arbitrage profits (free lunches) in the market.

However, in this framework the pricing or hedging performance is not independent of the model used for the underlying asset price. One important problem is that the pricing models are often derived under a perfect and complete market paradigm, whereas markets are actually incomplete and imperfect. Violations of the perfect market assumption generally preclude us from observing uniqueness of the asset's theoretical price, and we are often left only with bounds to characterize transaction prices. In an interesting study focusing on equity options, Green and Figlewski [15] thus explain why option writers will charge a volatility mark-up to protect their Profit and Loss against Model Risk.

The presence of Model Risk will also affect the performance of model based hedging strategies.

In a continuous-time and frictionless market, a market maker, for example, the seller of an option, can synthetically create an opposite position (called delta-hedging strategy) which eliminates his/her risk completely. If the hedger uses an alternative (wrong) option pricing model, his/her price for the option differs from the true (market) price and provide an incorrect hedge ratio. In the presence of Model Risk, even though we assume frictionless markets, the self-financing delta-hedging strategy does not replicate the final payoff of the option position. Bossy *and al.* [9] study the case of bond option hedging, the Model Risk is defined by the Profit and Loss of the seller of an option who believes that the true model is one of the univariate Markov models nested in the Heath-Jarrow-Morton [16] framework, whereas the true model is actually another model belonging to the same class. In a subsequent study, Akgun [akgun-00] extends the methodology developed by Bossy *et al.* [9] to include omitted jumps by the trader who uses the wrong pure diffusion univariate term structure model to hedge his derivatives exposure. In his setting, the jumps are driven by a finite state-space compound Poisson process. This in turn allows him to show that omitting jump risk can be fairly devastating, as evidenced by simulated forward Model Risk Profil and Loss probability distributions, especially when shorting in and at-the-money naked or spread option positions.

Indeed, it is important for regulators to measure the trading and the banking books interest-rate risk exposures correctly. The Basle Committee on Banking Supervision [1], [2] issued directives to help financial institutions evaluate the interest-rate risk exposures of their exchange traded and over-the-counter derivative activities, as well as for their on and off-balance sheet items. Regulators ask the banks and other financial institutions to set aside equity in order to cover market risk driven losses.

Proposition 6 of the Basle Committee Proposal [2] states that banks can calculate their market risk capital requirements as a function of their forecasted ten-days-ahead value-at-risk. The value-at-risk (VaR) is defined as the quantile at a 99 % confidence interval of the distribution of the future value of the considered activity. The aim is to estimate the potential loss that would not be exceeded with a 99 % probability over the next ten trading days. An important source of Model Risk arises from the approximation techniques that a bank adopts to incorporate non-linear payoff securities in the VaR model. Pritzker [22] investigates the trade-off between accuracy and the computational time for six alternative VaR computation methods. Among them, the delta-gamma Monte Carlo method provides the best trade-off but still leads to significant errors in the VaR figures, especially for deeply out of the money options. Under these directives, banks are allowed to apply their own internal risk measurement models to calculate the VaR which will in turn determine their regulatory capital charge. No particular type of model is prescribed, as long as the internal model captures the relevant market risks run by a financial institution.

Even if internal models are allowed, regulators have introduced a back-testing procedure to assess the accuracy of a given VaR model and penalties in the form of multipliers : the market risk capital charge is computed using the bank's own estimate of the value-at-risk, times a multiplier whose value depends on the number of exceptions over the last 250 days detected with the help of the back-testing procedure. The regulator has fixed the value of the multiplier between 3 and 4 in order to keep a security margin against possible model errors made in the computation of the VaR. Lopez [20] compares three commonly used back-testing procedures and shows that they all have very low power against alternative VaR models. Thus, even at the final stages of model assessment and compliance, the accuracy detection method can induce regulatory 'Model Risk and thus lead to over or under-capitalized financial institutions'.

Specifying a proper loss function to assess a model's accuracy is thus a first step to mitigate Model Risk. This loss function should depend on the specific applications associated with the model and be adapted to the time-horizon and the contractual features of the positions being valued or hedged, to the division and/or the responsibility levels involved (trading desk versus senior management), without leading to excessive risk taking behavior especially below the critical downside risk thresholds. The methodology of Bossy *and al.* [9] can be used to measure the risk implied by the choice of an erroneous univariate term structure model. It measures the distribution of the losses due to this error, including (but not reduced to) estimation errors. It also takes into account hedging errors in addition to pricing errors (that can be avoided by the calibration of a wrong model on true market data). This methodology is briefly discussed in Section 2.1 below. Quantile approximation from Talay and Zheng [24] is presented at the end of Section 2.

This methodology is however restricted to the comparison of one (potentially incorrect) model against one or several (possible true) models among a class of univariate Markov term structure models. This class does not contain all possible term structures models. We present a more general methodology and the results developed by Talay and Zheng [23] which aim at selecting a hedging strategy which minimizes the expected utility of the Loss due his/her model risk under the worst possible movements of nature (as characterized by forward rates' volatility trajectories). This methodology is discussed in Section 3.

This article is a synthesis of the following articles : Gibson, Lhabitant, Pistre and Talay [14], Bossy, Gibson, Lhabitant, Pistre and Talay [9], Talay and Zheng [23], [24]. Of course, our financial and numerical methodologies can easily be extended to a wide family of cases : European options with Black and Scholes type models, markets with transaction costs, etc.

2. MODEL RISK MEASUREMENT

We briefly recall the expression of the Profit and Loss obtained in Bossy *et al.* [9] for a trader who believes in a term structure model of the univariate Markov Heath-Jarrow-Morton family whereas the true term structure follows another model model in the same family. In reality the 'true' model is unknown. Thus one must consider this Model Risk analysis as being performed with respect to 'benchmark' models selected by the investor, the risk controller or the regulator.

2.1. Expression of the Profit and Loss in the Heath-Jarrow-Morton model

We are given a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ equipped with the augmented filtration $(\mathcal{F}_t)$, $t \in [0,T]$ generated by a real valued Brownian Motion $(W_t, t \in [0,T])$. We suppose that the yield curve of the financial market follows the Heath-Jarrow-Morton model and, as here our analysis is focused on volatility Model Risk, we also suppose that the premium risk process (λ_t) is null, thus we have : for all time T^*, the instantaneous forward rate $f(t,T^*)$ solves the stochastic differential equation

$$f(t,T^*) = f(0,T^*) + \int_0^t \sigma(s,T^*)\sigma^*(s,T^*)ds + \int_0^t \sigma(s,T^*)dW_s, \qquad (1)$$

with

$$\sigma^*(s,T^*) := \int_s^{T^*} \sigma(s,u)du.$$

In Bossy *et al.* [9], the Model Risk analysis is based on Monte Carlo simulations of the Profit and Losses of the self-financing strategies of a trader who aims at hedging a European option written on a bond of maturity T^0. In this subsection we give an outline of the method for the case where the function σ is deterministic.

Suppose that the trader does not know the map $\sigma(s,T)$. Instead, he or she chooses a deterministic model structure $\overline{\sigma}(s,T)$ and tries to hedge the contingent claim according to this model.

Let $\overline{V}_t$ be the value of the trader's portfolio at time t and V_t be the value of the perfectly hedging portfolio. The option seller's Profit and Loss is defined by :

$$P\&L_t := \overline{V}_t - V_t.$$

Given a price P_t, define the forward price P_t^F by

$$P_t^F := \frac{P_t}{B(t,T^0)},$$

where $B(t,T^0)$ is the price of the bond of maturity T^0 in the model driven by $\sigma(t,T^0)$.

It can be shown that the self financing constraint implies that

$$d\bar{V}_t^F = \frac{\partial \pi_{\overline{\sigma}}}{\partial x}(t, B^F(t,T))dB^F(t,T), \tag{2}$$

where $B^F(t,T)$ is the forward price of the discount bond, and that the forward Profit and Loss $P\&L_t^F$ satisfies

$$\begin{aligned} P\&L_t^F =& \overline{V}_0^F - \pi_{\overline{\sigma}}(0, B^F(0,T)) \\ &+ \pi_{\overline{\sigma}}(t, B^F(t,T)) - \pi_{\sigma}(t, B^F(t,T)) \\ &+ \frac{1}{2}\int_0^t \frac{\partial^2 \pi_{\overline{\sigma}}}{\partial x^2}(s, B^F(s,T))B^F(s,T)^2 \\ &\{(\overline{\sigma}^*(t,T^0) - \overline{\sigma}^*(t,T))^2 - (\sigma^*(t,T^0) - \sigma^*(t,T))^2\}ds, \end{aligned} \tag{3}$$

where $\pi_{\overline{\sigma}}$ is the solution to the following parabolic PDE parametered by the function $\overline{\sigma}$:

$$\begin{cases} \dfrac{\partial \pi_{\overline{\sigma}}(t,x)}{\partial t} + \dfrac{1}{2}x^2(\overline{\sigma}^*(t,T) - \overline{\sigma}^*(t,T^*))^2 \dfrac{\partial^2 \pi_{\overline{\sigma}}(t,x)}{\partial x^2} = 0, \\ \pi_{\overline{\sigma}}(T,x) = \Phi(x). \end{cases} \tag{4}$$

Thus the gamma of the position is shown to be essentially in the quantity of Model Risk induced by the position of the trader. This means that limiting the Model Risk of a trader implies limiting the gamma of the position, and that the Model Risk exposure of an option position is not similar to its interest rate exposure and thus has to be managed seperately.

As justified by Artzner *and al.*[5], quantiles of the negative part of $P\&L_{T^0}$, $\mathbb{E}[U(P\&L_{T^0})]$ where U is a utility function, are good candidates of Model Risk measurements. Bossy *and al.* [9] discuss numerical results obtained by Monte Carlo methods for simple and agregate strategies. In our next subsection we analyse the accuracy of such Monte Carlo methods. As the process $(B^F(t,T), P\&L_t^F)$ is the solution of a stochastic differential equation (see Equation (7) below), we focus our attention to the accuracy of the Monte Carlo method to compute the quantiles of diffusion processes.

2.2. Quantile approximation of a diffusion process by Monte Carlo methods

Given a random variable X and $0 < \delta < 1$, the quantile of level δ is the smallest $\rho(\delta)$ such that

$$\mathbb{P}[X \leqslant \rho(\delta)] = \delta.$$

The case where $X = X_T$, $X.$ solution to a stochastic differential equation, is of special interest for us.

Let (X_t) be a real valued process, solution to

$$\begin{cases} dX_t = b(X_t)dt + \sigma(X_t)dW_t, \\ X_0 = x, \end{cases}$$

where (W_t) is a r-dimensional Brownian motion and b, σ are smooth functions with bounded derivatives. The Euler scheme for (5) is

$$X^n_{(p+1)T/n} = X^n_{pT/n} + b(X^n_{pT/n})\frac{T}{n} + \sigma(X^n_{pT/n})(W_{(p+1)T/n} - W_{pT/n}).$$

Define $\rho^n(\delta)$ by

$$\mathbb{P}[X^n_T \leqslant \rho^n(\delta)] = \delta.$$

We suppose that (5) satisfies the technical assumption (M) explicited in Talay & Zheng [24], or the same uniform hypoellipticity condition as in Bally & Talay [7]. We do not rewrite here these technical conditions because they require too much material to be stated. We only emphasize that it can be shown that the so called Condition (M) is generally satisfied by Equation (7).

THÉORÈME 2.1. — *Under one of the above conditions, there exist strictly positive constants $C(T)$ and $q_T(\delta)$ such that*

$$|\rho(\delta) - \rho^n(\delta)| \leqslant \frac{C(T)}{q_T(\delta)n}, \ \forall n.$$

See Talay & Zheng [24] for the proof of Theorem 2.1., and the extension of the results to one dimensional marginal distributions of diffusion processes. The proof shows that

$$q_T(\delta) := \inf_{y \in (\rho(\delta)-1, \rho(\delta)+1)} p_{X_T}(y),$$

where p_{X_T} is the density of the distribution of X_T.

As classical estimates show that the standard deviation of the statistical error of the Monte Carlo method, that is, the error due to the approximation of the expectation by the average over the simulations, is governed by $C(T)/(p_{X_T}(\rho(\delta))\sqrt{N})$, N being the number of simulations. To get estimates on the discretization step and the number of simulations which are necessary to obtain a desired accuracy with a given confidence interval one needs an accurate lower bound of the density of X_T. For the strictly uniform elliptic generators, see, *e.g.*, Azencott [6]. In the degenerate case, under restrictive assumption on b, see Kusuoka & Stroock [kusuoka-stroock-87]. Such assumptions are not satisfied in our Model Risk study, and therefore we need the supplementary results of our next subsection.

2.3. Application to Model Risk

Define π_σ as the solution of the same parabolic problem as (4) with σ instead of $\overline{\sigma}$. Set

$$\begin{cases} u_1(t) := \sigma^*(t, T^0), \\ u_2(t) := (\sigma^*(t, T^0) - \sigma^*(t, T)), \\ \varphi(t, x) := \dfrac{\partial \pi_{\overline{\sigma}}}{\partial x}(t, x) - \dfrac{\partial \pi_\sigma}{\partial x}(t, x). \end{cases} \tag{6}$$

Thus the forward price of the bond and the forward Profit and Loss $P\&L_t^F$ satisfy

$$\begin{cases} dB^F(t, T) = B^F(t, T) u_1(t) u_2(t) dt + B^F(t, T) u_2(t) dW_t, \\ dP\&L_t^F = \varphi(t, B^F(t, T)) dB^F(t, T). \end{cases} \tag{7}$$

We are interested in the quantile of $P\&L_{T^0} = P\&L_{T^O}^F$, that is

$$\mathbb{P}[P\&L_{T^0} \leqslant \rho(\delta)] = \delta.$$

In Talay & Zheng [24] one proves that the conclusion of Theorem 2.1 applies in this situation. In particular, one gets that the law of $P\&L_{T^0}$ has a density p_{T^0} under the following assumptions : $|u_2(t)| \geqslant a > 0$ for all t in $[0, T^0]$, the functions $u_1(t)$, $u_2(t)$ are bounded and $B^F(0, T)\varphi(0, B^F(0, T)) \neq 0$. In addition, p_{T^0} is strictly positive on its support.

In view of our comments for Theorem 2.1, it would be useful to obtain an accurate pointwise lower bound estimate for p_{T^0}. This is also done in [24].

3. MODEL RISK MANAGEMENT AGAINST WORST CASE VOLATILITY

The previous methodology is restricted to the comparison of the potentially incorrect models against the potentially true (but unknown) or benchmark models among a class of univariate Markov term structure models. The following methodology is far more general, since it allows to optimize the choice of the trader's strategy against 'all' possible actual volatility processes.

3.1. Motivation

The objective is to propose a new strategy for the trader which, in a sense, guarantees good performances whatever is the unknown process $\sigma(\cdot, \cdot)$. The construction corresponds to a 'worst case' worry and, in this sense, can be viewed as a continuous time and rigorous extension of discrete time strategies based upon prescriptions issued from VaR analyses at the beginning of each period. Roughly speaking, the idea can be expressed by the following graph :

$$\boxed{\text{Trader} = \text{MinimizerofRisk.}}$$

$$\boxed{\text{Market} = \text{MaximizerofRisk.}}$$

$$\boxed{\text{TradervsMarket.}}$$

Thus the Model Risk control problem is set up as a two players (Trader versus Market) zero-sum stochastic differential game problem. We notice that our approach is related to, but different from, Cvitanic and Karatzas dynamic measure of risk(see [cvitanic-karatzas-99]). Moreover, the solution at time 0 of our stochastic game problem can be viewed as a 'reserve position' for a financial institution.

3.2. The stochastic differential game and the Hamilton Jacobi Bellman Isaacs equation

Let (π_t) be the delta process chosen by the trader. The self financing constraint implies that one has

$$\begin{cases} dB^F(t,T) = B^F(t,T)u_1(t)u_2(t)dt + B^F(t,T)u_2(t)dW_t, \\ d\overline{V}^F(t) = \pi(t)B^F(t,T)u_1(t)u_2(t)dt + \pi(t)B^F(t,T)u_2(t)dW_t, \end{cases} \tag{8}$$

where $u_1(t)$ and $u_2(t)$ are defined as in (6).

We adopt the definition of admissible controls and strategies of Fleming & Souganidis [12]. The set of all admissible controls for the market on $[t,T]$ is denoted by $Ad_u(t)$ and the set of all admissible strategies for the investor on $[t,T]$ is denoted by $Ad_\Pi(t)$. These admissible controls and strategies take value in compact sets K_u and K_π respectively.

For given $\Pi \in Ad_\Pi(r)$ and $u_. \in Ad_u(r)$, we define the objective function as

$$J(r,x,y,\Pi,u_.) := \mathbb{E}_{r,x,y}[F(f(B^F(T^0,T)) - \overline{V}^F_{T^0})]$$

where F is a utility function and f is the profile of the payoff function [1]. The function F should be chosen according to the definition of measures of risk introduced in Artzner *et al.* [5].

DÉFINITION 3.1. — *The value function of the Model Risk control problem with initial data (r,x,y) is defined by*

$$V(r,x,y) := \inf_{\Pi \in Ad_\Pi(r)} \sup_{u_. \in Ad_u(r)} J(r,x,y,\Pi,u_.). \tag{9}$$

We have the following result :

1. For the sake of simplicity our notation does not emphasize that the process $(B^F(t,T), \overline{V}^F_t)$ is parametered by $(u^1(t), u^2(t), \pi_t)$.

THÉORÈME 3.2. — *Suppose that*

$$|F(f(x)-y)-F(f(\overline{x})-\overline{y})| \leqslant P(|x|,|y|,|\overline{x}|,|\overline{y}|)(|x-\overline{x}|+|y-\overline{y}|),$$

where $P(|x|,|y|,|\overline{x}|,|\overline{y}|)$ is a polynomial function.
Then the semi-value function $V(r,x,y)$ defined in (9) is the unique viscosity solution in the space,

$$S := \{\varphi(t,x,y) \textit{ is continuous}, \ \exists \overline{A}>0, \lim_{x^2+y^2\to\infty} \varphi(t,x,y)(-\overline{A}|\log(x^2+y^2)|^2) = 0, \ \forall t \in [0,T^O]\}$$

to the Hamilton-Jacobi-Bellman-Isaacs Equation,

$$\begin{cases} \dfrac{\partial v}{\partial t}(t,x,y) + \mathcal{H}^-(D^2v(t,x,y), Dv(t,x,y), t, x) = 0 \text{ in } [0,T^O)\times\mathbb{R}^2, \\ v(T^O,x,y) = F(f(x)-y), \end{cases} \tag{10}$$

where

$$\mathcal{H}^-(A,p,t,x) := \max_{u\in K_u}\min_{\pi\in K_\pi}\left[\frac{1}{2}u_2^2x^2A_{11} + u_2^2x^2\pi A_{12} + \frac{1}{2}u_2^2x^2\pi^2A_{22} + p_1u_1u_2x + p_2u_1u_2\pi x\right], \tag{11}$$

for all 2×2 symmetric matrix A and all vector p in $\mathbb{R}^2$.
Moreover, $V(r,x,y)$ satisfies the Dynamic Programming Principle, that is,

$$V(r,x,y) = \inf_{\Pi\in Ad_\Pi(r)} \sup_{u\ \in Ad_u(r)} \mathbb{E}_{r,x,y}[V(t,x_t,y_t)]. \tag{12}$$

For the proof, see Talay & Zheng [23].

The value function $V(r,x,y)$ and the optimal strategy can be solved numerically by the finite difference method. In our numerical tests we consider the Profit and Loss of the seller of a European call option. The utility function is

$$F(x) := (f(x)-y)_+.$$

The maturity T^0 of the option is 6 monthes and the option is written on a discount bond of maturity T equal to 5 years. The trader uses two bonds to hedge this option : the bond of maturity 6 monthes and the bond of maturity 5 years. The strike of the option is $K = 0.509156$.

We set $K_\pi := [-c_1,c_1] = [-1,1]$. The constant c_1 has been chosen after having computed perfectlly replicating strategies in the cases of Ho–Lee and Vasicek models. In addition, we fix $u_2\in[0,c_2]$ and $u_1\in[-c_3,c_3]$ with $c_2 = 0.6$ and $c_3 = 0.07$.

We comment Fig. 2. The graph of optimal strategy $H^*(t,x,y)$ consists in three parts :

- When x (the forward price of the bond) is comparatively small with respect to y (the forward price of the portfolio), then $H(t,x,y) = 0$. It is a conservative strategy.
- When x is comparatively small with respect to y, $H(t,x,y) = c_1$, the maximum value. It is a risky strategy.
- In the other cases, $H(t,x,y)$ takes values between 0 and c_1. It is similar to the classical hedging strategy.

Moreover, we observe from Fig. 3. that the optimal volatility u_2^* is a bang-bang control.

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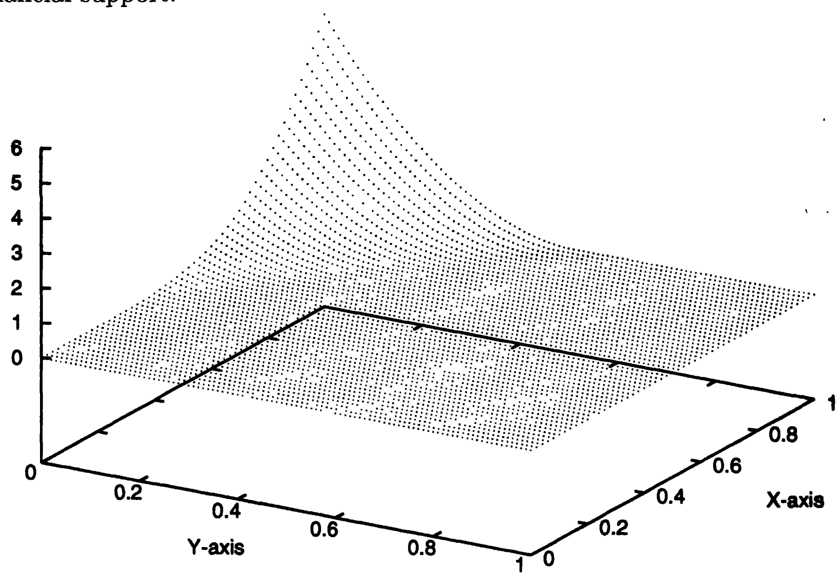

FIG 1. — The Value Function : Graph of $V(0,x,y)$

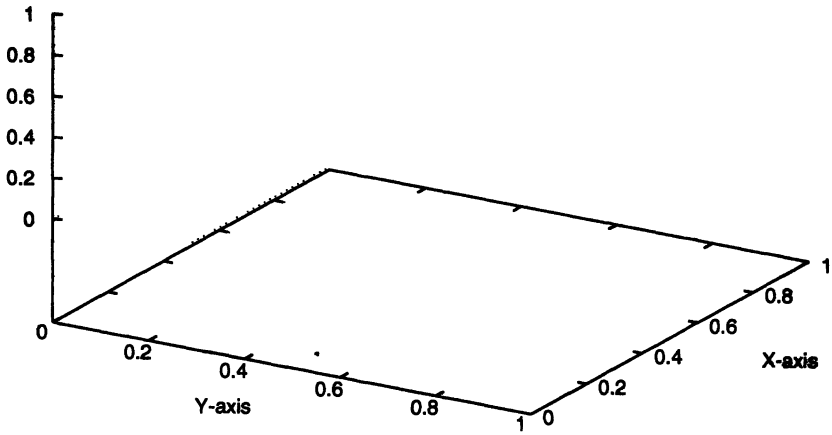

FIG 2. — The Optimal Strategy : Graph of $H^*(0, x, y)$

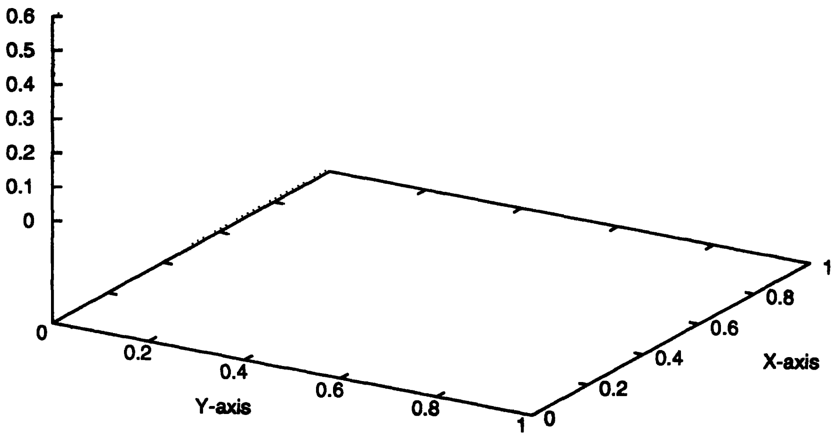

FIG 3. — The Optimal Control : Graph of $u_2^*(0, x, y)$

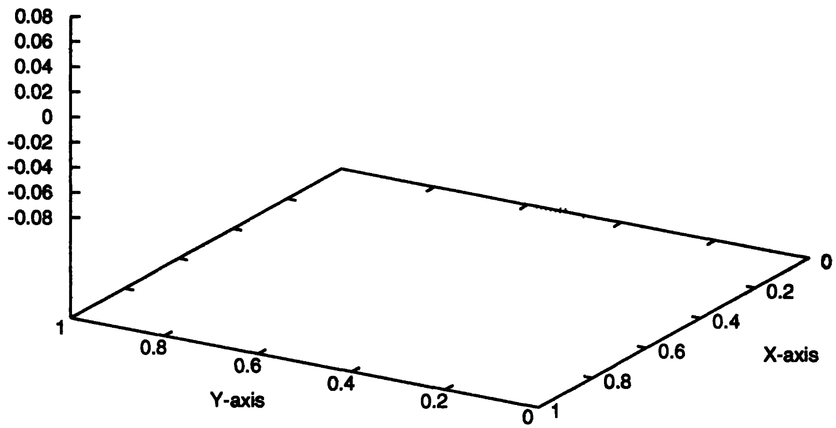

FIG 4. — The Optimal Control : Graph of $u_2^*(0, x, y)$

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