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LOWER SPACE BOUNDS FOR ACCEPTING SHUFFLE LANGUAGES

ANDRZEJ SZEPIETOWSKI

Abstract. In [6] it was shown that shuffle languages are contained in one-way-NSPACE(log n) and in P. In this paper we show that nondeterministic one-way logarithmic space is in some sense the lower bound for accepting shuffle languages. Namely, we show that there exists a shuffle language which is not accepted by any deterministic one-way Turing machine with space bounded by a sublinear function, and that there exists a shuffle language which is not accepted with less than logarithmic space even if we allow two-way nondeterministic Turing machines.

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1. INTRODUCTION

The operations shuffle and shuffle closure have been introduced to describe sequentialized execution histories of concurrent processes [7, 8]. Together with other operations they describe various classes of languages which have been extensively studied (see [1, 3-5, 10]). Here, we consider the class of shuffle languages which emerges from the class of finite languages through regular operations (union, concatenation, Kleene star) and shuffle operations (shuffle and shuffle closure). In [6] it was shown that shuffle languages are contained in the class one-way-NSPACE(log n) and thus in the class P (i.e. they are accepted in polynomial time by deterministic Turing machines). For every shuffle expression $E$, a shuffle automaton was constructed which accepts the language generated by $E$ and it was shown that the computations of the automaton can be simulated by a one-way nondeterministic Turing machine in logarithmic space.

In this paper we show that nondeterministic one-way logarithmic space is in some sense the lower bound for accepting shuffle languages. Namely, we show that there exists a shuffle language which is not accepted by any deterministic one-way
Turing machine with space bounded by a sublinear function, and that there exists a shuffle language which is not accepted with less than logarithmic space even if we allow two-way nondeterministic Turing machines.

2. SHUFFLE LANGUAGES

Let $\Sigma$ be any fixed alphabet and $\lambda$ the empty word. The shuffle operation $\circ$ is defined inductively as follows:

- $u \circ \lambda = \lambda \circ u = \{u\}$, for $u \in \Sigma^*$ and
- $au \circ bv = a(u \circ bv) \cup b(au \circ v)$, for $u, v \in \Sigma^*$ and $a, b \in \Sigma$.

For any languages $L_1, L_2 \subset \Sigma^*$ the shuffle $L_1 \circ L_2$ is defined as

$$L_1 \circ L_2 = \bigcup_{u \in L_1, v \in L_2} u \circ v.$$ 

For any language $L$, the shuffle closure operator is defined by:

$$L^\circ = \bigcup_{i=0}^{\infty} L^{\circ i},$$

where $L^\circ = \{\lambda\}$ and $L^{\circ i} = L^{\circ i-1} \circ L$.

**Definition 1.** Each $a \in \Sigma$, $\lambda$ and $\emptyset$ are shuffle expressions. Besides, if $S_1$, $S_2$ are shuffle expressions, then $(S_1 \cdot S_2)$, $S_1^\ast$, $(S_1 + S_2)$, $(S_1 \circ S_2)$ and $S_1^\otimes$ are shuffle expressions, and nothing else is a shuffle expression.

The language $L(S)$ generated by a shuffle expression $S$ is defined as follows.

- $L(a) = \{a\}$, $L(\lambda) = \{\lambda\}$, $L(\emptyset) = \emptyset$.
- If $L(S_1) = L_1$ and $L(S_2) = L_2$, then $L((S_1 \cdot S_2)) = L_1 \cdot L_2$, $L((S_1 + S_2)) = L_1 \cup L_2$, $L(S_1^\ast) = L_1^\ast$, $L((S_1 \circ S_2)) = L_1 \circ L_2$, and $L(S_1^\otimes) = L_1^\otimes$.

A language $L$ is a shuffle language if there exists a shuffle expression $E$ such that $L = L(E)$. We shall also use the following notation, for arbitrary string $z$: $|z|$ denotes the length of $z$, $|z|_e$ the number of occurrences of a symbol $e$ in $z$, $z_i$ the $i$-th symbol of $z$, and $z^R$ the reverse of $z$ ($z$ written backwards).

3. TURING MACHINES

We consider the Turing machine model with a read-only input tape and a separate two-way, read-write work tape. The number of tape cells used on the work tape, called space, is our measure of computational complexity. A Turing machine is called one-way if its input head cannot move to the left.

We use so called weak mode of space complexity. Let $L(n)$ be a function on natural numbers. A Turing machine is said to be weakly $L(n)$ space-bounded if for every accepted input of length $n$, at least one accepting computation uses no more than $L(n)$ space. But our results are also valid for strong mode of space complexity, which requires that for every input of length $n$, all computations are $L(n)$ space bounded. We shall use the following notation: $DSPACE[L(n)]$
or \( NSPACE[L(n)] \) denotes the class of languages accepted by deterministic or nondeterministic \( L(n) \) space-bounded Turing machines, respectively. We add the prefix \textit{one-way} if we consider classes of languages accepted by one-way Turing machines.

By a configuration of a Turing machine \( M \) we shall mean a tuple \((q, \gamma, j)\), where \( q \) is the current state of \( M \), \( \gamma \) are the contents of the non-blank sector of the work tape, and \( j \) is the position of the work head, \( 1 \leq j \leq |\gamma| + 1 \) (we assume that \( M \) cannot write the blank symbol on its work tape). The space used by the configuration \((q, \gamma, j)\) is equal to \(|\gamma| \) – the number of non-blank cells on the work tape. It is easy to see that the number of all configurations with space bounded by \( k \) is less than \( r^k \), for some constant \( r > 1 \) (for more details see [9] or [2]).

4. LOWER BOUND FOR ONE-WAY TURING MACHINES

In this section we show that there exists a shuffle language which is not accepted by any deterministic one-way Turing machine in space bounded by a sublinear function.

\textbf{Theorem 2.} There exists a shuffle language \( L \) such that \( L \notin \text{one-way-} DSPACE[S(n)] \), for any \( S(n) = o(n) \).

\textit{Proof.} Consider the shuffle language

\[ L = (a + b)^*a(ac + bd)^\oplus d(c + d)^* + (a + b)^*b(ac + bd)^\oplus c(c + d)^* \]

and let \( h : \{a, b\}^* \rightarrow \{c, d\}^* \) be the isomorphism described by \( h(a) = c \) and \( h(b) = d \). First we shall prove the following.

\textbf{Lemma 3.} Let \( k \) be a positive number. For every \( u, v : u \in (a + b)^k \) and \( v \in (c + d)^k \), the concatenation \( uv \) belongs to \( L \) if and only if \( h(u)^R \neq v^R \) (\( v^R \) denotes the reverse of \( v \)).

\textit{Proof.} If \( uv \in L \) then \( uv \) can be decomposed into

\[ uv = u'a''v''d' \quad \text{ or } \quad uv = u'bv''c'v' \]

with \( u', u'' \in (a + b)^* \), \( v', v'' \in (c + d)^* \), and \( u''v'' \in (ac + bd)^\oplus \). We shall only deal with the first case. Note that in this case \( u = u'a'' \) and \( v = v''d' \).

Since \( u''v'' \in (ac + bd)^\oplus \), we have

\[ |u''v''|_a = |u''v''|_c \quad \text{ and } \quad |u''v''|_b = |u''v''|_d \]

(where \( |z|_e \) denotes the number of occurrences of a symbol \( e \) in a string \( z \)).

And because

\[ |u''v''|_a + |u''v''|_b = |u''| \quad \text{ and } \quad |u''v''|_c + |u''v''|_d = |v''| \]
we have
|u''| = |v''|
and hence
|u'| = |v'|.
Let $i = |u'| + 1 = |v'| + 1$. Then the words $h(u)$ and $v^R$ disagree on the $i$-th symbol, 
$(h(u))_i = h(u_i) = h(a) = c$ and $(v^R)_i = d$ (where $z_i$ denotes the $i$-th symbol of a word $z$). Thus $h(u) \neq v^R$.

Suppose now that $h(u) \neq v^R$ and that $i$ is the last index, where $h(u)$ and $v^R$ disagree. We can assume that $u_i = a$ and $(v^R)_i = d$. Then $u$ and $v$ can be decomposed in the following way: $u = u'au''$, $v = v''dv'$, and $h(u'') = (v'')^R$. (It is possible that $u'' = v'' = \lambda$.) In this situation $u''v'' \in (ac + bd)\otimes$ and thus $uv \in L$. 
This ends the proof of the lemma.

Suppose now, for a contradiction, that $L$ is accepted by a one-way deterministic Turing machine $M$ with space weakly bounded by $S(n)$.

Let $k$ be a positive number. For every $u \in (a + b)^k$, let $conf(u)$ be the configuration reached by $M$ after reading $u$. Because there exists $v \in \{c, d\}^k$ such that the word $uv \in L$, then $conf(u)$ uses at most $S(2k)$ cells on the work tape. There are $2^k$ different words in $(a + b)^k$, and at most $r^{S(2k)}$ configurations with space bounded by $S(2k)$, for some constant $r > 1$. Since $\lim_{n \to \infty} \frac{S(n)}{n} = 0$, there exists $k$ such that $r^{S(2k)} < 2^k$, and there exist two different words $x, y \in (a + b)^k$, such that $conf(x) = conf(y) = \alpha$.

Consider now the accepting computation of $M$ on the word $x(h(y))^R$. By Lemma 3, $x(h(y))^R \in L$, because $h(x) \neq (h(y))^R = h(y)$. In this computation $M$ reaches the configuration $\alpha$ just after reading $x$. This means that $M$ also accepts the word $y(h(y))^R$ because $M$ reaches $\alpha$ after reading $y$ and afterwards it proceeds exactly like for $x(h(y))^R$ and accepts at the end. But, by Lemma 3, $y(h(y))^R$ does not belong to $L$, a contradiction.

5. LOWER BOUND FOR TWO-WAY TURING MACHINES

In this section we show that there exists a shuffle language which is not accepted by any nondeterministic two-way Turing machine in space bounded by a sublogarithmic function.

**Theorem 4.** There exists a shuffle language $L_1$ such that $L_1 \notin NSPACE[S(n)]$ for any $S(n) = o(\log n)$.

**Proof.** Consider the shuffle language

$L_1 = (ab)\otimes$.

The theorem follows from the fact that the class $NSPACE[S(n)]$ is closed under intersections with regular languages, and that the language

$L_1 \cap a^*b^* = \{a^nb^n \mid n \geq 0\}$
is not accepted by any nondeterministic Turing machine with space bounded by $S(n) = o(\log n)$ (see [9]).

REFERENCES


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