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PERIODS AND GAUSS-MANIN CONNECTION FOR THE

$$\text{MUMFORD CURVE } y_2^{r_2} y_1^{r_1} - y_2^{r_2} - y_1^{r_1} + \lambda = 0$$

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A method is introduced which allows to obtain explicit formulas for the periods $q(\lambda)$ of a family (C_λ) of smooth curves. It gives expressions for $q(\lambda)$ in the vicinity of a point λ_0 for which the curve C_{λ_0} is totally degenerate provided one knows Picard-Fuchs equations for differentials of the family (C_λ) .

Techniques from rigid analytic geometry are used, see [T]. We work with the notion of periods for p -adic Schottky groups as defined by Manin-Drinfeld, [MD]. The result can certainly be applied to the usual complex periods. In this approach it is basic that one has a canonical basis for the De Rham cohomology classes.

In this manuscript only one example is treated. The curves C_λ^r given by the equation in the title are prestable and totally degenerate for $\lambda = 1$. The p -adic Schottky uniformization is constructed in section 2. In section 3 a crucial formula for the Gauss-Manin connection is explained. The main application is the expression for the periods in proposition 4 of section 4. For elliptic curves the result is classical, see [F]. It is planned to give a more complete account of this method in a joint paper with F. Herrlich. The relation to the work of B. Dwork, [D], shall be included.

1. The curve C_λ^r

Let K be a field of characteristic 0 and $r = (r_1, r_2)$ a pair of integers ≥ 2 . Assume that there is a primitive root of unity ρ_i of

order r_i in K .

Let (y_1, y_2) be a system of inhomogeneous coordinates for $\mathbb{P} \times \mathbb{P}$, where $\mathbb{P}$ is the projective line over K and let λ be a parameter in K . The equation

$$y_2^{r_2} y_1^{r_1} - y_2^{r_2} - y_1^{r_1} + \lambda = 0$$

defines a projective curve C_λ^r in $\mathbb{P} \times \mathbb{P}$.

If u_i, v_i are homogeneous variables for $\mathbb{P}$ with $y_i = \frac{u_i}{v_i}$, then C_λ^r is the set of zeroes of the bihomogeneous equation

$$u_2^{r_2} u_1^{r_1} - u_2^{r_2} v_1^{r_1} - v_2^{r_2} u_1^{r_1} + \lambda v_2^{r_2} v_1^{r_1} = 0$$

The curve C_λ^r is non-singular if and only if $\lambda(\lambda-1) \neq 0$. The curve C_λ^r is a union of $r_1 \cdot r_2$ projective lines and prestable.

Let σ_1 (resp. σ_2) be the automorphism on $\mathbb{P} \times \mathbb{P}$ for which

$$y_1 \circ \sigma_1 = \rho_1 \cdot y_1, \quad y_2 \circ \sigma_1 = y_2.$$

(resp. $y_1 \circ \sigma_2 = y_1, \quad y_2 \circ \sigma_2 = \rho_2 \cdot y_2$).

The restriction $\sigma_i|_{C_\lambda^r}$ of σ_i onto C_λ^r is an automorphism of C_λ^r and $\sigma_1 \circ \sigma_2 = \sigma_2 \circ \sigma_1$. Let G denote the group generated by $\sigma_1|_{C_\lambda^r}$ and $\sigma_2|_{C_\lambda^r}$. It is canonically isomorphic to $\mathbb{Z}/r_1\mathbb{Z} \oplus \mathbb{Z}/r_2\mathbb{Z}$.

The field of K -rational functions of C_λ^r is generated by $y_1|_{C_\lambda^r}$ and $y_2|_{C_\lambda^r}$ if $\lambda \neq 0, \lambda \neq 1$. We will write in the sequel y_i instead of $y_i|_{C_\lambda^r}$ and define x_i to be $y_i^{r_i}$. Then

$$\frac{dx_1}{x_1-1} = - \frac{dx_2}{x_2-1}$$

because the rational functions x_1, x_2 satisfy the relation

$$x_2 x_1 - x_2 - x_1 + \lambda = 0$$

and thus

$$x_2 dx_1 + x_1 dx_2 - dx_2 - dx_1 = 0$$

$$(x_1-1)dx_2 + (x_2-1)dx_1 = 0.$$

Let $I := \{i = (i_1, i_2) \in \mathbb{Z}^2 : 1 \leq i_1 < r_1, 1 \leq i_2 < r_2\}$.

For $i = (i_1, i_2) \in I$ we define

$$\omega_i = \frac{dx_1}{y_1^{i_1} y_2^{i_2} (x_1-1)} = - \frac{dx_2}{y_1^{i_1} y_2^{i_2} (x_2-1)}$$

$$\omega'_i = \frac{dx_1}{y_1^{i_1} y_2^{i_2}}$$

Then the De Rham cohomology vectorspace $H_{DR}^1(C_\lambda^r)$ admits a direct decomposition

$$\bigoplus_{i \in I} \langle \omega_i, \omega'_i \rangle$$

where $\langle \omega_i, \omega'_i \rangle$ denotes the K -vectorspace of differentials generated by ω_i and ω'_i . In fact $\langle \omega_i, \omega'_i \rangle$ is the eigenspace of the canonical action of G on $H_{DR}^1(C_\lambda^r)$ with respect to the character $\chi : G \rightarrow K^*$ for which $\chi(\sigma_1) = \rho_1^{-i_1}$, $\chi(\sigma_2) = \rho_2^{-i_2}$.

As $\dim H_{DR}^1(C_\lambda^r) = 2(r_1-1)(r_2-1)$ the genus of C_λ^r is $(r_1-1)(r_2-1)$.

2. p-adic uniformization

Let now K be complete with respect to non-archimedean valuation $||$ and assume that $|\lambda-1| < 1$ and that $r_1 \cdot r_2$ is prime to the characteristic of the residue field. I want to show that C_λ^r is a Mumford curve. This will be achieved by constructing the non-archimedean or p-adic Schottky uniformization for C_λ^r .

Let z be a coordinate for $\mathbb{P}$, and $s \in K$, $|s-1| < 1$, $s \neq 1$ and let

$$\begin{aligned} \sigma_1(z) &= \rho_1 \cdot z \\ \sigma_2(z) &= \frac{(s-\rho_2)z + (\rho_2-1)s}{(1-\rho_2)z + (\rho_2 s-1)} \end{aligned}$$

Then σ_1, σ_2 are elliptic fractional linear transformation of $\mathbb{P}$ and σ_2 has the multiplier ρ_2 and the fixed points 1 and s . One can show that the group $\langle \sigma_1, \sigma_2 \rangle$ is discontinuous in the sense of [GP], Chap. I, §1, and that the commutator subgroup Γ of $\langle \sigma_1, \sigma_2 \rangle$ is a free group freely generated by $\{\gamma_i := \sigma_1^{i_1} \sigma_2^{i_2} \sigma_1^{-i_1} \sigma_2^{-i_2} : i \in I\}$, see [GH].

Let $z_1 := z$. $z_2 := \frac{z-s}{z-1}$. Then

$$z_1 = \frac{z_2 - s}{z_2 - 1}.$$

Let Γ_i the group generated by $\Gamma \cup \{\sigma_i\}$ $i = 1, 2$.

Define

$$y_1 := \prod_{\gamma \in \Gamma_2} \frac{z_1 \circ \gamma}{(z_1 \circ \gamma)(1)}$$

$$y_2 := \prod_{\gamma \in \Gamma_1} \frac{z_2 \circ \gamma}{(z_2 \circ \gamma)(\infty)}.$$

Both products converge on the domain Z of ordinary points for Γ .

They are both meromorphic on Z and are Γ -automorphic forms on Z with constant factors of automorphy, see [GP], Chap. II, §2.

A direct computation gives

$$y_1 \circ \sigma_1 = \rho_1 \cdot y_1$$

$$y_2 \circ \sigma_2 = \rho_2 \cdot y_2.$$

One can conclude that $y_1^{\Gamma_1}, y_2^{\Gamma_2}$ are $\langle \sigma_1, \sigma_2 \rangle$ -automorphic and that y_1, y_2 are Γ -invariant, see [GP], Chap. III, §1, for the notions.

Let $\lambda := y_1^{\Gamma_1}(s)$

Proposition 1: The mapping $z \mapsto (y_1(z), y_2(z))$

gives a bianalytic mapping between the Mumford curve Z/Γ and the curve C_λ^{Γ} .

Proof: see [GH].

Remark: The mapping

$$s \mapsto \lambda(s)$$

is a bianalytic mapping between $\{s \in K : |1-s| < 1\}$ and

$\{\lambda \in K : |1-\lambda| < 1\}$ with $\lambda(1) = 1$. Moreover $\lambda(s^{-1}) = \lambda(s)^{-1}$.

3. Gauss-Manin connection

There are canonical analytic Γ -automorphic forms with constant factors of automorphy such that $\alpha_i := \frac{du_i}{u_i}$ are analytic differentials on C_λ^r and such that $\{\alpha_i : i \in I\}$ is a basis of the K -vectorspace of analytic differentials on C_λ^r , see [GP], Chap. II, §4.

Let $q_{ij} := \frac{u_i \circ \gamma_j}{u_i} \in K^*$. The matrix $q := (q_{ij})$ is the period matrix of Γ with respect to the basis $\{\gamma_i : i \in I\}$, see [MD], §2. Also there are meromorphic functions ζ_i on Z such that $\zeta_i - \zeta_i \circ \gamma_j = \begin{cases} 1 & : i = j \\ 0 & : i \neq j \end{cases}$, see [G2], p. 387, and [G1], section 3.

The differentials $\beta_i := d\zeta_i$ are of the second kind and $\{\alpha_i : i \in I\} \cup \{\beta_i : i \in I\}$ is a basis of $H_{DR}^1(C_\lambda^r)$.

We consider now C^r as a family of curves by letting λ vary through $\{\lambda \in K : |\lambda - 1| < 1\}$. The Gauss-Manin connection ∇ of C^r is a connection

$$\nabla : H_{DR}^1 \rightarrow H_{DR}^1 \otimes \Omega$$

where H_{DR}^1 is the sheaf of De Rham cohomology classes of C^r as family of curves over $S = \{s \in K : |s-1| < 1\}$ and Ω is the sheaf of analytic differentials on S .

The main result of [G1] is a proof of

$$\text{Proposition 2: } \nabla(\alpha_i) = \sum_{j \in I} \beta_j \otimes \frac{dq_{ij}}{q_{ij}}$$

$$\nabla(\beta_i) = 0.$$

We want to apply this formula to the differential of the first kind

$$\omega_i = \frac{dx_1}{y_1^{i_1} y_2^{i_2} (x_1 - 1)}.$$

Proposition 3:

$$\omega_i = F_i(\lambda) \cdot \sum_{j \in I} \rho_1^{i_1 j_1} \rho_2^{i_2 j_2} \alpha_j$$

$$\text{with } F_i(\lambda) = \sum_{n=0}^{\infty} \frac{\left(\frac{i_1}{r_1}\right)_n \cdot \left(\frac{i_2}{r_2}\right)_n}{(n!)^2} (1-\lambda)^n \quad \text{and } (a)_n := \prod_{i=0}^{n-1} (a+i).$$

Proof: The method of proof consist in the following: It is well known that the cohomology class ω of $\frac{dx}{y}$, $y := x^a(x-1)^b(x-\lambda)^c$, satisfies the hypergeometric differential equation also known as Picard-Fuchs equation for ω :

$$\lambda(1-\lambda)\nabla_{\lambda}^2(\omega) + [a+c-(a+b+2c)\lambda]\nabla_{\lambda}(\omega) - (a+b+c-1)\omega = 0$$

see for instance [M], p. 378 or [D], Chap. I, p. 8.

$$\text{In our case } a = \frac{-i_1}{r_1}, b = -1 + \frac{i_2}{r_2}, c = \frac{i_2}{r_2}.$$

A straightforward computation shows that the above F_i is up to a constant the only power series solution of the above differential equation.

But ω_i being a differential of the first kind admits a representation

$$\omega_i = \sum_{j \in I} G_{ij} \alpha_j$$

with G_{ij} analytic in S .

Thus $\nabla_{\lambda}(\omega_i) \equiv \sum_{j \in I} \dot{G}_{ij} \alpha_j \pmod{H'}$ when H' is the subspace generated by $\{\beta_i : i \in I\}$. Thus each $\dot{G}_{ij} = c_j \cdot F_i$ with $c_j \in K$, where the dot over G_{ij} means the derivative with respect to λ . By considering the limit case for $s \rightarrow 1$ one obtains the above constants. For the details see [GH].

R. Coleman (Berkeley) has informed me that he has a completely different approach to this result.

4. Application to periods

The formulas for the Gauss-Manin connection and the Picard-Fuchs equation allow to derive an explicit expression for the logarithmic derivative of q_{ij} with respect to the variable λ in the domain

$$\{|\lambda-1| < 1\}.$$

Proposition 4:

$$\frac{\dot{q}_{ij}}{q_{ij}} = \sum_{k \in I} c_{ik} \cdot E_{kj}$$

with $c_{ik} = \frac{(\rho_1^{-i_1 k_1} - 1)(\rho_2^{-i_2 k_2} - 1)}{r_1 \cdot r_2}$

$$E_{kj} = \frac{A_{kj}}{(1-\lambda)\lambda^{\frac{k_1}{r_1} + \frac{k_2}{r_2}} \cdot F_k^2}$$

$$A_{kj} = 1 - \rho_1^{k_1 j_1} - \rho_2^{k_2 j_2} + \rho^{kj} = c_{-k,j} \cdot r_1 \cdot r_2$$

$$\lambda^a = (1 - (1-\lambda))^a := \sum_{n=0}^{\infty} \binom{a}{n} (1-\lambda)^n \cdot (-1)^n$$

$$F_k = \sum_{n=0}^{\infty} \frac{\left(\frac{k_1}{r_1}\right)_n \cdot \left(\frac{k_2}{r_2}\right)_n}{(n!)^2} (1-\lambda)^n$$

which is the hypergeometric function ${}_2F_1\left(\frac{k_1}{r_1}, \frac{k_2}{r_2}; 1; 1-\lambda\right)$, see [MOS], Chap. II, (2.1).

Remark: In the special case $r_1 = r_2 = 2$ the index set I consists of $(1,1)$ only and with $q := q_{11}$ one gets

$$\frac{\dot{q}}{q} = \frac{4}{(1-\lambda)\lambda \cdot {}_2F_1\left(\frac{1}{2}, \frac{1}{2}, 1; 1-\lambda\right)}$$

which is equivalent to a classical formula, see [F]. Be aware that λ is not the Legendre parameter as our equation is

$$y_2^2 y_1^2 - y_2^2 - y_1^2 + \lambda = 0.$$

We sketch now a proof of proposition 4.

1) Let $\omega_i^* := \frac{\omega_i}{F_i} = \sum_{j \in I} \rho^{ij} \alpha_j$. Then

$$\nabla_\lambda(\omega_i^*) = \sum_{k \in I} E_{ik} \cdot \beta_k$$

with $E_{ik} := \sum_{j \in I} \rho^{ij} \frac{\dot{q}_{jk}}{q_{jk}}$

Let L_i denote the operator

$$L_i = \lambda(1-\lambda)\nabla_\lambda^2 - [1 - (\frac{i_1}{r_1} + \frac{i_2}{r_2} + 1)(1-\lambda)]\nabla_\lambda - \frac{i_1 i_2}{r_1 r_2}.$$

It is known that

$$L_i(\omega_i) = 0.$$

Now

$$\nabla_\lambda(F_i \omega_i^*) = F_i \nabla_\lambda(\omega_i^*) + \dot{F}_i \omega_i^*$$

$$\nabla_\lambda^2(F_i \omega_i^*) = F_i \nabla_\lambda^2(\omega_i^*) + 2\dot{F}_i \nabla_\lambda(\omega_i^*) + \ddot{F}_i \omega_i^*$$

$$\text{and } \nabla_\lambda^2(\omega_i^*) = \sum_{k \in I} \dot{E}_{ik} \beta_k$$

Substituting into the equation $L_i(\omega_i) = 0$ and looking for the coefficient at β_k which must be zero gives

$$\frac{\dot{E}_{ik}}{E_{ik}} = -2 \frac{\dot{F}_i}{F_i} + \frac{1}{1-\lambda} - \frac{(\frac{i_1}{r_1} + \frac{i_2}{r_2})}{\lambda}.$$

Solving this differential equation gives

$$E_{ik} = \frac{A_{ik}}{(1-\lambda) \lambda^{\frac{i_1}{r_1} + \frac{i_2}{r_2} + 1} F_i^2}$$

with a constant $A_{ik} \in K$. E_{ik} is considered as a Laurent series in $(1-\lambda)$; its residue at 1 is just A_{ik} .

In a joint work with F. Herrlich we determined the constants A_{ik} . A careful study of the action of Γ on the Bruhat-Tits tree of $\mathbb{P}$ gives the result that the vanishing order $\text{ord } q_{ji}$ of q_{ji} at the

point $s = \lambda = 1$ is as follows:

$$\text{ord } \dot{q}_{ik} = \begin{cases} 4 : j = k \\ 2 : j \neq k \text{ and } j_1 = k_1 \text{ or } j_2 = k_2 \\ 1 : \text{otherwise} \end{cases}$$

Therefore the residue of $\frac{dq_{jk}}{q_{jk}}$ at $\lambda = 1$ is $\text{ord } q_{jk}$ and the residue of $E_{ik} d\lambda$ at $\lambda = 1$ is

$$\begin{aligned} & \sum_{j \in I} \rho^{ij} + \sum_{\substack{j \in I \\ j_1 = k_1}} \rho^{ij} + \sum_{\substack{j \in I \\ j_2 = k_2}} \rho^{ij} + \rho^{ik} \\ \text{As } \sum_{\substack{j \in I \\ j_1 = k_1}} \rho^{ij} &= \sum_{j_2=1}^{r_2-1} \rho^{i_1 j_1} \rho^{i_2 j_2} = -\rho^{i_1 k_1} \text{ and} \\ \sum_{\substack{j \in I \\ j_2 = k}} \rho^{ij} &= -\rho^{i_2 k_2} \text{ and } \sum_{j \in I} \rho^{ij} = 1 \text{ this residue is} \end{aligned}$$

indeed A_{ik} .

2) Let $\bar{\Gamma} = \Gamma/[\Gamma, \Gamma]$ be the commutator factor group of Γ ; it is a free $\mathbb{Z}$ -module generated by the images e_i of γ_i , $i \in I$.

Now G is canonically isomorphic to the factor group $\langle \sigma_1, \sigma_2 \rangle / \Gamma$ and thus acts on $\bar{\Gamma}$ by inner automorphisms; we consider $\bar{\Gamma}$ as G -module.

$$\begin{aligned} \text{As } \sigma_1 \gamma_i \sigma_1^{-1} &= \sigma_1 \cdot \sigma_1^{i_1} \sigma_2^{i_2} \sigma_1^{-i_1} \sigma_2^{-i_2} \cdot \sigma_1^{-1} \\ &= \sigma_1^{i_1+1} \sigma_2^{i_2} \sigma_1^{-i_1-1} \cdot \sigma_1 \cdot \sigma_2^{-i_2} \cdot \sigma_1^{-1} \\ &= \gamma_{i_1+1, i_2} \cdot \sigma_2^{i_2} \cdot \sigma_1 \cdot \sigma_2^{-i_2} \cdot \sigma_1^{-1} \\ &= \gamma_{i_1+1, i_2} \cdot \gamma_{1, i_2}^{-1} \end{aligned}$$

$$\begin{aligned} \text{and } \sigma_2 \gamma_i \sigma_2^{-1} &= \sigma_2 \sigma_1^{i_1} \sigma_2^{i_2} \sigma_1^{-i_1} \sigma_2^{-i_2} \cdot \sigma_2^{-1} \\ &= \sigma_2 \sigma_1^{+i_1} \sigma_2^{-1} \sigma_1^{-i_1} \cdot \sigma_1^{i_1} \cdot \sigma_2 \cdot \sigma_2^{i_2} \sigma_1^{-i_1} \sigma_2^{-i_2-1} \\ &= \gamma_{i_1, 1}^{-1} \gamma_{i_1, i_2+1} \end{aligned}$$

the action of G is known.

Let M be the submodule of the group ring $\mathbb{Z}[G]$ generated by

$$a_i = (\sigma_1^{i_1} - 1) \cdot (\sigma_2^{i_2} - 1)$$

for all $(i_1, i_2) \in I$. It is easy to verify that the mapping

$$\kappa : \bar{\Gamma} \rightarrow M$$

which sends e_i to a_i , $i \in I$, is indeed an isomorphism of G -modules.

In order to be able to work with a simpler basis we consider $K \otimes M$ and let

$$w_i := \sum_{j \in I} \rho^{+ij} \cdot a_j \in K \otimes M$$

where $i \cdot j$ is the multiplication in I considered as multiplicative semi-group in the ring $J = \mathbb{Z}/r_1\mathbb{Z} \oplus \mathbb{Z}/r_2\mathbb{Z}$ and $\rho^i := \rho_1^{i_1} \cdot \rho_2^{i_2}$ for $i \in \mathbb{Z}/r_1\mathbb{Z} \oplus \mathbb{Z}/r_2\mathbb{Z}$.

Then
$$w_i = \sum_{j \in J} \rho^{+ij} \sigma^j$$

with
$$\sigma^j = \sigma_1^{j_1} \cdot \sigma_2^{j_2} \quad \text{for } j \in J$$

and
$$\sigma_1 w_i = \sigma_1^{-i_1} \cdot w_i$$

$$\sigma_2 w_i = \rho_2^{-i_2} \cdot w_i$$

This shows that $\{w_i : i \in I\}$ is a basis of $K \otimes M$ and thus

$$a_i = \sum_{j \in I} c_{ij} w_j$$

with a matrix $c = (c_{ij})$, $c_{ij} \in K$, of determinant $\neq 0$. In fact c is the inverse of the matrix

$$(\rho^{+ij})_{i,j \in I}$$

A straight forward computation gives:
$$c_{ij} = \frac{\rho_1^{i_1 j_1} - 1}{r_1} \cdot \frac{\rho_2^{i_2 j_2} - 1}{r_2}$$

for any $i, j \in I$, $i = (i_1, i_2)$, $j = (j_1, j_2)$.

3) From 2) we get that

$$\alpha_i = \sum_{j \in I} c_{ij} \omega_j^*$$

Now
$$\nabla(\alpha_i) = \sum_{k \in I} \beta_k \frac{\dot{q}_{ik}}{q_{ik}}$$

$$= \sum_{j \in I} c_{ij} \left(\sum_{k \in I} E_{jk} \beta_k \right)$$

$$= \sum_{k \in I} \left(\sum_{j \in I} c_{ij} E_{jk} \right) \cdot \beta_k$$

and thus $\sum_{j \in I} c_{ij} E_{jk} = \frac{\dot{q}_{ik}}{q_{ik}}$ which completes the proof.

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