CAHIERS DE
TOPOLOGIE ET GÉOMÉTRIE DIFFÉRENTIELLE
CATÉGORIQUES

M. DOUPOVEC
I. KOLÁR

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Cahiers de topologie et géométrie différentielle catégoriques, tome 40, nº 1 (1999), p. 21-30
<http://www.numdam.org/item?id=CTGDC_1999__40_1_21_0>

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Volume XL-1 (1999)

ON THE JETS OF FIBRED MANIFOLD MORPHISMS

by M. DOUPOVEC and I. KOLÁR

Résumé. Le (r,s,q)–jet d'un morphisme de variétés fibrées f est déterminé par le r–jet de l'application f, par le s–jet de la restriction de f à la fibre et par le q–jet de l'application de base induite par f, $s \geq r \leq q$. Nous démontrons que les (r,s,q)–jets sont les seules images homomorphes de dimension finie de germes de morphismes de variétés fibrées satisfaisant deux conditions naturelles.

1. Introduction

About 1986, it was clarified that all product preserving bundle functors on the category $\mathcal{M}f$ of all manifolds and all smooth maps are the classical Weil bundles. Moreover, their natural transformations are in bijection with the homomorphisms of Weil algebras, see Chapter VIII of [4] for a survey. In particular, this explains some relations of the synthetic differential geometry, [2], to the classical Weil theory, [6]. Using that result, the second author deduced an abstract characterization of the jet spaces, [3]. He proved that under very weak assumptions the classical r–jets by C. Ehresmann, [1], are the only finite dimensional homomorphic images of germs of smooth maps.

In the present paper, we study the jets of fibered manifold morphisms from a similar point of view. In Section 2, we recall the concept of (r,s,q)–jet, $s \geq r \leq q$, [4], p. 126, which includes higher order contacts along the fiber and on the base. In Section 3 we formulate our main result: under similar assumptions as in [3], the (r,s,q)–jets are the only finite dimensional homomorphic images of germs of fibered manifold morphisms. The proof is essentially based on a recent result

The authors were supported by a grant of the GA ČR No 201/96/0079.

by W. M. Mikulski, [5], who has characterized all product preserving bundle functors on the category $\mathcal{FM}$ of all fibered manifolds and all fiber preserving maps in terms of homomorphisms of Weil algebras. In Section 4 we present an original proof of Mikulski's theorem. Furthermore, we would like to underline that the generalized velocities, which are defined in Section 6 by means of jets on fibered manifolds, are of independent geometric interest.

From a general point of view, our main result demonstrates that the (r,s,q)–jets of $\mathcal{FM}$–morphisms play an analogous role as the classical r–jets of smooth maps. –All manifolds and maps are assumed to be infinitely differentiable.

2. Jets of $\mathcal{FM}$–morphisms

Given two manifolds M, N and a smooth map $f : M \to N$, we can construct the r–jet $j^r_x f$ at $x \in M$. If we replace M by a fibered manifold $p : Y \to M$, we can consider a higher order contact along the fiber Y_x passing through $y \in Y$, $x = p(y)$. Thus, for two maps $f, g : Y \to N$ and an integer $s \geq r$, we define $j^{r,s}_y f = j^{r,s}_y g$ by

$$j^r_y f = j^r_y g \quad \text{and} \quad j^s_y(f|Y_x) = j^s_y(g|Y_x). \tag{1}$$

The space of all such (r,s)–jets is denoted by $J^{r,s}(Y,N)$, [4], p. 126.

If even N is a fibered manifold $\overline{p} : \overline{Y} \to \overline{M}$ and $f, g : Y \to \overline{Y}$ are two $\mathcal{FM}$–morphisms, whose base maps are denoted by $\underline{f}, \underline{g} : M \to \overline{M}$, we can require a higher order contact of the base maps as well. Hence we define

$$j^{r,s,q}_y f = j^{r,s,q}_y g$$

by (1) and by $j^q_x \underline{f} = j^q_x \underline{g}$. If $h : \overline{Y} \to \widetilde{Y}$ is another $\mathcal{FM}$–morphism, the formula

$$j^{r,s,q}_y (h \circ f) = (j^{r,s,q}_{f(y)} h) \circ (j^{r,s,q}_y f)$$

introduces a well defined composition of (r,s,q)–jets. The space of all (r,s,q)–jets of the $\mathcal{FM}$–morphisms of Y into $\overline{Y}$ is denoted by $J^{r,s,q}(Y,\overline{Y})$. The source and the target are indicated similarly to the classical case. Clearly, we have

$$J^{r,s,q}(Y, Y_1 \times Y_2) = J^{r,s,q}(Y, Y_1) \times_Y J^{r,s,q}(Y, Y_2) \tag{2}$$

for every three fibered manifolds Y, Y_1, Y_2.

Write $\mathbb{R}^{k,\ell} = (p_{k,\ell} : \mathbb{R}^k \times \mathbb{R}^\ell \to \mathbb{R}^k)$ for the product fibered manifold. If $m = \dim M$ and $m + n = \dim Y$, we introduce the principal bundle of all (r, s, q)–frames on Y by

$$P^{r,s,q}Y = \mathrm{inv} J^{r,s,q}_{0,0}(\mathbb{R}^{m,n}, Y),$$

where inv indicates the invertible (r, s, q)–jets and $(0, 0) \in \mathbb{R}^m \times \mathbb{R}^n$. Its structure group is

$$G^{r,s,q}_{m,n} = \mathrm{inv} J^{r,s,q}_{0,0}(\mathbb{R}^{m,n}, \mathbb{R}^{m,n})_{0,0}$$

and both multiplication in $G^{r,s,q}_{m,n}$ and the action on $P^{r,s,q}Y$ are given by the jet composition.

3. The main result

We are going to apply an abstract viewpoint similarly to [3]. Let $G(Y, \overline{Y})$ be the set of all germs of $\mathcal{FM}$–morphisms of Y into $\overline{Y}$. Consider a rule F transforming every pair Y, $\overline{Y}$ of fibered manifolds into a fibered manifold $F(Y, \overline{Y})$ over $Y \times \overline{Y}$ and a system of maps $\varphi_{Y,\overline{Y}} : G(Y, \overline{Y}) \to F(Y, \overline{Y})$ commuting with the projections $G(Y, \overline{Y}) \to Y \times \overline{Y}$ and $F(Y, \overline{Y}) \to Y \times \overline{Y}$ for all Y, $\overline{Y}$. Clearly, if we interpret the construction of (r, s, q)–jets as an operation on germs

$$j^{r,s,q}_y f = j^{r,s,q}(\mathrm{germ}_y f),$$

then $F = J^{r,s,q}$ and $\varphi = j^{r,s,q}$ is an example of such a pair. Analogously to [3], we formulate the following requirements I–IV.

I. Every $\varphi_{Y,\overline{Y}} : G(Y, \overline{Y}) \to F(Y, \overline{Y})$ is surjective.

II. If $W_1, \overline{W}_1 \in G_y(Y, \overline{Y})_{\bar{y}}$ and $W_2, \overline{W}_2 \in G_{\bar{y}}(\overline{Y}, \widetilde{Y})_{\tilde{y}}$ satisfy $\varphi(W_1) = \varphi(\overline{W}_1)$ and $\varphi(W_2) = \varphi(\overline{W}_2)$, then $\varphi(W_2 \circ W_1) = \varphi(\overline{W}_2 \circ \overline{W}_1)$.

By I and II, we have a well defined composition (denoted by the same symbol as the composition of germs and maps)

$$X_2 \circ X_1 = \varphi(W_2 \circ W_1)$$

for every $X_1 = \varphi(W_1) \in F_y(Y,\overline{Y})_{\overline{y}}$ and $X_2 = \varphi(W_2) \in F_{\overline{y}}(\overline{Y},\widetilde{Y})_{\widetilde{y}}$. Write $\varphi_y f$ for $\varphi(\mathrm{germ}_y f)$. For another pair $Z,\overline{Z}$ of fibered manifolds, every local $\mathcal{FM}$–isomorphism $f : Y \to Z$ and every $\mathcal{FM}$–morphism $g : \overline{Y} \to \overline{Z}$ induce a map $F(f,g) : F(Y,\overline{Y}) \to F(Z,\overline{Z})$ by

$$(3) \qquad F(f,g)(X) = (\varphi_{\overline{y}} g) \circ X \circ \varphi_{f(y)}(f^{-1}), \quad X \in F_y(Y,\overline{Y})_{\overline{y}},$$

where f^{-1} is constructed locally. We require

III. Each map $F(f,g)$ is smooth.

For the sets of germs, we have $G(Y, Y_1 \times Y_2) = G(Y,Y_1) \times_Y G(Y,Y_2)$. At the manifold level, the meaning of the following requirement was clarified in [3].

IV. (Product property) $F(Y, Y_1 \times Y_2)$ coincides with the fibered product $F(Y,Y_1) \times_Y F(Y,Y_2)$ over Y.

The main result of the present paper is the following assertion.

Theorem. *For every pair (F,φ) satisfying I–IV, there exist integers $s \geq r \leq q$ such that $(F,\varphi) = (J^{r,s,q}, j^{r,s,q})$.*

The proof will occupy the rest of the paper.

First of all, for every integers k,ℓ we define an induced bundle functor $F_{k,\ell}$ on $\mathcal{FM}$ by

$$F_{k,\ell}Y = F_{0,0}(\mathbb{R}^{k,\ell}, Y), \quad F_{k,\ell}f = F_{0,0}(1_{\mathbb{R}^{k,\ell}}, f) : F_{k,\ell}Y \to F_{k,\ell}\overline{Y}$$

for every $\mathcal{FM}$–morphism $f : Y \to \overline{Y}$, where the subscript $0,0$ indicates the restriction over $(0,0) \in \mathbb{R}^{k,\ell}$. By the product property of F, each functor $F_{k,\ell}$ preserves products.

4. Product preserving bundle functors on $\mathcal{FM}$

We recall that a bundle functor F on a local category $\mathcal{C}$ over manifolds transforms every $\mathcal{C}$–object Q into a fibered manifold FQ over the underlying manifold of Q and every $\mathcal{C}$–morphism $f : Q_1 \to Q_2$ into an $\mathcal{FM}$–morphism $Ff : FQ_1 \to FQ_2$ whose base map is the underlying smooth map of f. Moreover, F is assumed to have the localization property, [4], p. 170.

First of all we present one construction of a product preserving bundle functor on $\mathcal{FM}$. Let $\mu : A \to B$ be a Weil algebra homomorphism.

It induces two bundle functors T^A, T^B on $\mathcal{M}f$ and a natural transformation $\widetilde{\mu} : T^A \to T^B$, [4], Chapter VIII. For every fibered manifold $p : Y \to M$, we have $T^B p : T^B Y \to T^B M$. Then we take into account the map $\widetilde{\mu}_M : T^A M \to T^B M$ and construct the induced bundle

$$T^\mu Y = \widetilde{\mu}_M^* T^B Y,$$

which will be also denoted by $T^\mu Y = T^A M \times_{T^B M} T^B Y$. For every $\mathcal{FM}$-morphism $f : Y \to \overline{Y}$ over $\underline{f} : M \to \overline{M}$, we have $T^B f : T^B Y \to T^B \overline{Y}$ and we construct the induced map

$$T^\mu f : T^A \underline{f} \times_{T^B \underline{f}} T^B f : T^\mu Y \to T^\mu \overline{Y}.$$

This defines a bundle functor T^μ on $\mathcal{FM}$. Clearly, T^μ preserves products.

The following result is due to W. M. Mikulski, [5] (but our proof is original and shorter).

Lemma 1. *For every product preserving bundle functor H on $\mathcal{FM}$ there exists a Weil algebra homomorphism $\mu : A \to B$ such that $H = T^\mu$.*

Proof. Let pt denote one element manifold and $\mathrm{pt}_M : M \to \mathrm{pt}$ the unique map. There are two canonical injections $i_1, i_2 : \mathcal{M}f \to \mathcal{FM}$ defined by $i_1 M = (1_M : M \to M)$, $i_1 f = (f, f)$, $i_2 M = (\mathrm{pt}_M : M \to \mathrm{pt})$, $i_2 f = (f, 1_{\mathrm{pt}})$ and a natural transformation $t : i_1 \to i_2$, $t_M = (1_M, \mathrm{pt}_M) : i_1 M \to i_2 M$. Applying H, we obtain two bundle functors $H \circ i_1$, $H \circ i_2$ on $\mathcal{M}f$ and a natural transformation $H \circ t : H \circ i_1 \to H \circ i_2$. Clearly, both $H \circ i_1$ and $H \circ i_2$ preserve products. By the Weil theory, there exists a Weil algebra homomorphism $\mu : A \to B$ such that $H \circ i_1 = T^A$, $H \circ i_2 = T^B$ and $H \circ t = \widetilde{\mu}$. Consider a commutative diagram

(4)
$$\begin{array}{ccc} (p : Y \to M) & \xrightarrow{(1_Y, \mathrm{pt}_M)} & (\mathrm{pt}_Y : Y \to \mathrm{pt}) \\ {\scriptstyle (p, 1_M)}\downarrow & & \downarrow{\scriptstyle i_2 p} \\ (1_M : M \to M) & \xrightarrow{t_M} & (\mathrm{pt}_M : M \to \mathrm{pt}). \end{array}$$

One verifies easily that (4) is a pullback in $\mathcal{FM}$. We have assumed that H preserves products and has the localization property. But every fibered manifold is locally a product and the induced bundle of a product is a product, so that H preserves inducing of bundles. If we apply H to (4), we obtain a pullback diagram

$$\text{(5)}\qquad \begin{array}{ccc} HY & \xrightarrow{q_2} & T^B Y \\ {\scriptstyle q_1}\downarrow & & \downarrow{\scriptstyle T^B p} \\ T^A M & \xrightarrow{\tilde{\mu}_M} & T^B M. \end{array}$$

This proves our claim. $\square$

We describe the projections q_1 and q_2 in the case $H = F_{k,\ell}$. If we put $Y = \mathbb{R}^{k,\ell}$ into (4), we obtain a commutative diagram

$$\text{(6)}\qquad \begin{array}{ccc} \mathbb{R}^{k,\ell} & \longrightarrow & i_2\mathbb{R}^{k+\ell} \\ \downarrow & & \downarrow \\ i_1\mathbb{R}^k & \longrightarrow & i_2\mathbb{R}^k. \end{array}$$

In general, if we have an $\mathcal{FM}$–morphism $f : Y \to \overline{Y}$ with the base map $\underline{f} : M \to \overline{M}$ and we need distinguish the manifold map $f : Y \to \overline{Y}$ from the $\mathcal{FM}$–morphism itself, we write $(f, \underline{f})$ for the latter. The same notation is used for germs as well. Consider an $\mathcal{FM}$–morphism $(g, \underline{g}) : \mathbb{R}^{k,\ell} \to Y$ and construct $i_2 g : i_2\mathbb{R}^{k+\ell} \to i_2 Y$, $i_1\underline{g} : i_1\mathbb{R}^k \to i_1 M$, $i_2\underline{g} : i_2\mathbb{R}^k \to i_2 M$. These four morphisms relate (4) and (6) into a commutative cube. Applying F to this cube, we find

$$\text{(7)}\qquad q_1(\varphi(W, \underline{W})) = \varphi(i_1\underline{W}), \quad q_2(\varphi(W, \underline{W})) = \varphi(i_2 W),$$

where $W = \text{germ}_{0,0}\, g$ and $\underline{W} = \text{germ}_0\, \underline{g}$.

5. The base and the fiber constructions

Consider two manifolds M, N and the set $G(M, N)$ of all germs of smooth maps of M into N. The base modification (F^1, φ^1) of (F, φ) is constructed as follows. We set $F^1(M,N) = F(i_1 M, i_1 N)$ and we define $\varphi^1_{M,N} : G(M,N) \to F^1(M,N)$ by $\varphi^1_{M,N}(W) = \varphi_{i_1 M, i_1 N}(i_1 W)$. In the same way, we introduce the fiber modification (F^2, φ^2) of (F, φ) by setting $F^2(M,N) = F(i_2 M, i_2 N)$ and $\varphi^2_{M,N} : G(M,N) \to F^2(M,N)$, $\varphi^2_{M,N}(W) = \varphi_{i_2 M, i_2 N}(i_2 W)$. Then we can apply Theorem 1 of [3]. This yields

Lemma 2. *There exist integers q and s such that $(F^1, \varphi^1) = (J^q, j^q)$ and $(F^2, \varphi^2) = (J^s, j^s)$.*

6. Generalized velocities

Analogously to the classical functor T^r_k of (k,r)–velocities, we introduce

$$T^{r,s}_{k,\ell} N = J^{r,s}_{0,0}(\mathbb{R}^{k,\ell}, N), \quad T^{r,s}_{k,\ell} f(j^{r,s}_{0,0} g) = j^{r,s}_{0,0}(f \circ g)$$

for every manifold N and every smooth map $f : N \to \overline{N}$. Hence $T^{r,s}_{k,\ell}$ is a bundle functor on $\mathcal{M}f$ that preserves products. In the case $k = 0$ or $\ell = 0$, we obtain the classical velocities, so that $r = s$. By the general theory, the Weil algebra of $T^{r,s}_{k,\ell}$ is

$$\mathbb{D}^{r,s}_{k,\ell} := T^{r,s}_{k,\ell}\mathbb{R}.$$

Let $E(k+\ell)$ be the ring of all germs of smooth functions on $\mathbb{R}^{k+\ell}$ at 0, $\mathfrak{m}(k+\ell)$ be its maximal ideal, which is generated by the germs of all variables $x_1, \dots, x_{k+\ell}$, and $\mathfrak{m}(k) \subset \mathfrak{m}(k+\ell)$ be the ideal generated by the germs of $x_1, \dots, x_k$. By the definition of $T^{r,s}_{k,\ell}$, the ideal generating $\mathbb{D}^{r,s}_{k,\ell}$ is

$$\left\langle \mathfrak{m}(k+\ell)^{s+1}, \mathfrak{m}(k)\mathfrak{m}(k+\ell)^r \right\rangle. \tag{8}$$

Indeed, this ideal kills all derivatives of the order greater than s and all derivatives of the order greater than r with at least one entry of $x_1, \dots, x_k$.

There is a canonical map

$$\nu_N : T^q_k N \to T^{r,s}_{k,\ell} N, \quad s \geq r \leq q \tag{9}$$

defined as follows. For $j_0^q\varphi \in T_k^q N$, $\varphi : \mathbb{R}^k \to N$, we construct $\varphi \circ p_{k,\ell} : \mathbb{R}^{k,\ell} \to N$ and we set

$$\nu_N(j_0^q\varphi) = j_{0,0}^{r,s}(\varphi \circ p_{k,\ell}). \tag{10}$$

Since $\varphi \circ p_{k,\ell}$ is constant along each fiber of $\mathbb{R}^{k,\ell}$, this construction is independent of $s \geq r$.

For a fibered manifold $p : Y \to M$, we define

$$T_{k,\ell}^{r,s,q}Y = J_{0,0}^{r,s,q}(\mathbb{R}^{k,\ell}, Y).$$

Consider $T_{k,\ell}^{r,s}p : T_{k,\ell}^{r,s}Y \to T_{k,\ell}^{r,s}M$ and the map $\nu_M : T_k^q M \to T_{k,\ell}^{r,s}M$. By definition, we verify directly

$$T_{k,\ell}^{r,s,q}Y = \nu_M^* T_{k,\ell}^{r,s}Y = T_k^q M \times_{T_{k,\ell}^{r,s}M} T_{k,\ell}^{r,s}Y. \tag{11}$$

The jet composition defines a right action of $G_{m,n}^{r,s,q}$ on $T_{m,n}^{r,s,q}\overline{Y}$ for every fibered manifold $\overline{Y}$. Clearly, one can express $J^{r,s,q}(Y,\overline{Y})$ as an associated fibre bundle

$$J^{r,s,q}(Y,\overline{Y}) = P^{r,s,q}Y[T_{m,n}^{r,s,q}\overline{Y}], \tag{12}$$

$m = \dim M$, $m + n = \dim Y$.

7. The end of the proof

By Lemma 1, $F_{k,\ell} = T^{\mu_{k,\ell}}$ for a Weil algebra homomorphism $\mu_{k,\ell} : A_{k,\ell} \to B_{k,\ell}$. In Lemma 2, we have constructed an integer s.

Lemma 3. *For every $k > 0$ and every ℓ, there exists an integer $r_{k,\ell} \leq s$ such that $B_{k,\ell} = \mathbb{D}_{k,\ell}^{r_{k,\ell},s}$.*

Proof. Every smooth function f on $\mathbb{R}^{k+\ell}$ can be interpreted as an $\mathcal{FM}$-morphism $f : \mathbb{R}^{k,\ell} \to i_2\mathbb{R}$ over $\mathrm{pt}_{\mathbb{R}^{k,\ell}}$. By (7), the ideal $\mathcal{N}_{k,\ell} \subset E(k+\ell)$ defining $B_{k,\ell}$ is the set of all germs W of functions satisfying $\varphi(W) = \varphi(\hat{0})$, where $\hat{0}$ is the germ of the zero function at $(0,0) \in \mathbb{R}^{k+\ell}$. Hence $\mathcal{N}_{k,\ell}$ has the following *substitution property*:

$$W \in \mathcal{N}_{k,\ell} \text{ and } h \in G_{0,0}(\mathbb{R}^{k,\ell}, \mathbb{R}^{k,\ell})_{0,0} \quad \text{implies} \quad W \circ h \in \mathcal{N}_{k,\ell}. \tag{13}$$

By the construction in Lemma 2, s is the smallest integer satisfying $(x_{k+\ell})^{s+1} \in \mathcal{N}_{k,\ell}$. Take h of the form

$$\overline{x}_{k+\ell} = c_1 x_1 + \cdots + c_{k+\ell} x_{k+\ell}, \tag{14}$$

$\overline{x}_i = 0$ otherwise. By the substitution property,

$$(c_1 x_1 + \cdots + c_{k+\ell} x_{k+\ell})^{s+1} \in \mathcal{N}_{k,\ell}$$

with arbitrary $c_1, \ldots, c_{k+\ell}$. This implies $\mathfrak{m}(k+\ell)^{s+1} \subset \mathcal{N}_{k,\ell}$. Further, since $\mathcal{N}_{k,\ell}$ is of finite codimension, there exists a smallest integer $r_{k,\ell}$ such that $x_1(x_{k+\ell})^{r_{k,\ell}} \in \mathcal{N}_{k,\ell}$. If we take h of the form (14),

$$\overline{x}_1 = c_1 x_1 + \cdots + c_k x_k \tag{15}$$

and $\overline{x}_i = 0$ otherwise, then we deduce $\mathfrak{m}(k)\mathfrak{m}(k+\ell)^{r_{k,\ell}} \subset \mathcal{N}_{k,\ell}$. Hence

$$\left\langle \mathfrak{m}(k+\ell)^{s+1}, \mathfrak{m}(k)\mathfrak{m}(k+\ell)^{r_{k,\ell}} \right\rangle \subset \mathcal{N}_{k,\ell}. \tag{16}$$

Conversely, the ideal on the left hand side has the substitution property and is determined by $(x_{k+\ell})^{s+1}$ and $x_1(x_{k+\ell})^{r_{k,\ell}}$, so that it is equal to $\mathcal{N}_{k,\ell}$. Since s is minimal, we have $r_{k,\ell} \leq s$. Comparing with (8), we prove the lemma. □

Lemma 4. *For $k \geq 1$, we have $r_{k,\ell} = r_{1,0}$.*

Proof. For every two germs $(f, \underline{f})$, $(g, \underline{g}) \in G_{0,0}(\mathbb{R}^{k,\ell}, Y)$, $\varphi(f, \underline{f}) = \varphi(g, \underline{g})$ implies $j^{r_{k,\ell}} f = j^{r_{k,\ell}} g$ with maximal $r_{k,\ell}$ by Lemma 3. Clearly, every germ $(h, \underline{h}) \in G_{0,0}(\mathbb{R}^{1,0}, \mathbb{R}^{k,\ell})_{0,0}$ is the germ of a curve on $\mathbb{R}^{k,\ell}$. We have $\varphi((f, \underline{f}) \circ (h, \underline{h})) = \varphi((g, \underline{g}) \circ (h, \underline{h}))$, which yields $j^{r_{1,0}}(f \circ h) = j^{r_{1,0}}(g \circ h)$. Taking into account the basic properties of jets, we deduce $j^{r_{1,0}} f = j^{r_{1,0}} g$. Since $r_{k,\ell}$ is maximal, we have $r_{k,\ell} \geq r_{1,0}$. Conversely, if we use Lemma 3 of [3], we prove in the same way $r_{1,0} \geq r_{k,\ell}$. □

For $k = 0$, we have the classical velocities, so that $r_{0,k} = s$. Write $r = r_{1,0}$. By the base part of Lemma 2, we obtain $T^{A_{k,\ell}} = T^q_k$ independently of ℓ. By construction, $q \geq r$.

Lemma 5. *We have* $F_{k,\ell} = T_{k,\ell}^{r,s,q}$.

Proof. It remains to prove that the natural transformation $\mu_{k,\ell}$ coincides with ν from (9). Since $i_1 N = (1_N : N \to N)$, every $(W, \underline{W}) \in G_{0,0}(\mathbb{R}^{k,\ell}, i_1 N)$ is of the form $(\underline{W} \circ p_{k,\ell}, \underline{W})$. If we put $Y = i_1 N$ and $H = F_{k,\ell}$ into (5) and use (7), we obtain

$$\widetilde{\mu}_N(j^q(\underline{W})) = j^{r,s}(\underline{W} \circ p_{k,\ell}).$$

□

Now it suffices to deduce that each $F(Y, \overline{Y})$ is the associated fiber bundle $P^{r,s,q}Y[T_{m,n}^{r,s,q}\overline{Y}]$. Obviously, every $W \in G_y(Y, \overline{Y})_{\overline{y}}$ and $V \in \mathrm{inv}G_{0,0}(\mathbb{R}^{m,n}, Y)_y$ determine $W \circ V \in G_{0,0}(\mathbb{R}^{m,n}, \overline{Y})$. Applying φ, we obtain the standard situation of the smooth associated bundles. This proves the theorem.

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Miroslav Doupovec, Institute of Mathematics, FS VUT Brno, Technická 2, 616 69 Brno, Czech Republic
E-mail address: doupovec@mat.fme.vutbr.cz

Ivan Kolář, Institute of Algebra and Geometry, Masaryk University, Janáčkovo nám. 2a, 662 96 Brno, Czech Republic
E-mail address: kolar@math.muni.cz