CAHIERS DE
TOPOLOGIE ET GÉOMÉTRIE DIFFÉRENTIELLE
CATÉGORIQUES

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Cahiers de topologie et géométrie différentielle catégoriques, tome 21, nº 2 (1980), p. 191-224

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CAHIERS DE TOPOLOGIE
ET GEOMETRIE DIFFERENTIELLE

Vol. XXI-2 (1980)

A GENERALIZED DUALITY THEOREM FOR STRUCTURE FUNCTORS

by Manfred B. WISCHNEWSKY

dedicated to Charles Ehresmann

INTRODUCTION.

One aim of this paper is to introduce a new concept for functors which is in fact a class of concepts - called structure functors, resp. structure functor sequences - containing as special instances some of the most important concepts introduced during the last two decades in category theory as well as a whole host of new concepts.

Structure functors appear everywhere. Examples are the q-functors in the sense of Ehresmann [6], proclusion functors in the sense of Wyler [41], reflective or coreflective subcategories of sketched structures in the sense of *A*. and C. Ehresmann [2], in particular of locally presentable categories in the sense of Gabriel-Ulmer [7], reflective or coreflective subcategories of topological categories (Brümmer [4,5], Herrlich [10, 11], Herrlich-Strecker [12], Hoffmann [13,14,15], Hušek [18], Kennison [19], Roberts [23], Tholen [26,27,28], Wischnewsky [31,32], Wolff [37,38], Wyler [39,40]), reflective or coreflective subcategories of Eilenberg-Moore categories (Lawvere [20], Linton [21]) as well as compositions of the functors in question.

The breakthrough in connection with these new concepts is given by the notion «connectedness with respect to a sequence of functors», which turns out to be the key for solving several fundamental problems in connection with structure functors (cp. [30,34,35,36]).

The duality theorem for structure functor sequences proved in this paper is extremely general. The usefulness as well as the importance can be seen from the corollaries derived from. It contains as special instances the duality theorems for topological functors (Antoine [1], Roberts [23]), for (E,M)-topological functors (Hoffmann [13]), for semi-topological functors (Tholen [28]), for locally orthogonal Q-functors (cp.

Wolff [38], Tholen [30]) as well as for topologically-algebraic structure functors [33].

In several subsequent papers [34, 35, 36] the theory of structure functors is completed.

Finally one should mention that special instances of this new notion - called «topologically-algebraic structure functors» (Wischnewsky [33]) describe completely all full reflective or coreflective restrictions of semitopological functors. The previous attempts by the author [32] as well as W. Tholen [30] to describe this particular class of functors, by introducing the notions (Φ, Γ)-structure functor [32], resp. equivalently (Φ, Γ)-concrete functor [30] failed. Both of these notions turned out to be not general enough (cp. [33]). A systematic study of Top-algebraic structure functors can be found in [33].

Finally I would like to thank W. Tholen in particular for many stimulating discussions.

0. NOTATIONS.

Let $S: \underline{A} \to \underline{X}$ be a functor. A *S-cone* is a triple $(X, \psi, D(\underline{A}))$ where X is an $\underline{X}$-object, $D(\underline{A}): \underline{D} \to \underline{A}$ is an $\underline{A}$-diagram ($\underline{D}$ may be void or large) and $\psi: \Delta X \to SD(\underline{A})$ is a functorial morphism (Δ denotes the canonical functor into the functor category). Often $(X, \psi, D(\underline{A}))$ shall be abbreviated by ψ. *Cone(S)* denotes the class of all S-cones. If $\underline{D} = 1$ (one point category) then ψ is called a *S-morphism* denoted by (A, a) where A is an $\underline{A}$-object and $a: X \to SA$ is an $\underline{X}$-morphism.

The dual notions are *S-cocones* and *S-comorphism*. The corresponding classes are denoted by *Co-Cone(S)*, resp. *Co-Mor(S)*.

Epi(S) denotes the class of all S-epimorphisms $e: X \to SA$ where a S-morphism (A, e) is a S-epimorphism if the equation

$$(Sp)e = (Sq)e \quad \text{for } \underline{A}\text{-morphisms } p, q: A \rightrightarrows B,$$

implies $p = q$. The dual notion is S-monomorphism. The class of all S-monomorphisms is denoted by *Mono(S)*.

Iso(S) denotes the class of all S-isomorphisms, i.e., of all objects (A, a) in *Mor(S)* with an isomorphism in $\underline{X}$.

$Init(S)$ denotes the class of all *S-initial cones*, i.e. of all $\underline{A}$-cones $\alpha : \Delta A \to D(\underline{A})$ such that for any $\underline{A}$-cone $\beta : \Delta B \to D(\underline{A})$ and $\underline{X}$-morphism $x : SB \to SA$ with $S\beta = (S\alpha)(\Delta x)$ there exists a unique $\underline{A}$-morphism $a : B \to A$ with $\beta = \alpha(\Delta a)$ and $Sa = x$.

1. CONNECTEDNESS.

1.1. DEFINITION.

Let $S_i : \underline{A}_i \to \underline{X}$, $i = 0, 1, \dots, n$ be functors with the same codomain $\underline{X}$.

$(A_i)_n$ [1] with $A_i \in Ob(\underline{A}_i)$, $i = 0, 1, \dots, n$ *is connected in* $\underline{X}$ *with respect to* $(S_i)_n$ (or for short $(S_i)_n$*-connected*) if for each $i = 1, 2, \dots, n$ there exists a $\underline{X}$-morphism

$$S_{i-1}(A_{i-1}) \to S_i(A_i) \quad \text{or} \quad S_i(A_i) \to S_{i-1}(A_{i-1}) \text{ [2]}.$$

We denote this by $S_{i-1}(A_{i-1}) - S_i(A_i)$ for $i = 1, 2, \dots, n$. The corresponding chain

$$S_0(A_0) \overset{x_1}{\text{——}} S_1(A_1) \overset{x_2}{\text{——}} \dots \overset{x_n}{\text{——}} S_n(A_n)$$

is denoted by $(A_i, x_i)_n$ where $x_0 := id(S_0(A_0))$.

Let $(A_i, x_i)_n$ and $(B_i, y_i)_n$ be two $(S_i)_n$-chains. A morphism $(a_i)_n : (A_i, x_i) \to (B_i, y_i)$ is a $(n+1)$-tuple of morphisms a_i, $i = 0, 1, \dots, n$ with $a_i : A_i \to B_i$ in $\underline{A}_i$ for all i, such that each cell in the following diagram commutes:

$$\begin{tikzcd}
S_0(A_0) \arrow[r, dash, "x_1"] \arrow[d, "S_0(a_0)"'] & \dots \arrow[r, dash] & S_{i-1}(A_{i-1}) \arrow[r, dash, "x_i"] \arrow[d, "S_{i-1}(a_{i-1})"'] \arrow[dr, phantom, "="] & S_i(A_i) \arrow[r, dash] \arrow[d, "S_i(a_i)"] & \dots \arrow[r, dash, "x_n"] & S_n(A_n) \arrow[d, "S_n(a_n)"] \\
S_0(B_0) \arrow[r, dash, "y_1"'] & \dots \arrow[r, dash] & S_{i-1}(B_{i-1}) \arrow[r, dash, "y_i"'] & S_i(B_i) \arrow[r, dash] & \dots \arrow[r, dash, "y_n"'] & S_n(B_n)
\end{tikzcd}$$

FIGURE 1

This defines the category $Chs(S_i)_n$ of all $(S_i)_n$-chains. If the

[1] $(X_i)_n := (X_0, X_1, \dots, X_n)$.

[2] This notion generalizes the notion «path-connected» in a category.

codomain-category $\underline{X}$ is a functor-category $[\underline{D}, \underline{C}]$ [1], we call the $(S_i)_n$-chains sometimes *$(S_i)_n$-functorial chains* or n-functorial chains if there is no confusion.

1.2. REMARKS.

1. If all S_i, $i = 0, \dots, n$ are identity functors, then the chains in $\underline{X}$ represent path-connected objects in the usual sense.

2. If $n = 1$ then the category $Chs(S_0, S_1)$ contains the comma categories $(S_0 \downarrow S_1)$ and $(S_1 \downarrow S_0)$ in the sense of Lawvere (cp. [22], II.6). Hence this notion is at the same time a generalization of Lawvere's comma categories [22].

3. If $n = 1$, $S_0 = S$ and $S_1 = Id$ then the category $Chs(S_0, S_1)$ contains the categories $Mor(S)$ and $Co\text{-}Mor(S)$ of all S-morphisms, resp. S-comorphisms.

1.3. Let

$$S_i : \underline{A}_i \to \underline{X}, \quad i = 0, 1, \dots, n,$$

be functors. Then we obtain a sequence of induced functors for every diagram category $\underline{D}$ ($\underline{D}$ may be void or large) by

$$S_i(\underline{D}) = [\underline{D}, S_i] : [\underline{D}, \underline{A}_i] \to [\underline{D}, \underline{X}].$$

Let σ_1, σ_2 be subsets of the index-set $\{0, 1, \dots, n\}$. Denote by

$$Chs(S_i; \sigma_1, \sigma_2)_n$$

the category of all $(S_i)_n$-functorial chains

$$S_0 D(\underline{A}_0) \overset{\psi_1}{\longrightarrow} S_1 D(\underline{A}_1) \overset{\psi_2}{\longrightarrow} \dots \overset{\psi_n}{\longrightarrow} S_n D(\underline{A}_n)$$

where:

$S_i = Id$ for $i \in \sigma_1$,

$D(\underline{A}_i) : \underline{D} \to \underline{A}_i$, $i \notin \sigma_2$, is an arbitrary functor, and

$D(\underline{A}_i) := \Delta A_i$ is a constant (= diagonal) functor for $i \in \sigma_2$.

Sometimes we omit σ_1 in the above notation.

1) may be void or large

If all functors $D(\underline{A}_i)$, $i = 0, 1, \dots, n$ are constant functors, then the functorial chain is called a *constant functorial chain.* The subcategory of all constant $(S_i)_n$-chains is denoted by $\Delta\text{-}Chs(S_i)_n$. Morphisms from constant $(S_i)_n$-chains to arbitrary $(S_1; \sigma_1, \sigma_2)$-chains are called co-extensions (dually extensions). If furthermore the constant functorial chain is pointwise in a class Σ of $(S_i)_n$-chains, the coextension is called a Σ-*coextension* (more exactly a $(S_i)_n$-Σ-*coextension*). Let $\Sigma \subset Chs(S_i)_n$ be a class of $(S_i)_n$-chains. We say that a $(S_i; \sigma_1, \sigma_2)_n$-functorial chain is a $(S_i; \sigma_1, \sigma_2)_n$-$\Sigma$-*chain* if it is pointwise in Σ (sometimes it is just called a Σ-functorial chain if there is no confusion).

2. SEMIFINAL STRUCTURE FUNCTOR SEQUENCES.

In order to define the notion «structure functor sequences» we need at first the following notion.

2.1. DEFINITION.

Let $\underline{C}$ be an arbitrary category and Π be a class of $\underline{C}$-objects. An object $A \in Ob(\underline{C})$ is called Π-*initial* if it is initial with respect to all objects in $\Pi \cup \{A\}$, i.e. for all objects $C \in \Pi \cup \{A\}$ there exists exactly one $\underline{C}$-morphism $A \to C$. The dual notion is Π-*final*. In particular the set of $\underline{C}$-morphisms $\underline{C}(A, A)$ has precisely one element. If $\Pi = Ob(\underline{C})$ then one has the usual notions initial, resp. final (= terminal) object in a category.

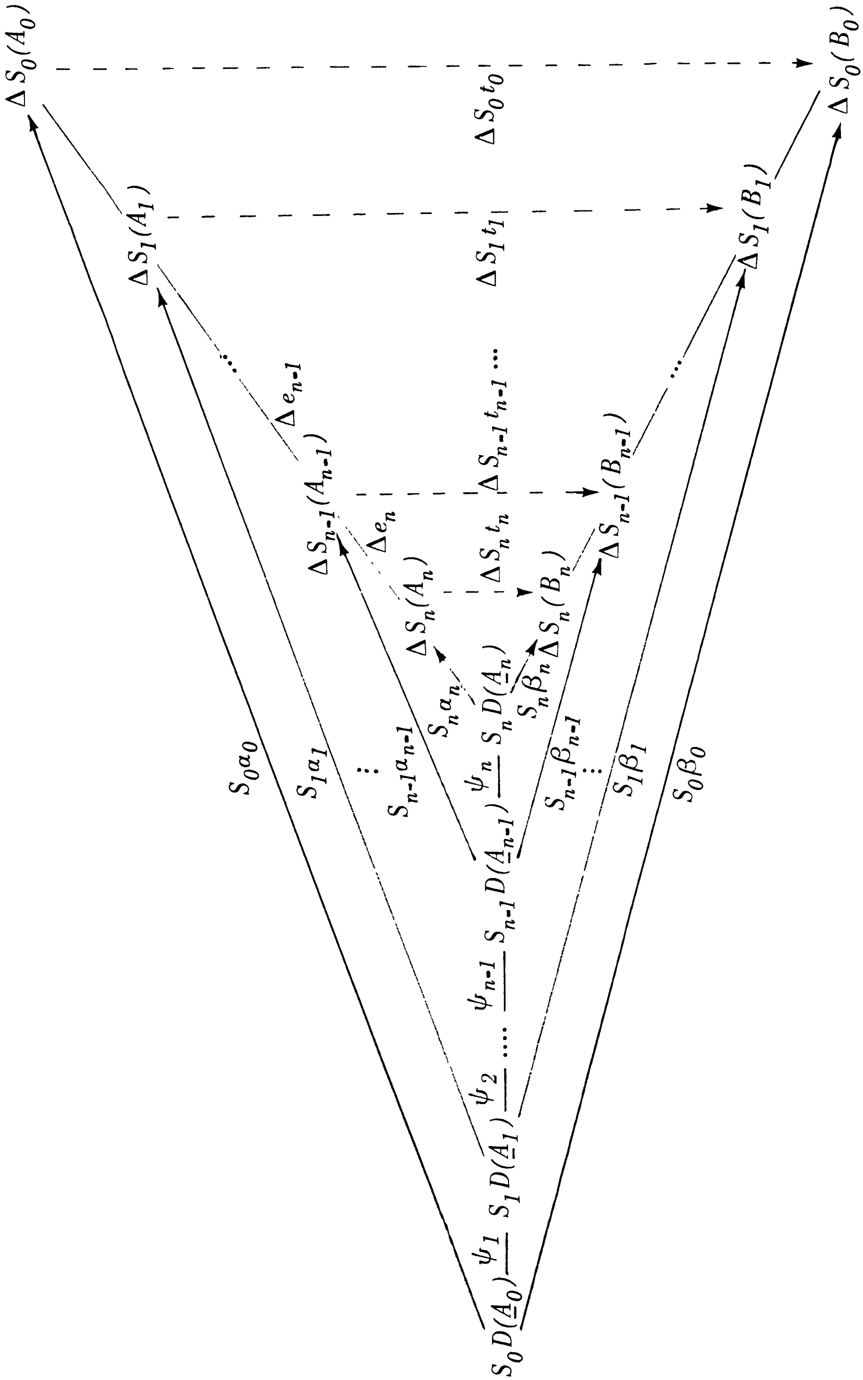
Let $(S_i)_n$ be a sequence of functors with the same codomain and σ_1, σ_2 be subsets of the index-set $\{0, 1, \dots, n\}$. Let Ω, Γ, Σ be classes of $(S_i)_n$-chains. Denote by $\Phi(\Omega)$ the class of all $(S_i; \sigma_1, \sigma_2)_n$-functorial chains being pointwise in Ω.

Given $(S_i; \sigma_1, \sigma_2)_n$, let ρ be another subset of the index-set $\{0, 1, \dots, n\}$. Let

$$(a_i)_n : (D(\underline{A}_i), \psi_i)_n \to (D^*(\underline{A}_i), \psi_i^*)_n$$

be a morphism between functorial chains. The morphism $(a_i)_n$ is called a ρ-*morphism* if for all $l \in \rho$, $a_l = Id$. This defines the subcategory of

FIGURE 2


$\Delta S_0(A_0)$
$\Delta S_0 t_0$
$\Delta S_0(B_0)$
$\Delta S_1(A_1)$
$\Delta S_1 t_1$
$\Delta S_1(B_1)$
Δe_{n-1}
$\Delta S_{n-1} t_{n-1}$
$\Delta S_{n-1}(A_{n-1})$
$\Delta S_{n-1}(B_{n-1})$
Δe_n
$\Delta S_n t_n$
$\Delta S_n(A_n)$
$\Delta S_n(B_n)$
$S_n \alpha_n$
$S_n \beta_n$
$S_0 \alpha_0$
$S_1 \alpha_1$
$S_{n-1} \alpha_{n-1}$
$S_{n-1} \beta_{n-1}$
$S_1 \beta_1$
$S_0 \beta_0$
$S_0 D(\underline{A}_0) \xrightarrow{\psi_1} S_1 D(\underline{A}_1) \xrightarrow{\psi_2} \cdots \xrightarrow{\psi_{n-1}} S_{n-1} D(\underline{A}_{n-1}) \xrightarrow{\psi_n} S_n D(\underline{A}_n)$

all ρ-morphisms, i.e. the subcategory of all $(S_i; \sigma_1, \sigma_2)$-functorial chains with ρ-morphisms as morphisms. This subcategory is denoted by

$$Chs(S_i; \sigma_1, \sigma_2, \rho)_n .$$

2.2. DEFINITION *(Semifinal structure functor sequences).*

Notation as above. Let furthermore ρ be a subset of the index-set. The sequence $(S_i)_n$ is a $(\Phi(\Omega), \Gamma, \Sigma)_{\sigma_1, \sigma_2, \rho}$*-semifinal structure functor sequence* if for any functorial chain in $\Phi(\Omega)$ there exists a Σ-extension which is a ρ-morphism and which is initial with respect to the subclass of all ρ-extensions of the given functorial chain lying in Γ. The corresponding initial object is called a *semifinal extension* (with respect to $(\Phi(\Omega), \Gamma, \Sigma)_{\sigma_1, \sigma_2, \rho}$).

2.3. REMARKS.

There are several possible generalizations of the notion introduced here. I will not pursue these possibilities here.

1° The sequence $(S_i)_n$ has $(\Phi(\Omega), \Gamma, \Sigma)_{\sigma_1, \sigma_2, \rho}$-semifinal extension only for a subclass $S(\Omega) \subset \Phi(\Omega)$. In the examples in mind these subclasses are defined by restrictions of the index-categories $\underline{D}$. In most cases $\underline{D} = 1$ (e.g. q-functors in the sense of Ehresmann [6], or proclusion functors in the sense of Wyler [41]).

2° The second possibly important generalization is given by the property that in Definition 2.2 the universal object is not a Σ-extension but an arbitrary functorial chain and that the universal property is valid with respect to a class $S(\Gamma) \subset \Phi(\Gamma)$ and is not necessarily unique [1].

The generalizations 1 and 2 can be obviously combined to the following generalization.

3° Let $S(\Omega) \subset \Phi(\Omega)$, $S(\Gamma) \subset \Phi(\Gamma)$ and $S(\Sigma) \subset \Phi(\Sigma)$ be classes of functorial chains. The sequence $(S_i)_n$ is a $(S(\Omega), S(\Gamma), S(\Sigma))_{\sigma_1, \sigma_2, \rho}$-*semifinal structure functor sequence* if, for any functorial chain in $S(\Omega)$, $(D_i(\underline{A}_i), \psi_i)$, exists a ρ-morphism

1) See Appendix for the corresponding « weak» notions.

$$(\alpha_i)_n : (D_i(\underline{A}_i), \psi_i) \to (\hat{D}_i(\underline{A}_i), \hat{\psi}_i) \quad \text{with} \quad (\hat{D}_i(\underline{A}_i), \hat{\psi}_i) \in S(\Sigma)$$

such that with respect to the (meta-)class of all ρ-morphisms

$$(\beta_i)_n : (D_i(\underline{A}_i), \psi_i) \to (C_i(\underline{A}_i), \phi_i) \quad \text{with} \quad (C_i(\underline{A}_i), \phi_i) \in S(\Gamma)$$

the chain $(\hat{D}_i(\underline{A}_i), \hat{\psi}_i)$ is initial (in the sense of Definition 2.1).

4° The definition 3 can be generalized once again by assuming that the sequence $(S_i)_n$ can not be described by just one triple (Ω, Γ, Σ) but by a sequence of triples

$$(\Omega_j, \Gamma_j, \Sigma_j)_{\sigma_1^j, \sigma_2^j, \rho_j} \quad \text{for } j = 1, \dots, n,$$

This leads to the notion $(S(\Omega_j), S(\Gamma_j), S(\Sigma_j))_{\sigma_1^j, \sigma_2^j, \rho_j}$-*semifinal structure functor.*

5° In all the previous definitions we assumed that the semifinal extensions are also ρ-morphisms. But there is a wider concept by assuming that the extensions are ρ'-morphisms for another subset ρ' of the index-set. It is obvious that ρ and ρ' must be compatible in a natural way if they are equivalent. For this more general case we will write

$$(S(\Omega_j), S(\Gamma_j), S(\Sigma_j))_{\sigma_1^j, \sigma_2^j, \rho_j, \rho_j'}.$$

3. SEMIFINAL STRUCTURE FUNCTOR SEQUENCES.

The «dual» notion of a semifinal structure functor sequence is the notion semiinitial structure functor sequence. Hence we have the following:

3.1. DEFINITION *(Seminitial structure functor sequence).*

Notation as in Definition 2.2.

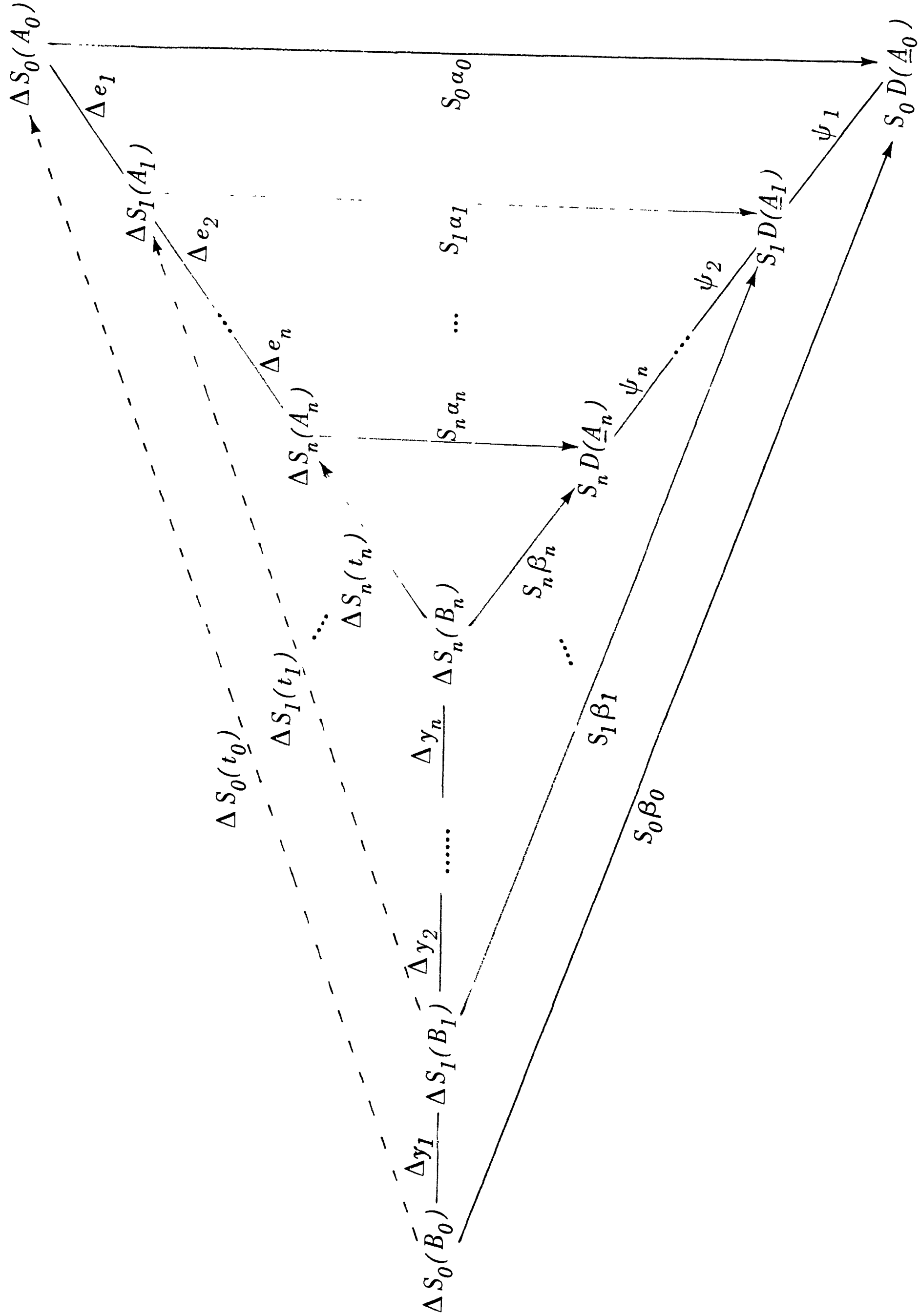
The sequence $(S_i)_n$ is a $(\Phi(\Omega), \Gamma, \Sigma)_{\sigma_1, \sigma_2, \rho}$-*semiinitial structure functor sequence* if for any functorial chain $(D(\underline{A}_i), \psi_i)_n$ in $\Phi(\Omega)$ there exists a ρ-coextension

$$(a_i)_n : (\Delta A_i, \Delta e_i)_n \to (D(\underline{A}_i), \psi_i) \quad \text{with} \quad (A_i, e_i)_n \in \Sigma \text{ [1)]}$$

such that $(a_i)_n$ is final with respect to all ρ-coextensions

1) Remember that $e_i : S_{i-1} A_{i-1} \to S_i A_i$ is an $\underline{X}$-morphism.

FIGURE 3


$\Delta S_0(A_0)$
$S_0\alpha_0$
$S_0D(\underline{A}_0)$
Δe_1
$\Delta S_1(A_1)$
$S_1\alpha_1$
$S_1D(\underline{A}_1)$
ψ_1
Δe_2
ψ_2
Δe_n
ψ_n
$\Delta S_n(A_n)$
$S_n\alpha_n$
$S_nD(\underline{A}_n)$
$\Delta S_n(t_n)$
$\Delta S_n(B_n)$
$S_n\beta_n$
$\Delta S_1(t_1)$
$S_1\beta_1$
$\Delta S_0(t_0)$
Δy_n
$S_0\beta_0$
Δy_2
$\Delta S_1(B_1)$
Δy_1
$\Delta S_0(B_0)$

$$(\beta_i)_n : (\Delta B_i, \Delta y_i)_n \rightarrow (D(\underline{A}_i), \psi_i) \quad \text{with } (B_i, y_i) \in \Gamma ,$$

i.e.

1° for any $(S_i)_n$-chain $(B_i, y_i)_n$ in Γ and ρ-functorial chain-morphism $(\beta_i)_n : (\Delta B_i, \Delta y_i)_n \rightarrow (D(\underline{A}_i), \psi_i)_n$ there exists a unique morphism

$$(t_i)_n : (B_i, y_i)_n \rightarrow (A_i, e_i)_n \quad \text{with } (\beta_i)_n = (a_i)_n (\Delta t_i)_n .$$

2° for any morphism $(g_i)_n : (A_i, e_i)_n \rightarrow (A_i, e_i)_n$ the equation $(a_i)_n = (a_i)_n (\Delta g_i)_n$ implies

$$g_i = Id(A_i) \quad \text{for } i = 0, 1, \dots, n .$$

The corresponding universal object is called *semiinitial coextension.*

Furthermore one should remark that one has similar possibilities to generalize the notion «semiinitial structure functor sequence» as for semifinal structure functor sequences (Remark 2.3).

4. THE DUALITY THEOREM.

4.1. DEFINITION.

1. Let $\underline{C}$ be an arbitrary category and Ω and Γ be classes of $\underline{C}$-objects. The pair (Ω, Γ) is called *pointed* if each set $\underline{C}(A, B)$, $A \in \Omega$, $B \in \Gamma$ has at most one element.

2. Let Φ, Π be classes of $Chs(S_i; \sigma_1, \sigma_2)$-objects. (Φ, Π) is called *ρ-pointed* if it is pointed in the category $Chs(S_i; \sigma_1, \sigma_2, \rho)_n$.

This condition is fundamental for the proof of the generalized duality theorem. In many concrete cases it is automatically fulfilled (cp. the examples in Section 5).

Now we are able to prove the main theorem in this paper.

4.2. THEOREM *(Duality Theorem for structure functor sequences).*

Let $S_i : \underline{A}_i \rightarrow \underline{X}$, $i = 0, 1, \dots, n$ be a sequence of functors. Let σ_1 be the subset of all indices l with $S_l = Id$. Let σ_2 and ρ be further subsets of the class of indices $\{0, 1, \dots, n\}$. Let Ω, Γ, Σ be classes of $(S_i)_n$-chains such that $\Sigma \subset \Gamma$ and (Ω, Γ) be ρ-pointed. Then the following two assertions are equivalent:

(i) $(S_i)_n$ is a $(\Phi(\Omega), \Gamma, \Sigma)_{\sigma_1, \sigma_2, \rho}$-semifinal structure functor

sequence.

(ii) $(S_i)_n$ *is a* $(\Phi(\Gamma),\Omega,\Sigma)_{\sigma_1,\sigma_2,\rho}$*-semiinitial structure functor sequence.*

PROOF. (ii)$\Rightarrow$(i). Given a $(S_i)_n$-functorial chain in $\Phi(\Omega)$:

$$S_0 D(\underline{A}_0) \xrightarrow{\psi_1} S_1 D(\underline{A}_1) \xrightarrow{\psi_2} \dots \xrightarrow{\psi_{n-1}} S_{n-1} D(\underline{A}_{n-1}) \xrightarrow{\psi_n} S_n D(\underline{A}_n)$$

i.e., $(\psi_i)_n$ is pointwise in Ω, denote by $\underline{C}(\Gamma)$ the subcategory of $Chs(S_i,\rho)_n$ consisting of all $(S_i)_n$-chains $(B_i,x_i)_n$ in Γ such that there exists a ρ-morphism

$$(\beta_i)_n : (D(\underline{A}_i),\psi_i) \to (\Delta B_i, \Delta x_i)_n .$$

Since $(\psi_i)_n$ is pointwise in Ω, $(B_i,x_i)_n$ is in Γ and (Ω,Γ) is ρ-pointed, the ρ-morphism $(\beta_i)_n$ is uniquely determined by the $(S_i)_n$-chain $(B_i,x_i)_n$. Hence we will denote it by $(\beta_i(B_j,x_j)_n)_n$. By the assignment

$$C(\underline{A}_l) : \underline{C}(\Gamma) \to \underline{A}_l : (B_i,x_i)_n \to B_l, \quad l = 0,1,\dots,n,$$

we obtain a sequence of functors. By the assignments

$$\kappa_i : S_{i-1}C(\underline{A}_{i-1}) \longrightarrow S_i C(\underline{A}_i):$$

$$\kappa_i(B_l,x_l) \;:= x_i : S_{i-1}(\underline{B}_{i-1}) \longrightarrow S_i(\underline{B}_i), \quad i = 1,\dots,n,$$

we obtain a $(S_i)_n$-functorial chain in $\Phi(\Gamma)$:

$$S_0 C(\underline{A}_0) \xrightarrow{\kappa_1} S_1 C(\underline{A}_1) \xrightarrow{\kappa_2} \dots \xrightarrow{\kappa_n} S_n C(\underline{A}_n).$$

Let $d \in ObDom(D(\underline{A}_1))$ be an arbitrary object. We obtain a ρ-functorial morphism

$$(\gamma_{i,d})_n : (\Delta D(\underline{A}_i)d,\ \Delta\psi_i\, d) \to (C(\underline{A}_i),\kappa_i)_n$$

by the assignments:

$$\gamma_{i,d} : \Delta D(\underline{A}_i)d \to C(\underline{A}_i) : \underline{C}(\Gamma) \to \underline{A}_i ,$$

$$\gamma_{i,d}(B_l,x_l)_n := \beta_i(B_l,x_l)_n(d) : D(\underline{A}_i)d \to C(\underline{A}_i)(B_l,x_l) = B_i,$$

$i = 0,1,\dots,n$. This is a well-defined assignment since $(\beta_i(B_l,x_l)_n)_n$ is uniquely determined by $(B_l,x_l)_n \in \underline{C}(\Gamma)$. The functoriality of the $\gamma_{i,d}$ follows from the functoriality of the $\beta_i(B_l,x_l)$ and the ρ-pointedness of (Ω,Γ), i.e. let

$$(f_l)_n : (B_l, x_l)_n \to (\hat{B}_l, \hat{x}_l)_n$$

be a morphism in $\underline{C}(\Gamma)$, then the following diagram is commutative:

$$\begin{tikzcd}
 & C(\underline{A}_i)(B_l, x_l) = B_i \arrow[dd, "f_i"] \\
D(\underline{A}_i)d \arrow[ur, "\gamma_{i,d}(B_l, x_l)_n"] \arrow[dr, "\gamma_{i,d}(\hat{B}_l, \hat{x}_l)_n"'] \arrow[r, phantom, "="] & \\
 & C(\underline{A}_i)(\hat{B}_l, \hat{x}_l)_n = \hat{B}_i
\end{tikzcd}$$

FIGURE 4

Since the sequence $(S_i)_n$ is a $(\Phi(\Gamma), \Omega, \Sigma)_{\sigma_1, \sigma_2, \rho}$-semiinitial structure functor sequence by assumption we obtain a $(S_i)_n$-chain $(A_i, a_i)_n$ in Σ, a $(S_i)_n$-functorial chain $(\tau_i)_n : (\Delta A_i, \Delta a_i)_n \to (C(\underline{A}_i), \kappa_i)_n$ and a unique morphism

$$(a_i(d)) : (D(\underline{A}_i)d, \psi_i d)_n \to (A_i, a_i)_n .$$

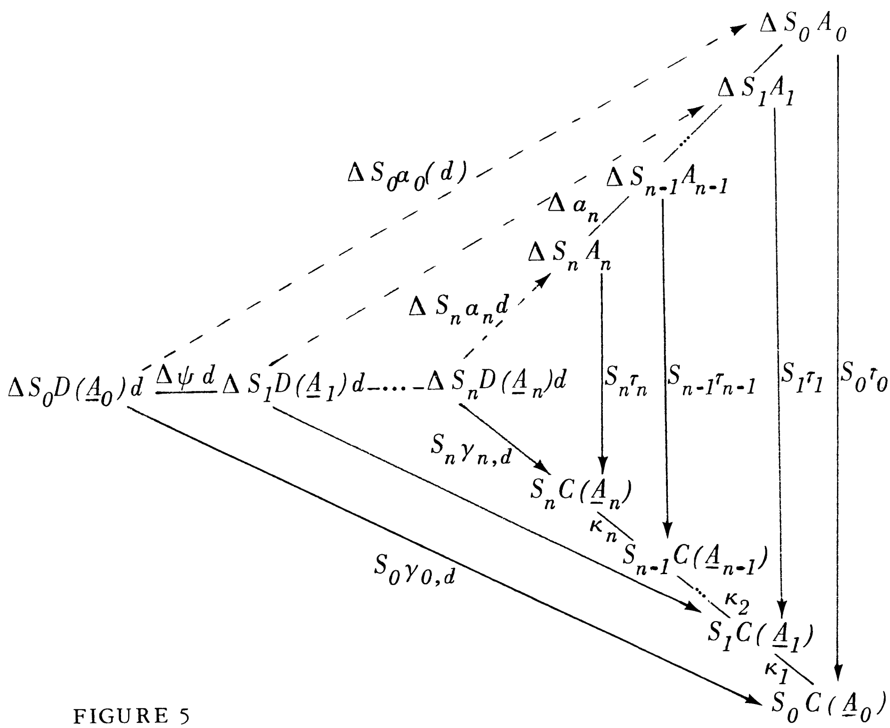


FIGURE 5

The uniqueness of the chain-morphism

$$(a_i(d))_n : (D(\underline{A}_i)d,\ \psi_i d) \to (A_i, a_i)$$

implies that the assignment

$$a_i : D(\underline{A}_i) \to \Delta A_i : \underline{D} \to \underline{A}_i : d \to a_i(d) : D(\underline{A}_i)d \to A_i$$

defines a functorial morphism. We claim that the extension

$$(a_i)_n : (D(\underline{A}_i), \psi_i)_n \to (\Delta A_i, \Delta a_i)_n$$

is the $(\Phi(\Omega), \Gamma, \Sigma)_{\sigma_1, \sigma_2, \rho}$-semifinal extension of the $\Phi(\Omega)$-chain $(D(\underline{A}_i), \psi_i)_n$ looked for. Let $(B_i, x_i)_n$ be a Γ-chain and

$$(\beta_i)_n : (D(\underline{A}_i), \psi_i)_n \to (\Delta B_i, \Delta x_i)_n$$

be a ρ-functorial morphism. Then $(B_i, x_i)_n$ is in $\underline{C}(\Gamma)$. Let

$$t_l := \tau_l\,(B_i, x_i)_n : A_l \to C(\underline{A}_l)(B_i, x_i)_n = B_l .$$

Since

$$(\tau_l)_n : (\Delta A_l, \Delta a_l)_n \to (C(\underline{A}_l), \kappa_l)_n$$

is a functorial chain-morphism, the sequence $(t_l)_n : (A_l, a_l)_n \to (B_l, x_l)_n$ is a chain-morphism. Hence $(t_l)_n$ is a morphism in $\underline{C}(\Gamma)$ since $(A_l, a_l)_n$ is in Σ and $\Sigma \subset \Gamma$. Let

$$(s_l)_n : (A_l, a_l)_n \to (B_l, x_l)_n$$

be another chain-morphism in $\underline{C}(\Gamma)$. Since

$$(\tau_i)_n : (\Delta A_i, \Delta a_i)_n \to (C(\underline{A}_i), \kappa_i)_n$$

is a functorial chain-morphism over $\underline{C}(\Gamma)$, we obtain for each $i = 0, 1, \dots, n$ the following commutative diagram :

$$\begin{tikzcd}
 & A_i \arrow[d, "s_i"] \\
A_i \arrow[ur, "\tau_i(A_l, a_l)_n"] \arrow[dr, "t_i = \tau_i(B_l, x_l)_n"'] \arrow[r, phantom, "="] & {} \\
 & B_i
\end{tikzcd}$$

FIGURE 6

Since $(A_i, a_i)_n$ is a semiinitial extension of $(C(\underline{A}_i), \kappa_i)_n$ and

$$(\tau_i(A_l, a_l)_n) : (A_i, a_i)_n \to (A_i, a_i)_n$$

is a chain-morphism it follows from the definition of semiinitial structure functor sequence, resp. from Definition 3.3, that

$$r_i\,(A_l, a_l)_n = Id(A_i) \quad \text{for all } i = 0, 1, \dots, n.$$

Hence $t_i = s_i$ for all $i = 0, 1, \dots, n$.

(i)$\Rightarrow$ (ii). Start with $(D(\underline{A}_i), \psi_i)_n \in \Phi(\Gamma)$, run through the proof (ii)$\Rightarrow$ (i) and dualize at the corresponding parts or interchange Ω and Γ and interprete the result in the dual of the comma-category of all extensions over $(D(\underline{A}_i), \psi_i)_n$. This completes the proof.

4.3. REMARK.

Let ρ' be another subset of the index-set $\{0, 1, \dots, n\}$. Then the semifinal extensions are ρ'-extensions iff the semiinitial extensions are ρ'-extensions.

This duality theorem is extremely general. The usefulness as well as the importance of this theorem is shown by a whole host of corollaries deduced from. The most important special instances are discussed in the sequel.

The interpretations of the classical examples in the language introduced in this paper is carried through in details only for the first example. For all subsequent examples we will restrict ourselves just to ad hoc definitions. The corresponding obvious translations are left out.

5. APPLICATIONS.

5.1. Duality for semitopological functors.

Semitopological functors represent exactly the full reflective restrictions of topological functors (Tholen-Wischnewsky, Oberwolfach 1977; Tholen [27]; Herrlich-Strecker [12]). They were first [1] introduced cum grano salis by Hoffmann [13, 14, 15], Tholen [27, 28], Wischnewsky [31]. Hoffmann used at least the semifinal [13] (up to the «wrong universe») as well as the locally $\underline{Q}$-orthogonal characterisation [14], up to the assumption that $\underline{Q}$ is closed under composition - a condition which implies that the functor in question is already topologically-algebraic in

1) V. Trnkova, Automata and categories, *Lecture Notes in Comp. Sc.* 32, Springer.

the sense of Hong [17]; but he was not aware of the equivalence of these two definitions as well as the fact that functors fulfilling one of these definitions are reflective restrictions of topological functors (at least in the same universe).

Semifinal functors (with respect to the right universe) as well as even a generalization were used in [31] in order to study adjoint liftings along topologically-algebraic functors (and more general for semitopological functors).

The key for all further generalizations was Tholen's duality Theorem [1] generalizing the corresponding duality Theorem for topological functors, resp. (E,M)-topological functors (Antoine [1], Roberts [23], Hoffmann [13]).

Hoffmann [13], Tholen [27] as well as the author in this paper used an idea going back to Herrlich [10] Lemma 6.1. One has to read Herrlich's diagram [10] page 133 just in «both directions».

The correct definition of locally orthogonal $\underline{Q}$-functors as well as the characterization as semitopological functors were first given by Tholen [28] and independently at the same time by the author (unpublished). The characteristic diagram for the internal dual of locally orthogonal $\underline{Q}$-functors was first given by the author supposing that this class of functors describing all left extension functors (cp. Guitart [8], Rosicky [24]) is larger than that of semitopological functors. But in fact H. Wolff could show [38] using the corresponding external characterization which he had obtained independently from Tholen-Wischnewsky [2] that these functors are semitopological. Hence by applying the previous results this is nothing else that another duality Theorem for semitopological functors (more exactly for locally orthogonal $\underline{Q}$-functors) containing Tholen's original one as special instance. By applying our general duality Theorem this

1) In connection with this duality theorem he introduced the notion semiinitial factorization.

2) THOLEN, W. & WISCHNEWSKY, M.B., Structure functors II: External characterizations (preprint), *J. Pure and Appl. Algebra* 15 (1979), 75-92.

is a trivial consequence from the definition.

Let $n = 2$,

$$S_0 := S: \underline{A} \to \underline{X} \quad \text{and} \quad S_1 = S_2 = Id: \underline{X} \to \underline{X}.$$

Then we have $\sigma_1 = \{1, 2\}$ and $\sigma_2 = \{2\}$. Let

$$Id(S) \subset \underline{Q} \subset Epi(S)$$

be a class of S-morphisms closed under composition with isomorphisms from the left. Let Ω be the class of all S-double-morphisms of type

$$(A,\ SA \xleftarrow{q} Y \xrightarrow{x} X) \quad \text{with} \quad (A, q) \in \underline{Q}.$$

Let $\Gamma = Mor(S)$ and $\Sigma = \underline{Q}$. Let $\rho = \rho' = \{2\}$. (Ω, Γ) is ρ-pointed, since $\underline{Q} \subset Epi(S)$. Hence we obtain:

5.1.1. DEFINITION (Hoffmann [13], Tholen [27], Tholen-Wischnewsky [29], Wischnewsky [31], Wolff [38]).

Let $S: \underline{A} \to \underline{X}$ be a functor.

1. S is a $(\Phi(\Omega), Mor(S), \underline{Q})_{1,2,2}$-*semifinal structure functor* (for short semifinal functor) iff for every S-double-cone

$$SD(\underline{A}) \xleftarrow{\psi} D(\underline{X}) \xrightarrow{\phi} \Delta X, \quad D(\underline{A}): \underline{D} \to \underline{A} \quad \text{and} \quad D(\underline{X}): \underline{D} \to \underline{X},$$

with ψ being pointwise in $\underline{Q}$, there exists a S-co-cone $\alpha: D(\underline{A}) \to \underline{A}$ and a S-morphism $(A,\ e: X \to SA)$ in $\underline{Q}$ with $(\Delta e)\phi = (S\alpha)\psi$ such that, for every S-co-cone $\beta: D(\underline{A}) \to \Delta B$ and every S-morphism

$$(B,\ b: X \to SB) \quad \text{with} \quad (\Delta b)\phi = (S\beta)\psi$$

there exists a unique $\underline{A}$-morphism $t: A \to B$ with

$$b = (St)e \quad \text{and} \quad \beta = (\Delta t)\alpha.$$

$$\begin{tikzcd}
 & & & \Delta SA \arrow[dd, dashed, "\Delta St"] \\
SD(\underline{A}) \arrow[urrr, "S\alpha"] \arrow[drrr, "S\beta"'] & D(\underline{X}) \arrow[l, "\psi"'] \arrow[r, "\phi"] & \Delta X \arrow[ur, "\Delta e"] \arrow[dr, "\Delta b"'] & \\
 & & & \Delta SB
\end{tikzcd}$$

FIGURE 7

2. S is a $(\Phi(Mor(S)), \Omega, \underline{Q})_{1,2,2}$*-semiinitial structure functor* (= locally orthogonal $\underline{Q}$-functor) iff for every S-cone $(D(\underline{A}), \phi : \Delta X \to SD(\underline{A}))$ there exist a S-morphism $(A, e : X \to SA)$ in $\underline{Q}$ and a functorial morphism $a : \Delta A \to D(\underline{A})$ with $\phi = (Sa)(\Delta e)$ such that for every S-double morphism in Ω

$$(B,\ b: Y \to SB,\ y: Y \to X)$$

and S-cone $\beta : \Delta B \to D(\underline{A})$ with $\phi(\Delta y) = (S\beta)(\Delta b)$ there exists exactly one $\underline{A}$-morphism $\omega : B \to A$ with $\beta = a(\Delta\omega)$ and $ey = (S\omega)b$.

$$\begin{tikzcd}
 & & & \Delta SA \arrow[dd, "Sa"] \\
\Delta SB \arrow[urrr, dashed, "\Delta\omega"] \arrow[drrr, "S\beta"'] & \Delta Y \arrow[l, "\Delta b"'] \arrow[r, "\Delta y"] & \Delta X \arrow[ur, "\Delta e"] \arrow[dr, "\phi"] & \\
 & & & SD(\underline{A})
\end{tikzcd}$$

FIGURE 8

5.1.2. COROLLARY *(Duality Theorem for locally orthogonal $\underline{Q}$-functors - Wolff [38]).*

Let $S : \underline{A} \to \underline{X}$ be a functor. Then there are equivalent:

(i) S is a locally orthogonal Q-functor, i. e. a semiinitial structure functor (in the sense of Definition 5.1.1).

(ii) S is a semifinal structure functor (in the sense of Def. 5.1.1).

The definition of locally orthogonal $\underline{Q}$-functors delivers just another characterization of semitopological functors. W. Tholen's original duality Theorem for semitopological functors is obtained by taking $\Omega = Co\text{-}Mor(S)$ in the previous Definition 5.1.1.

5.1.3. COROLLARY *(Duality Theorem for semitopological functors - Tholen [27]).*

Let $S : \underline{A} \to \underline{X}$ be a functor. Then there are equivalent:

(i) Every S-co-cone has a semifinal extension, i. e. S is a

$(\Phi(Co\text{-}Mor(S)), Mor(S), \underline{Q})_{1,1,1}$-semifinal structure functor.

(ii) Every S-cone has a semiinitial coextension, i. e. S is a

$(\Phi(Mor(S)), Co\text{-}Mor(S), \underline{Q})_{1,1,1}$-semiinitial structure functor.

By taking for $\underline{Q}$ the class $Iso(S)$ of all S-isomorphisms we obtain the classical duality Theorem for topological functors.

5.1.4. COROLLARY *(Duality Theorem for topological functors - Antoine [1], Roberts [23]).*

Let $S: \underline{A} \to \underline{X}$ be a functor. Then there are equivalent:

(i) S is a topological functor.

(ii) S^{op} is a topological functor.

This Corollary implies immediately a representation Theorem as well as a duality Theorem for co-semitopological functors. By the representation Theorem for semitopological functors as full reflective restriction of topological functors and the above Corollary, the co-semitopological functors are exactly the full coreflective restrictions of topological functors, i. e. we obtain the following :

5.1.5. COROLLARY *(Representation, resp. duality Theorem for co-semitopological functors - Tholen [30], Wischnewsky [32]).*

Let $S: \underline{A} \to \underline{X}$ be a functor, $Id(S) \subset \underline{P} \subset Mono(S)$ be a subclass of S-monomorphisms closed under composition with $\underline{A}$-isomorphisms from the right. Let $\Omega = Co\text{-}Mor(S)$, $\Gamma = \underline{P}$ and Σ be the class of all S-double morphisms

$$(P, X \xleftarrow{y} Y \xrightarrow{p} SP) \text{ with } (P, p) \in \underline{P}.$$

Then the following assertions are equivalent (where $\sigma_1 = \{1, 2\}$):

(i) S is co-semitopological.

(ii) S is a full coreflective restriction of a topological functor.

(iii) S is a $(\Phi(Co\text{-}Mor(S)), \Sigma, \underline{P})_{\sigma_1, 2, 2}$-semifinal structure functor.

(iv) S is a $(\Phi(\Sigma), Co\text{-}Mor(S), \underline{P})_{\sigma_1, 2, 2}$-semiinitial structure functor.

$$\begin{tikzcd}
 & & \Delta SP \arrow[dl, "\Delta p"'] \arrow[dd, "S\pi"] \\
\Delta SB \arrow[urr, dashed, "\Delta St"] \arrow[r, "\Delta b"] \arrow[drr, "S\beta"'] & \Delta X \arrow[d, "\phi"] & \\
 & D(\underline{X}) & SD(\underline{A}) \arrow[l, "\psi"']
\end{tikzcd}$$

FIGURE 9

«Semiinitial characterization of co-semitopological functors».

Herrlich-Strecker [12] and Börger-Tholen [3] showed that the following concepts are equivalent:

(a) topologically-algebraic functors (Y.H. Hong [16], S.S. Hong [17]),

(b) $\underline{M}$-functors (Tholen [27], Wischnewsky [31]),

(c) orthogonal $\underline{Q}$-functors (Tholen [26]),

(d) locally orthogonal $\underline{Q}$-functors and $\underline{Q}$ is closed under composition (Tholen [28]).

By the characterization (d) we obtain immediately diagrammatically the same duality theorems as for locally orthogonal $\underline{Q}$-functors only with the additional assumption that $\underline{Q}$ is compositive. But there is another duality theorem for topologically-algebraic functors. A similar one can be obtained for locally orthogonal $\underline{Q}$-functors. In this case one has to replace the class *Semi-Univ(S)* by the class of all S-semifinal morphisms.

Recall from Herrlich-Strecker [12] that a S-morphism $e: X \to SA$ is called *semi-universal* provided for any initial cone $\mu : \Delta A' \to D(\underline{A})$, any $\underline{A}$-cone $\alpha : \Delta A \to D(\underline{A})$ and any S-morphism

$$f: X \to SA' \quad \text{with} \quad (S\mu)(\Delta f) = (S\alpha)(\Delta e),$$

there exists a unique $\underline{A}$-morphism $g: A \to A'$ such that the following diagram commutes:

$$\begin{tikzcd}
\Delta X \arrow[r, "\Delta e"] \arrow[d, "\Delta f"'] & \Delta SA \arrow[d, "S\alpha"] \arrow[dl, dashed, "\Delta Sg"'] \\
\Delta SA' \arrow[r, "S\mu"'] & SD(\underline{A})
\end{tikzcd}$$

FIGURE 10

i.e., e is in $Init^{\perp}(S)$, the class of all S-morphisms being orthogonal to all S-initial cones (in the sense of cone-factorizations). We denote the class of all semi-universal S-morphisms by *Semi-Univ(S)*.

The following definition of a topologically-algebraic functor differs from the characterization given in (a), (b), (c), (d). The equivalence is easily shown. For the rest of part 5.1 we assume that the functor in question $S: \underline{A} \to \underline{X}$ is faithful (without any loss of generality!).

5.1.6. DEFINITION.

Let $S: \underline{A} \to \underline{X}$ be a functor.

1. S *is a semiinitial topologically-algebraic functor* iff for every S-cone $\psi: \Delta X \to SD(\underline{A})$, $D(\underline{A}): \underline{D} \to \underline{A}$ arbitrary, there exist a S-semi-universal morphism $(A, e: X \to SA)$ and a $\underline{A}$-cone $\alpha: \Delta A \to D(\underline{A})$ with $\psi = (S\alpha)\Delta e$ such that for any chain of type

$$SB \xrightarrow{x} S\hat{B} \xleftarrow{\hat{e}} X \quad \text{with } x \in X(SB, S\hat{B}) \text{ and } \hat{e} \in \textit{Semi-Univ}(S),$$

any S-cone $\hat{\beta}: \Delta \hat{B} \to D(\underline{A})$ and any S-cone $\beta: \Delta B \to D(\underline{A})$ such that

$$\psi = (S\hat{\beta})(\Delta \hat{e}) \quad \text{and} \quad (S\beta) = (S\hat{\beta})(\Delta x),$$

there exist unique morphisms $\hat{t}: \hat{B} \to A$ and $t: B \to A$ with

$$St = (S\hat{t})x, \quad e = (S\hat{t})\hat{e}, \quad \hat{\beta} = \alpha(\Delta \hat{t}) \quad \text{and} \quad \beta = \alpha(\Delta t).$$

$$\begin{tikzcd}
 & & & \Delta SA \arrow[d, "Sa"] \\
\Delta SB \arrow[r, "\Delta x"] \arrow[urrr, dashed, bend left, "\Delta St"] \arrow[drrr, "S\beta"'] & \Delta S\hat{B} \arrow[urr, dashed, "\Delta S\hat{t}"] \arrow[drr, "S\hat{\beta}"] & \Delta X \arrow[l, "\Delta \hat{e}"'] \arrow[ur, "\Delta e"] \arrow[dr, "\psi"] & \\
 & & & SD(\underline{A})
\end{tikzcd}$$

FIGURE 11

2. S *is a semifinal topologically-algebraic functor* if for all functors $D(\underline{A}), \hat{D}(\underline{A}): \underline{D} \to \underline{A}$, and all functorial morphisms

$$\psi: SD(\underline{A}) \to S\hat{D}(\underline{A}) \quad \text{and} \quad \hat{\psi}: \Delta X \to S\hat{D}(\underline{A}),$$

$\hat{\psi}$ pointwise in *Semi-Univ*(S), there exist a semi-universal morphism $(A, e: X \to SA)$ and S-co-cones $\hat{a}: \hat{D}(\underline{A}) \to \Delta A$ and $a: D(\underline{A}) \to \Delta A$ with

$$\Delta e = (S\hat{a})\hat{\psi} \quad \text{and} \quad Sa = (S\hat{a})\psi$$

such that, for any S-morphism $(B, b: X \to SB)$ and S-co-cones

$$\hat{\beta}: \hat{D}(\underline{A}) \to \Delta B \quad \text{and} \quad \beta: D(\underline{A}) \to \Delta B$$

with $\Delta b = (S\hat{\beta})\hat{\psi}$ and $S\beta = (S\hat{\beta})\psi$, there exists a unique morphism

$t: A \to B$ with

$$b = (St)e, \quad \hat{\beta} = (\Delta t)\hat{a} \quad \text{and} \quad \beta = (\Delta t)a.$$

$$\begin{tikzcd}
& & & \Delta SA \arrow[dd, dashed, "\Delta St"] \\
SD(A) \arrow[r, "\psi"] \arrow[urrr, "Sa"] \arrow[drrr, "S\beta"'] & S\hat{D}(A) \arrow[urr, "S\hat{a}"] \arrow[drr, "S\hat{\beta}"'] & \Delta X \arrow[l, "\hat{\psi}"'] \arrow[ur, "\Delta e"'] \arrow[dr, "\Delta b"] & \\
& & & \Delta SB
\end{tikzcd}$$

FIGURE 12

Applying our general duality Theorem we obtain the following

5.1.7. COROLLARY *(Duality Theorem for topologically-algebraic functors).*

Let $S: \underline{A} \to \underline{X}$ be a functor. Then the following assertions are equivalent:

(i) S is a topologically-algebraic functor (in the sense of Y.H. Hong).

(ii) S is a semiinitial topologically-algebraic functor.

(iii) S is a semifinal topologically-algebraic functor.

5.1.8. REMARKS.

1. By taking $\hat{e} = e$ in Definition 5.1.6 (1) it is obvious that the $\underline{A}$-cone $a: \underline{A} \to D(\underline{A})$ is a S-initial cone.

2. By the equivalence of the notions topologically-algebraic functor, M-functor, and orthogonal $\underline{Q}$-functor the above corollary is also a duality Theorem for orthogonal $\underline{Q}$-functors, resp. M-functors.

3. By taking $\hat{e} = e$ in Definition 5.2.1, 1 and the remark that S-semiuniversal morphisms are S-epimorphisms (Börger-Tholen [3], Herrlich-Strecker [12]) it follows immediately that every topologically-algebraic functor is semitopological ($\Rightarrow$ Definition 5.1.1, 2).

4. By taking $S = Id: \underline{A} \to \underline{A}$ we obtain duality theorems for cone-factorizations (E, M) in a category $\underline{A}$.

5.2. Duality for full coreflective restrictions of semitopological functors.

It is well-known that the composition of semitopological functors is again a semitopological functor. Thus in particular full reflective restrictions of semitopological functors are again semitopological. Hence for the study of full reflective restrictions of semitopological functors one can apply again the theory of semitopological functors. In contrast to this property full coreflective restrictions of semitopological functors are in general no longer semitopological, e.g. the corresponding restriction has in general no left adjoint. Hence similar questions and problems do arise as for semitopological functors. Both Tholen [30] and the author [31, 32] introduced generalizations of the notion semitopological functor. These were called «concrete functors» by Tholen [30], semifinal factorization functor [31], resp. structure functor [32] by the author. But an analysis of these notions in connection with several examples as well as the problem mentioned above showed that these definitions are not general enough. This led to the notion «structure functor» as it is defined in this paper. By this new notion not only the problem started with could be solved [33] but it turned out that this notion is far more general.

5.2.1. DEFINITION (Wischnewsky [33]).

Let $S: \underline{A} \to \underline{X}$ be a functor. Let

$$S = (\underline{A} \xrightarrow{\hat{Q}} \underline{B} \xrightarrow{Q} \underline{X})$$

be an arbitrary factorization. Let

$$Id(Q) \subset \Phi \subset Mor(Q), \quad Id(\hat{Q}) \subset \Gamma \subset Co\text{-}Mor(\hat{Q})$$
$$\text{and } Id(\hat{Q}) \subset \Pi \subset Co\text{-}Mor(\hat{Q})$$

be classes of Q-morphisms, resp. $\hat{Q}$-comorphisms. Let $n = 3$,

$$S_0 = S, \quad S_1 = Q \quad \text{and} \quad S_2 = S_3 = Id: \underline{X} \to \underline{X}.$$

Then $\sigma = \{2, 3\}$. Let

$$\sigma_1 = \{2, 3\}, \ \sigma_2 = \{3\} \ \text{ and } \ \rho = \rho' = \{3\}.$$

Let $\gamma(\Pi, \Phi)$ be a subclass of the class of all $(S_i)_3$-chains of type

$$SA \xrightarrow{Qp} QB \xleftarrow{f} Y \xrightarrow{x} X \quad \text{with } (A, p) \in \Pi \text{ and } (B, f) \in \Phi$$

containing all chains of type

$$SA \xrightarrow{Qg} QB \xrightarrow{x} X$$

with arbitrary $(A, g) \in \Gamma$ and $(B, x) \in \mathit{Co\text{-}Mor}(Q)$. Let $\Omega(\Gamma)$ be the class of all $(S_i)_3$-chains

$$(A, B, g : \hat{Q}A \to B, x : X \to QB) \text{ with } (A, g) \in \Gamma .$$

(S, Q, Id, Id) is a $(\Phi(\Omega(\Gamma)), \gamma(\Pi, \Phi), \Omega(\Gamma))_{\sigma_1, 3, 3}$-*semiinitial structure functor* iff, for every functorial chain in $\Phi(\Omega(\Gamma))$:

$$\Delta X \xrightarrow{\phi} QD(\underline{B}) \xleftarrow{Q\gamma} SD(\underline{A})$$

with γ pointwise in Γ there exist a $(S_i)_3$-chain

$$X \xrightarrow{e} QB \xleftarrow{Qg} SA \quad \text{with } (A, g) \in \Gamma$$

and functorial morphisms

$$\beta : \Delta B \to D(\underline{B}) \text{ and } a : \Delta A \to D(\underline{A})$$

with

$$(Q\beta)(\Delta e) = \phi \text{ and } \beta(\Delta g) = \gamma(\hat{Q}a)$$

such that

1. for any $(S_i)_3$-chain in $\gamma(\Pi, \Phi)$

$$SA' \xrightarrow{Qp} QB' \xleftarrow{f} Y \xrightarrow{x} X$$

and any pair of functorial morphisms $\beta' : \Delta B \to D(\underline{B})$, $a' : \Delta A' \to D(\underline{A})$ with

$$\gamma(\hat{Q}a') = \beta'(\Delta p) \text{ and } (Q\beta')(\Delta f) = \phi(\Delta x)$$

there exists a unique pair of morphisms $a : A' \to A$ and $b : B' \to B$ with

$$bp = g(\hat{Q}q), \quad (Qb)f = ex, \quad a' = a(\Delta a), \quad \beta' = \beta(\Delta b) .$$

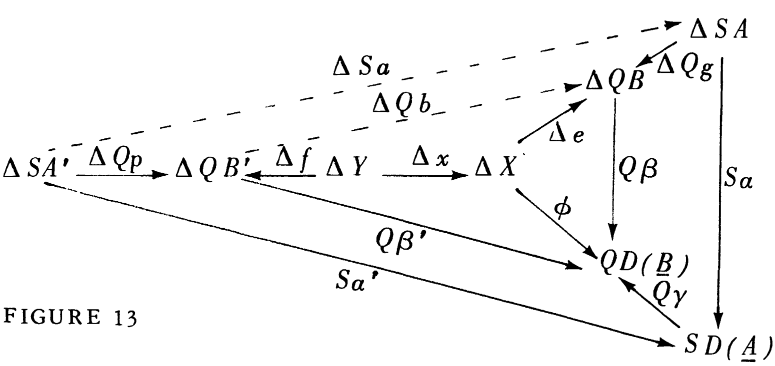


FIGURE 13

2. For any pair of morphisms $s: A \to A$ and $t: B \to B$ the equations

$$e = (Qt)e, \quad \beta = \beta(\Delta t), \quad \alpha = \alpha(\Delta s), \quad tg = g(\hat{Q}s)$$

imply $s = Id(A)$ and $t = Id(B)$.

Let $S: \underline{A} \to \underline{X}$ be a functor. If there exist a factorization $S = Q\hat{Q}$, and classes

$$Id(Q) \subset \Phi \subset Mor(Q), \quad Id(\hat{Q}) \subset \Gamma \subset Co\text{-}Mor(\hat{Q}),$$
$$\Pi \subset Co\text{-}Mor(\hat{Q}) \quad \text{with} \quad Id(\hat{Q}) \subset \Pi$$

and $\gamma(\Pi, \Phi)$ such that (S, Q, Id, Id) is a semiinitial structure functor in the sense of Definition 5.2.1, then S is called a *topologically-algebraic structure functor* or for short a Top-algebraic structure functor (with respect to $(\hat{Q}, Q, \Phi, \Gamma, \Pi, \gamma(\Pi, \Phi)))$.

If $\underline{X}$ is the category of all sets, then all reflective or coreflective restrictions of monadic functors or topological functors are topologically-algebraic functors.

5.2.2. THEOREM [1] *(Representation Theorem, Wischnewsky [33]).*

Let S be a functor. Then the following assertions are equivalent:

(i) S is a full reflective or coreflective restriction of a semitopological functor.

(ii) S is a topologically-algebraic structure functor.

If one takes in particular the trivial factorization $S = IdS$, i.e.

$$\hat{Q} = S \quad \text{and} \quad Q = Id \quad \text{and} \quad \Pi = Id(S)\ ^{2)},$$

then the topologically-algebraic structure functors with respect to $(S, Id, \Phi, \Gamma, Id(S))$ are the (Φ, Γ)-structure functors in the sense of [32] Part 1, resp. the (Φ, Γ)-concrete functors in the sense of Tholen [30]. Hence from the above Theorem we obtain the following Corollary:

5.2.3. COROLLARY *(Tholen [30]).*

Let S be a (Φ, Γ)-structure functor in the sense of [32], resp.

[1] This theorem can be sharpened [33].

[2] Take in this case for $\gamma(Id(S), \Phi)$ the class of all S-double comorphisms of type $SA \xleftarrow{f} Y \xrightarrow{x} X$ with $(A, f) \in \Phi$.

a (Φ, Γ)*-concrete functor in the sense of* [30] [1]*. Then* S *is a full coreflective restriction of a semitopological functor.*

Applying our general duality Theorem we obtain a duality Theorem for topologically-algebraic structure functors.

5.2.4. COROLLARY *(Duality Theorem for topologically-algebraic structure functors).*

Let $S: \underline{A} \to \underline{X}$ *be a functor and* $(\hat{Q}, Q, \Phi, \Gamma, \Pi)$ *as in 5.2.1. Then there are equivalent:*

(i) (S, Q, Id, Id) *is a* $(\Phi(\Omega(\Gamma)), \gamma(\Pi, \Phi), \Omega(\Gamma))_{\sigma_1, 3, 3}$*-semiinitial structure functor sequence, i. e.* S *is a topologically-algebraic structure functor.*

(ii) (S, Q, Id, Id) *is a* $(\Phi(\gamma(\Pi, \Phi)), \Omega(\Gamma), \Omega(\Gamma))_{\sigma_1, 3, 3}$*-semifinal structure functor sequence represented by the following diagram*

$$\begin{tikzcd}
& & & & & \Delta S A \arrow[dl, "\Delta Q g"'] \arrow[dddd, dashed, "\Delta S a"] \\
& & & & \Delta Q B \arrow[dd, dashed, "\Delta Q b"] & \\
S D(\underline{A}) \arrow[r, "Q\pi"] \arrow[uurrrrr, "S a"] \arrow[ddrrrrr, "S a'"'] & Q D(\underline{B}) \arrow[urrr, "Q\beta"] \arrow[drrr, "Q\beta'"'] & D(\underline{X}) \arrow[l, "\phi"'] \arrow[r, "\psi"] & \Delta X \arrow[ur, "\Delta e"] \arrow[dr, "\Delta x"'] & & \\
& & & & \Delta Q B' & \\
& & & & & \Delta S A' \arrow[ul, "\Delta Q g'"]
\end{tikzcd}$$

FIGURE 14

5.3. Duality for arbitrary sequences of reflective or coreflective restrictions (over topological functors).

The basic idea of 5.2 can be expressed as follows: «Start with an arbitrary factorization

$$\underline{A} \xrightarrow{\ Q\ } \underline{B} \xrightarrow{\ \hat{Q}\ } \underline{X}$$

of the functor in question. If this factorization $(Q, \hat{Q})$ fulfills a certain universal property then we can construct from it a factorization

[1] The idea looking at functors of this type goes back to W. Tholen.

$$\underline{A} \xrightarrow{E} \underline{C} \xrightarrow{T} \underline{X},$$

where E and T belong to a certain class of functors (in 5.2, T is semi-topological and E is a coreflective restriction of T). This idea is heavily used in [34] in order to study (and to characterize) compositions of structure functors, i.e.

$$(*)\quad (S: \underline{A} \to \underline{X}) = (\underline{A}_n \xrightarrow{E_n} \underline{A}_{n-1} \xrightarrow{E_{n-1}} \cdots \xrightarrow{E_2} \underline{A}_1 \xrightarrow{E_1} \underline{X})$$

where each E_i, $i = 1, \ldots, n$ is a structure functor. In the following part we will restrict ourselves to the important special instance where

$E_1 \equiv$ topological functor and

$E_i \equiv$ (reflective or coreflective) full embedding for $i = 2, \ldots, n$.

The examples considered in 5.1 and 5.2 are special instances of compositions of structure functors of this type. The idea expressed above can be considered as an inductive step. Hence we obtain the following principle: Start with an arbitrary factorization

$$(S: \underline{A} \to \underline{X}) = (\underline{A} = \underline{B}_n \xrightarrow{Q_n} \underline{B}_{n-1} \xrightarrow{Q_{n-1}} \cdots \xrightarrow{Q_2} \underline{B}_1 \xrightarrow{Q_1} \underline{X})$$

which fulfills the «semifinal (resp. semiinitial) structure functor sequence property» with respect to:

$$S_0 := S = Q_1 Q_2 \ldots Q_n, \quad S_1 := Q_1 Q_2 \ldots Q_{n-1},$$
$$\ldots \quad S_{n-1} = Q_1, \quad S_n = Id(\underline{X}).$$

Then construct (by induction) the factorization (*). This idea leads to the following definition:

5.3.1. DEFINITION (Wischnewsky [34]).

Let $S: \underline{A} \to \underline{X}$ be a functor. Let

$$(S: \underline{A} \to \underline{X}) = (\underline{A} = \underline{B}_0 \xrightarrow{Q_n} \underline{B}_1 \longrightarrow \cdots \longrightarrow \underline{B}_{n-1} \xrightarrow{Q_1} \underline{X})$$

be a factorization of S. Let γ_1, γ_2 be a partition of the index-set $\{2, \ldots, n\}$. Let

$$Id(Q_{n-i+1}) \subset \Gamma_i \subset \begin{cases} Mor(Q_{n-i+1}) & \text{if } n-i+1 \in \gamma_1 \\ Co\text{-}Mor(Q_{n-i+1}) & \text{if } n-i+1 \in \gamma_2 \end{cases}$$

and $\Gamma_n \subset Mor(Q_1)$ [1]. Let

$$S_i := Q_1 \dots Q_{n-i} \text{ for } i = 0, \dots, n\text{-}1 \quad \text{and} \quad S_n = Id: \underline{X} \to \underline{X}.$$

S is called a (γ_1, γ_2)-(topologically-algebraic) [2] *structure functor* if, for all $(S_i)_n$-functorial chains (*)

$$S_0 D(\underline{B}_0) \xrightarrow{\psi_1} S_1 D(\underline{B}_1) \xrightarrow{\psi_2} \dots \xrightarrow{\psi_{n-1}} S_{n-1} D(\underline{B}_{n-1}) \xrightarrow{\psi_n} \Delta X$$

where

$$\psi_i = S_i \phi_i \quad \text{for} \quad i = 1, \dots, n\text{-}1$$

and ϕ_i being pointwise in Γ_i for all $i = \{1, \dots, n\text{-}1\}$, ψ_n arbitrary, there exist a sequence $((A_i, g_i))_n$ (**) with

$$g_0 = Id(A_0), \quad g_i \in \Gamma_i \text{ for } i = 1, \dots, n \quad \text{and} \quad A_n = X,$$

and $\underline{B}_i$-co-cones $\alpha_i : D(\underline{B}_i) \to \Delta A_i$ rendering the corresponding diagrams commutative (cp. Figure 15) such that for every sequence $(B_i, f_i)_n$ (***) with

$$B_n = X, \quad f_i \in \Gamma_i \text{ if } n - i + 1 \in \gamma_2 \quad \text{and} \quad f_i \in Mor(Q_{n-i+1}) \text{ otherwise}$$

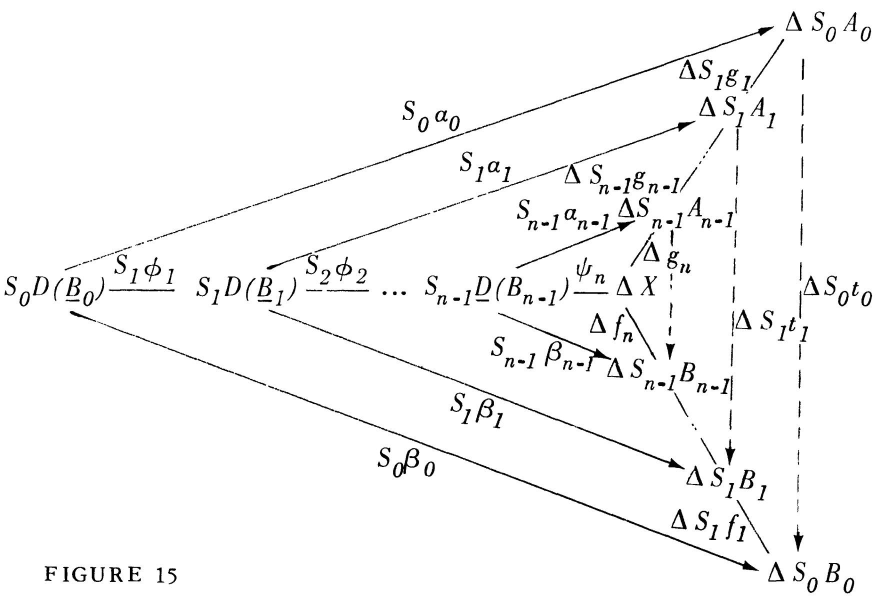


FIGURE 15

[1] The Γ_i must fulfill the usual compatibility criterions with respect to compositions with isomorphisms from the left or right.

[2] In the following we will omit the adjective « topologically-algebraic ».

and $\underline{B}_i$-co-cones $\beta_i : D(\underline{B}_i) \to \Delta B_i$ rendering the corresponding diagrams commutative, there exists a unique sequence

$$t_i : A_i \to B_i \text{ for } i = 0, \dots, n-1 \text{ and } t_n = Id(X)$$

such that

$$\beta_i = (\Delta t_i)_i \quad \text{for } i = 0, \dots, n-1$$

and the following cells commute :

$$\begin{tikzcd} A_i \arrow[r, "g_i"] \arrow[d, "t_i"'] \arrow[dr, phantom, "="] & Q_{n-i+1} A_{i-1} \arrow[d, "Q_{n-i+1} t_{n-i+1}"] \\ B_i \arrow[r, "f_i"] & Q_{n-i+1} B_{i-1} \end{tikzcd}$$

FIGURE 16

The class of $(S_i)_n$-chains of type (*) (domain $D(\underline{B}_i) = 1$!) is denoted by $\Omega(\Gamma_i)$, of type (**) $\Sigma(\Gamma_i)$, and of type (***) by $\Gamma(\Gamma_i)$.

Hence a (γ_1, γ_2)-structure functor is a $(\Phi(\Omega(\Gamma_i)), \Gamma(\Gamma_i), \Sigma(\Gamma_i))$-semifinal structure functor in the language of Section 2. The pair (γ_1, γ_2) is called the *index of* S.

From the previous results, it is clear how one has to define a $(\Phi(\Gamma(\Gamma_i)), \Omega(\Gamma_i), \Sigma(\Gamma_i))$*-semiinitial structure functor.*

5.3.2. THEOREM *(Representation Theorem* [34].

Let $S : \underline{A} \to \underline{X}$ *be a functor. Then there are equivalent:*

(i) S *is a* (γ_1, γ_2)*-structure functor.*

(ii) There exists a factorization of S *:*

$$(S : \underline{A} \to \underline{X}) = (\underline{A} = \underline{A}_n \xrightarrow{E_n} \underline{A}_{n-1} \xrightarrow{E_{n-1}} \dots \longrightarrow \underline{A}_1 \xrightarrow{E_1} \underline{X})$$

with $E_1 \equiv$ *(semi-)topological functor and*

$$E_i \equiv \begin{cases} \text{full reflective embedding } (\text{if } i \in \gamma_1) \\ \text{full coreflective embedding } (\text{if } i \in \gamma_2). \end{cases}$$

5.3.3. THEOREM *(Duality Theorem for* (γ_1, γ_2)*-structure functors.*

The following assertions are equivalent:

1º S *(more exactly* $(S_i)_n$*) is a* $(\Phi(\Omega(\Gamma_i)), \Gamma(\Gamma_i), \Sigma(\Gamma_i))$*-semi-*

final structure functor.

2^o S *is a* $(\Phi(\Gamma(\Gamma_i), \Omega(\Gamma_i), \Sigma(\Gamma_i))$*-semiinitial structure functor.*

5.3.4. REMARK.

The class of all (γ_1, γ_2)-structure functors for arbitrary γ_1, γ_2 has many nice properties. So for instance it is closed under composition and external duality, and so on (cp. [34]).

5.4. The connexion between internal and external duality of topologically-algebraic structure functors.

The external (= categorical) dual notions of semiinitial, resp. semifinal structure functor sequences are co-semiinitial, resp. co-semifinal structure functor sequences which are obviously again structure functor sequences in the sense of Definition 2.2, resp. 3.1, but with respect to a different set of characteristic data. The duality between semiinitial and semifinal structure functor sequences given in Theorem 4.2 is called an internal duality to distinguish it from external duality. The striking result of this paragraph is now that there is a strong connection between internal and external duality, at least at the level of topologically-algebraic structure functors. Using the representation Theorem 5.3.2, we will show that the internal duality can be considered as a sort of «truncated» external duality.

We assume that a given functor $S: A \to X$ has a representation of the form (*)

$$(S: \underline{A} \to \underline{X}) = (\underline{A} = \underline{A}_n \xrightarrow{E_{n-1}} \underline{A}_{n-1} \longrightarrow \cdots \xrightarrow{E_0} \underline{A}_0 \xrightarrow{T} \underline{X}$$

where T is topological and the functors E_i are by turns full reflective or coreflective embeddings. We call n the *length* of the factorization (*) of S. $TAS(n)$ denotes the «class» of all topologically-algebraic functors which have a factorization of at least length n. The minimum of all lengths of a given topologically-algebraic functor S is called the *index* of S and denoted by $ind(S)$. Hence topological functors have index 0 and semitopological functors index 1 (if they are not topological).

Let n be even. Let

$$TAS_r(n) \subset TAS(n), \quad \text{resp.} \quad TAS_c(n) \subset TAS(n)$$

be the subclass of $TAS(n)$ consisting of all S having a factorization (*) with E_0 reflective, resp. E_0 coreflective. Hence $S \in TAS(n)$ has a representation of the form (**):

$$(S: A \to X) = (A \xrightarrow{\text{corefl.}} . \xrightarrow{\text{refl.}} . \xrightarrow{\text{corefl.}} \ldots \xrightarrow{\text{refl.}} \ldots \xrightarrow{\text{top.}} X)$$

The external (categorical) dualization Op gives a bijection

$$Op: TAS_r(n) \Longleftrightarrow TAS_c(n).$$

Hence using the representation Theorem 5.3.2 which gives an internal characterization of functors of type (*) we obtain by categorical dualization an internal characterization of the elements in $TAS_c(n)$ which is dual to the internal characterization in 5.3.2. Now $TAS_r(n-1)$ can be considered as a subclass of $TAS_r(n)$ as well as of $TAS_c(n)$ assuming that E_{n-1} is the identity, resp. E_0 is the identity. But this implies that each $S \in TAS_r(n-1)$ has two different internal characterizations. It is now easy to see that these are just the semiinitial, resp. the semifinal characterization. If n is odd, then a similar consideration by interchanging «r» and «c» leads to the same result. Hence the internal duality in Theorem 5.3.3 can be considered as a truncated external duality since it can be obtained from the external duality simply by restricting corresponding data.

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APPENDIX. LOCALIZABLE STRUCTURE FUNCTORS, BASIC DEFINITIONS

Originally the author planed [1] an extended paper on localizable structure functors motivated by Y. Diers paper [2] on localizable algebraic functors. But in the meantime W. Tholen submitted a paper on Mac Neille completions of concrete categories with local properties [3] where all basic definitions and theorems are given for the special instance of localizable semitopological functors as well as an excellent list of examples. Hence I will restrict myself to the fundamental definitions for localizable structure functors. The major difference between the special instance of localizable semitopological functors and arbitrary structure functors lies in the fact that arbitrary structure functors admit locally as well as globally corresponding generalizations, where at the level of semitopological functors these different types of notions coincide. By this notion we are able to describe internally the composition e.g. of localizing right adjoints with locally left adjoints. The idea behind is simply to weaken the notion: «Π-initial object» in definition 2.1. I will use Tholen's terminology in [3].

A.1. DEFINITION. Let $\underline{C}$ be an arbitrary category, Π a class of $\underline{C}$-objects and A be a $\underline{C}$-object.

1° A is called *locally* Π-*initial* if for all objects B, C in $\Pi \cup \{A\}$, and all $\underline{C}$-morphisms $g: A \to C$, $h: B \to C$ there exists a unique $f: A \to B$ with $hf = g$.

2° A is called *strongly locally* Π-*initial* if for all B, C, g, h as above there exists a unique $f: A \to B$.

3° A is called *prequasi*-Π-*initial* if for all objects B in $\Pi \cup \{A\}$, and all $\underline{C}$-morphisms $u, v: A \to B$ there exists a unique $j: A \to A$ with $uj = v$.

4° A is called *pre*-Π-*initial* if it is prequasi-Π-initial and if j can always be chosen as the identity.

1) Announced in [35].

2) Y. DIERS, Catégories localisables, Thèse, Univ. Paris VI, 1977.

3) W. THOLEN, Mac Neille completion of concrete categories with local properties, submitted.

5° A is called *weakly* Π-*initial* if for every $B \to A$ there exists at least one morphism $A \to B$.

6° A is called *quasi*-Π-*initial* if A is both weakly Π-initial and pre-quasi-Π-initial.

The following diagram (Tholen in [3]) for $\Pi = Ob\,\underline{C}$) shows the hierarchy

$$\begin{tikzcd}
 & \Pi\text{-initial} \arrow[dl] \arrow[dr] \arrow[d] & \\
\text{strongly locally } \Pi\text{-initial} \arrow[rr] \arrow[dd] & \text{quasi-}\Pi\text{-initial} \arrow[dd] & \text{pre-}\Pi\text{-initial} \arrow[dd] \\
 & & \\
\text{locally } \Pi\text{-initial} \arrow[rr] & \text{weakly-}\Pi\text{-initial} & \text{prequasi-}\Pi\text{-initial}
\end{tikzcd}$$

A.2. DEFINITION. Let $\underline{C}$ be an arbitrary category and Π be a class of $\underline{C}$-objects.

- $\underline{C}$ has *(strongly) locally* Π-*initial objects* if for all C, there exist a (strongly) Π-initial object A and a morphism $g: A \to C$ in C.

- $\underline{C}$ is called *(strongly)* Π-*localizing* if $\underline{C}$ has (strongly) locally Π-initial objects and if there exists only a set of non-isomorphic (strongly) locally Π-initial objects in $\underline{C}$.

A.3. DEFINITION *(Globally localizable structure functors)*. Notations as in Definition 2.2. The sequence $(S_i)_n$ is called a *(strongly) localizable* $(\Phi(\Omega),\Gamma,\Sigma)_{\sigma_1,\sigma_2,\rho}$-*semifinal structure functor sequence* if for any functorial chain being pointwise in Ω the category of all ρ-extensions has (strongly) locally Π-initial objects where Π is the class of all Γ-extensions and where (strongly) locally Π-initial objects are Σ-extensions.

A.4. REMARKS. 1° In the same way one defines *(strongly) localizable* $(\Phi(\Omega),\Gamma,\Sigma)_{\sigma_1,\sigma_2,\rho}$-*semiinitial structure functors*. There exists a corresponding duality theorem. Hence we speak just of (strongly) localizable structure functors.

2° *Locally localizable structure functors* are obtained if each object $S_i(A_i)$ in Fig. 2 fulfills a different «weak universal property» in the sense of Definition A.1, 1-6; hence the different types of such functors.

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