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*Compositio Mathematica*, tome 101, n° 2 (1996), p. 217-224

[http://www.numdam.org/item?id=CM\\_1996\\_\\_101\\_2\\_217\\_0](http://www.numdam.org/item?id=CM_1996__101_2_217_0)

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# Compact Kähler manifolds with hermitian semipositive anticanonical bundle

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Received 21 October 1994; accepted in final form 9 March 1995

**Abstract.** This note states a structure theorem for compact Kähler manifolds with semipositive Ricci curvature: any such manifold has a finite étale covering possessing a De Rham decomposition as a product of irreducible compact Kähler manifolds, each one being either Ricci flat (torus, symplectic or Calabi-Yau manifold), or Ricci semipositive without non trivial holomorphic forms. Related questions and conjectures concerning the latter case are discussed.

**Key words:** Compact Kähler manifold, semipositive Ricci curvature, complex torus, symplectic manifold, Calabi-Yau manifold, Albanese map, fundamental group, Bochner formula, De Rham decomposition, Cheeger-Gromoll theorem, nef line bundle, Kodaira-Iitaka dimension, rationally connected manifold

## 1. Main results

This short note is a continuation of our previous work [DPS93] on compact Kähler manifolds  $X$  with semipositive Ricci curvature. Our purpose is to state a splitting theorem describing the structure of such manifolds, and to raise some related questions. The foundational background will be found in papers by Lichnerowicz [Li67], [Li71], and Cheeger-Gromoll [CG71], [CG72]. Recall that a *Calabi-Yau manifold*  $X$  is a compact Kähler manifold with  $c_1(X) = 0$  and finite fundamental group  $\pi_1(X)$ , such that the universal covering  $\tilde{X}$  satisfies  $H^0(\tilde{X}, \Omega_{\tilde{X}}^p) = 0$  for all  $1 \leq p \leq \dim X - 1$ . A *symplectic manifold*  $X$  is a compact Kähler manifold admitting a holomorphic symplectic 2-form  $\omega$  (of maximal rank everywhere); in particular  $K_X = \mathcal{O}_X$ . We denote here as usual

$$\Omega_X = \Omega_X^1 = T_X^*, \quad \Omega_X^p = \Lambda^p T_X^*, \quad K_X = \det(T_X^*).$$

The following structure theorem generalizes the structure theorem for Ricci-flat manifolds (due to Bogomolov [Bo74a], [Bo74b], Kobayashi [Ko81] and Beauville [Be83]) to the Ricci semipositive case.

**STRUCTURE THEOREM.** *Let  $X$  be a compact Kähler manifold with  $-K_X$  hermitian semipositive. Then*

- (i) *The universal covering  $\tilde{X}$  admits a holomorphic and isometric splitting*

$$\tilde{X} \simeq \mathbb{C}^q \times \prod X_i$$

*with  $X_i$  being either a Calabi-Yau manifold or a symplectic manifold or a manifold with  $-K_{X_i}$  semipositive and  $H^0(X_i, \Omega_{X_i}^{\otimes m}) = 0$  for all  $m > 0$ .*

- (ii) *There exists a finite étale Galois covering  $\hat{X} \rightarrow X$  such that the Albanese variety  $\text{Alb}(\hat{X})$  is a  $q$ -dimensional torus and the Albanese map  $\alpha : \hat{X} \rightarrow \text{Alb}(\hat{X})$  is a locally trivial holomorphic fibre bundle whose fibres are products  $\prod X_i$  of the type described in (i), all  $X_i$  being simply connected.*
- (iii) *We have  $\pi_1(\hat{X}) \simeq \mathbb{Z}^{2q}$  and  $\pi_1(X)$  is an extension of a finite group  $\Gamma$  by the normal subgroup  $\pi_1(\hat{X})$ . In particular there is an exact sequence*

$$0 \rightarrow \mathbb{Z}^{2q} \rightarrow \pi_1(X) \rightarrow \Gamma \rightarrow 0,$$

*and the fundamental group  $\pi_1(X)$  is almost abelian.*

Recall that a line bundle  $L$  is said to be hermitian semipositive if it can be equipped with a smooth hermitian metric of semipositive curvature form. A sufficient condition for hermitian semipositivity is that some multiple of  $L$  is spanned by global sections; on the other hand, the hermitian semipositivity condition implies that  $L$  is numerically effective (nef) in the sense of [DPS94], which, for  $X$  projective algebraic, is equivalent to saying that  $L \cdot C \geq 0$  for every curve  $C$  in  $X$ . Examples contained in [DPS94] show that all three conditions are different (even for  $X$  projective algebraic). By Yau’s solution of the Calabi conjecture (see [Au76], [Yau78]), a compact Kähler manifold  $X$  has a hermitian semipositive anticanonical bundle  $-K_X$  if and only if  $X$  admits a Kähler metric  $g$  with  $\text{Ricci}(g) \geq 0$ . The isometric decomposition described in the theorem refers to such Kähler metrics.

In view of ‘standard conjectures’ in minimal model theory it is expected that projective manifolds  $X$  with no nonzero global sections in  $H^0(X, \Omega_X^{\otimes m})$ ,  $m > 0$ , are rationally connected. We hope that most of the above results will continue to hold under the weaker assumption that  $-K_X$  is nef instead of hermitian semipositive. However, the technical tools needed to treat this case are still missing.

We would like to thank the DFG-Schwerpunktprogramm ‘Komplexe Mannigfaltigkeiten’ and the Institut Universitaire de France and for making our work possible.

## 2. Bochner formula and holomorphic differential forms

Our starting point is the following well-known consequence of the Bochner formula.

LEMMA. *Let  $X$  be a compact  $n$ -dimensional Kähler manifold with  $-K_X$  hermitian semipositive. Then every section of  $\Omega_X^{\otimes m}$ ,  $m \geq 1$  is parallel with respect to the given Kähler metric.*

*Proof.* The Lemma is an easy consequence of the Bochner formula

$$\Delta(\|u\|^2) = \|\nabla u\|^2 + Q(u),$$

where  $u \in H^0(X, \Omega_X^{\otimes m})$  and  $Q(u) \geq m\lambda_0 \|u\|^2$ . Here  $\lambda_0$  is the smallest eigenvalue of the Ricci curvature tensor. For details see for instance [Ko83].  $\square$

The following definition of a modified Kodaira dimension  $\kappa_+(X)$  is taken from Campana [Ca93]. As the usual Kodaira dimension  $\kappa(X)$ , this is a birational invariant of  $X$ . Other similar invariants have also been considered in [BR90] and [Ma93].

DEFINITION. Let  $Y$  be a compact complex manifold. We define

- (i)  $\kappa_+(Y) = \max\{\kappa(\det \mathcal{F}) : \mathcal{F} \text{ is a subsheaf of } \Omega_Y^p \text{ for some } p > 0\}$ ,
- (ii)  $\kappa_{++}(Y) = \max\{\kappa(\det \mathcal{F}) : \mathcal{F} \text{ is a subsheaf of } \Omega_Y^{\otimes m} \text{ for some } m > 0\}$ .

Here we let as usual  $\det \mathcal{F} = (\Lambda^r \mathcal{F})^{**}$ , where  $r = \text{rank } \mathcal{F}$  and  $\kappa$  is the usual Iitaka dimension of a line bundle.

Clearly, we have  $-\infty \leq \kappa(Y) \leq \kappa_+(Y) \leq \kappa_{++}(Y)$  where  $\kappa(Y) = \kappa(K_Y)$  is the usual Kodaira dimension. It would be interesting to know whether there are precise relations between  $\kappa_+(Y)$  and  $\kappa_{++}(Y)$ , as well as with the weighted Kodaira dimensions defined by Manivel [Ma93]. The above lemma implies:

PROPOSITION. *Let  $X$  be a compact Kähler manifold with  $-K_X$  hermitian semipositive. Then  $\kappa_{++}(X) \leq 0$ .*

*Proof.* Assume that  $\kappa_{++}(X) > 0$ . Then we can find an integer  $m > 0$  and a subsheaf  $\mathcal{F} \subset \Omega_X^{\otimes m}$  with  $\kappa(\det \mathcal{F}) > 0$ . Hence there is some  $\mu \in \mathbb{N}$  and  $s \in H^0(X, (\det \mathcal{F})^\mu)$  with  $s \neq 0$ . Since  $\kappa(\det \mathcal{F}) > 0$ ,  $s$  must have zeroes. Hence the induced section  $\tilde{s} \in H^0(X, \Omega_X^{\otimes \mu r m})$  has zeroes too,  $r$  being the rank of  $\mathcal{F}$ . This contradicts the previous Lemma.  $\square$

COROLLARY. *Let  $X$  be a compact Kähler manifold with  $-K_X$  hermitian semipositive. Let  $\phi : X \rightarrow Y$  be a surjective holomorphic map to a normal compact Kähler space. Then  $\kappa(Y) \leq 0$ . (Here  $\kappa(Y) = \kappa(\hat{Y})$ , where  $\hat{Y}$  is an arbitrary desingularization of  $Y$ .)*

*Proof.* This follows from the inequalities  $0 \geq \kappa_+(X) \geq \kappa_+(Y) \geq \kappa(Y)$ . For the second inequality, which is easily checked by a pulling-back argument, see [Ca93].  $\square$

### 3. Proof of the structure theorem

We suppose here that  $X$  is equipped with a Kähler metric  $g$  such that  $\text{Ricci}(g) \geq 0$ , and we set  $n = \dim_{\mathbb{C}} X$ .

(i) Let  $(\tilde{X}, g) \simeq \prod (X_i, g_i)$  be the De Rham decomposition of  $(\tilde{X}, g)$ , induced by a decomposition of the holonomy representation in irreducible representations. Since the holonomy is contained in  $U(n)$ , all factors  $(X_i, g_i)$  are Kähler manifolds with irreducible holonomy and holonomy group  $H_i \subset U(n_i)$ ,  $n_i = \dim X_i$ . By Cheeger-Gromoll [CG71], there is possibly a flat factor  $X_0 = \mathbb{C}^q$  and the other factors  $X_i, i \geq 1$ , are compact. Also, the product structure shows that  $-K_{X_i}$  is hermitian semipositive. It suffices to prove that  $\kappa_{++}(X_i) = 0$  implies that  $X_i$  is a Calabi-Yau manifold or a symplectic manifold. In view of Section 2, the condition  $\kappa_{++}(X_i) = 0$  means that there is a nonzero section  $u \in H^0(X_i, \Omega_{X_i}^{\otimes m})$  for some  $m > 0$ . Since  $u$  is parallel by the lemma, it is invariant under the holonomy action, and therefore the holonomy group  $H_i$  is not the full unitary group  $U(n_i)$  (indeed, the trivial representation does not occur in the decomposition of  $(\mathbb{C}^{n_i})^{\otimes m}$  in irreducible  $U(n_i)$ -representations, all weights being of length  $m$ ). By Berger’s classification of holonomy groups [Bg55] there are only two remaining possibilities, namely  $H_i = \text{SU}(n_i)$  or  $H_i = \text{Sp}(n_i/2)$ . The case  $H_i = \text{SU}(n_i)$  leads to  $X_i$  being a Calabi-Yau manifold. The remaining case  $H_i = \text{Sp}(n_i/2)$  implies that  $X_i$  is symplectic (see e.g. [Be83]).

(ii) Set  $X' = \prod_{i \geq 1} X_i$ . The group of covering transformations acts on the product  $\tilde{X} = \mathbb{C}^q \times X'$  by holomorphic isometries of the form  $x = (z, x') \mapsto (u(z), v(x'))$ . At this point, the argument is slightly more involved than in Beauville’s paper [Be83], because the group  $G'$  of holomorphic isometries of  $X'$  need not be finite ( $X'$  may be for instance a projective space); instead, we imitate the proof of ([CG72], Theorem 9.2) and use the fact that  $X'$  and  $G' = \text{Isom}(X')$  are compact. Let  $E_q = \mathbb{C}^q \rtimes U(q)$  be the group of unitary motions of  $\mathbb{C}^q$ . Then  $\pi_1(X)$  can be seen as a discrete subgroup of  $E_q \times G'$ . As  $G'$  is compact, the kernel of the projection map  $\pi_1(X) \rightarrow E_q$  is finite and the image of  $\pi_1(X)$  in  $E_q$  is still discrete with compact quotient. This shows that there is a subgroup  $\Gamma$  of finite index in  $\pi_1(X)$  which is isomorphic to a crystallographic subgroup of  $\mathbb{C}^q$ . By Bieberbach’s theorem, the subgroup  $\Gamma_0 \subset \Gamma$  of elements which are translations is a subgroup of finite index. Taking the intersection of all conjugates of  $\Gamma_0$  in  $\pi_1(X)$ , we find a normal subgroup  $\Gamma_1 \subset \pi_1(X)$  of finite index, acting by translations on  $\mathbb{C}^q$ . Then  $\tilde{X} = \tilde{X}/\Gamma_1$  is a fibre bundle over the torus  $\mathbb{C}^q/\Gamma_1$  with  $X'$  as fibre and  $\pi_1(X') = 1$ . Therefore  $\tilde{X}$  is the desired finite étale covering of  $X$ .

(iii) is an immediate consequence of (ii), using the homotopy exact sequence of a fibration. □

**COROLLARY 1.** *Let  $X$  be a compact Kähler manifold with  $-K_X$  hermitian semipositive. If  $\tilde{X}$  is indecomposable and  $\kappa_+(X) = 0$ , then  $X$  is Ricci-flat.*

**COROLLARY 2.** *Let  $X$  be a compact Kähler manifold with  $-K_X$  hermitian semipositive. Then, if  $\widehat{X} \rightarrow X$  is an arbitrary finite étale covering*

$$\begin{aligned} \kappa_+(X) = -\infty &\iff \kappa_{++}(X) = -\infty \\ &\iff \forall \widehat{X} \rightarrow X, \forall p \geq 1, \quad H^0(\widehat{X}, \Omega_{\widehat{X}}^p) = 0. \end{aligned}$$

*If  $\kappa_+(X) = -\infty$ , then  $\chi(X, \mathcal{O}_X) = 1$  and  $X$  is simply connected.*

*Proof.* The equivalence of all three properties is a direct consequence of the structure theorem. Now, any étale covering  $\widehat{X} \rightarrow X$  satisfies  $\kappa_+(\widehat{X}) = \kappa_+(X) = -\infty$ , hence  $\chi(\widehat{X}, \mathcal{O}_{\widehat{X}}) = \chi(X, \mathcal{O}_X) = 1$  (by Hodge symmetry we have  $h^p(X, \mathcal{O}_X) = 0$  for  $p \geq 1$ , whilst  $h^0(X, \mathcal{O}_X) = 1$ ). However, if  $d$  is the covering degree, the Riemann-Roch formula implies  $\chi(\widehat{X}, \mathcal{O}_{\widehat{X}}) = d \chi(X, \mathcal{O}_X)$ , hence  $d = 1$  and  $X$  must be simply connected. □

**4. Related questions for the case  $-K_X$  nef**

In order to make the structure theorem more explicit, it would be necessary to characterize more precisely the manifolds for which  $\kappa_+(X) = -\infty$ . We expect these manifolds to be rationally connected, even when  $-K_X$  is just supposed to be nef.

**CONJECTURE.** *Let  $X$  be a compact Kähler manifold such that  $-K_X$  is nef and  $\kappa_+(X) = -\infty$ . Then  $X$  is rationally connected, i.e. any two points of  $X$  can be joined by a chain of rational curves.*

Campana even conjectures this to be true without assuming  $-K_X$  to be nef.

Another hope we have is that a similar structure theorem might also hold in the case  $-K_X$  nef. A small part of it would be to understand better the structure of the Albanese map. We proved in [DPS93] that the Albanese map is surjective when  $\dim X \leq 3$ , and if  $\dim X \leq 2$  it is well-known that the Albanese map is a locally trivial fibration. It is thus natural to state the following

**PROBLEM.** *Let  $X$  be a compact Kähler manifold with  $-K_X$  nef. Is the Albanese map  $\alpha: X \rightarrow \text{Alb}(X)$  a smooth locally trivial fibration?*

The following simple example shows, even in the case of a locally trivial fibration, that the structure group of transition automorphisms need not be a group of isometries, in contrast with the case  $-K_X$  hermitian semipositive.

**EXAMPLE 1** (see [DPS94], Example 1.7). Let  $C = \mathbb{C}/(\mathbb{Z} + \mathbb{Z}\tau)$  be an elliptic

curve, and let  $E \rightarrow C$  be the flat rank 2 bundle associated to the representation  $\pi_1(C) \rightarrow \text{GL}_2(\mathbb{C})$  defined by the monodromy matrices

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

Then the projectivized bundle  $X = \mathbb{P}(E)$  is a ruled surface over  $C$  with  $-K_X$  nef and not hermitian semipositive (cf. [DPS94]). In this case, the Albanese map  $X \rightarrow C$  is a locally trivial  $\mathbb{P}_1$ -bundle, but the monodromy group is not relatively compact in  $\text{GL}_2(\mathbb{C})$ , hence there is no invariant Kähler metric on the fibre.

**EXAMPLE 2.** The following example shows that the picture is unclear even in the case of surfaces with  $\kappa_+(X) = -\infty$ . Let  $\mathbf{p} = (p_1, \dots, p_9)$  be a configuration of 9 points in  $\mathbb{P}_2$  and let  $\pi: X_{\mathbf{p}} \rightarrow \mathbb{P}_2$  be the blow-up of  $\mathbb{P}_2$  with center  $\mathbf{p}$ . Here some of the points  $p_i$  may be infinitely near: as usual, this means that the blowing-up process is made inductively, each  $p_i$  being an arbitrary point in the blow-up of  $\mathbb{P}_2$  at  $(p_1, \dots, p_{i-1})$ . There is always a cubic curve  $C$  containing all 9 points ( $C$  is even unique if  $\mathbf{p}$  is general enough). The only assumption we make is that  $C$  is nonsingular, and we let  $C = \{Q(z_0, z_1, z_2) = 0\} \subset \mathbb{P}_2$ ,  $\text{deg } Q = 3$ . Then  $C$  is an elliptic curve and  $-K_{X_{\mathbf{p}}} = \pi^* \mathcal{O}(3) - \sum E_i$  where  $E_i = \pi^{-1}(p_i)$  are the exceptional divisors. Clearly  $Q$  defines a section of  $-K_{X_{\mathbf{p}}}$ , of divisor equal to the strict transform  $C'$  of  $C$ , hence  $-K_{X_{\mathbf{p}}} \simeq \mathcal{O}(C')$ , and  $(-K_{X_{\mathbf{p}}})^2 = (C')^2 = C^2 - 9 = 0$ . Therefore  $-K_{X_{\mathbf{p}}}$  is always nef.

It is easy to see that  $-mK_{X_{\mathbf{p}}}$  may be generated or not by sections according to the choice of the 9 points  $p_i$ . In fact, if  $p'_i$  is the point of  $C'$  lying over  $p_i$ , we have

$$-K_{X_{\mathbf{p}}}|_{C'} = \pi^*(\mathcal{O}(3))|_{C'} \otimes \mathcal{O}\left(-\sum p'_j\right) = \pi^*\left(\mathcal{O}(3)|_C \otimes \mathcal{O}\left(-\sum p_j\right)\right).$$

Since  $C' \simeq C$  is an elliptic curve and  $-K_{X_{\mathbf{p}}}|_{C'}$  has degree 0, there are nonzero sections in  $H^0(C', -mK_{X_{\mathbf{p}}}|_{C'})$  if and only if  $L_{\mathbf{p}} = \mathcal{O}(3)|_C \otimes \mathcal{O}(-\sum p_j)$  is a torsion point in  $\text{Pic}^0(C)$  of order dividing  $m$ . Such sections always extend to  $X_{\mathbf{p}}$ . Indeed, we may assume that  $m$  is exactly the order. Then  $\mathcal{O}(-C') \otimes \mathcal{O}(-mK_{X_{\mathbf{p}}}) = \mathcal{O}((m-1)C')$  admits a filtration by its subsheaves  $\mathcal{O}(kC')$ ,  $0 \leq k \leq m-1$ , and the  $H^1$  groups of the graded pieces are  $H^1(X_{\mathbf{p}}, \mathcal{O}_{X_{\mathbf{p}}}) = 0$  for  $k = 0$  and

$$H^1(C', \mathcal{O}(kC')|_{C'}) = H^0(C', \mathcal{O}(-kC')) = 0 \text{ for } 0 < k < m.$$

Therefore  $H^1(X_{\mathbf{p}}, \mathcal{O}(-C') \otimes \mathcal{O}(-mK_{X_{\mathbf{p}}})) = 0$ , as desired. In particular,  $-K_{X_{\mathbf{p}}}$  is hermitian semipositive as soon as  $L_{\mathbf{p}}$  is a torsion point in  $\text{Pic}^0(C)$ . In this case, there is a polynomial  $R_m$  of degree  $3m$  vanishing of order  $m$  at all points  $p_i$ , such that the rational function  $R_m/Q^m$  defines an elliptic fibration  $\varphi: X_{\mathbf{p}} \rightarrow \mathbb{P}_1$ ; in this fibration  $C$  is a multiple fibre of multiplicity  $m$  and we have  $-mK_{X_{\mathbf{p}}} =$

$\varphi^* \mathcal{O}_{\mathbb{P}^1}(1)$ . An interesting question is to understand what happens when  $L_{\mathbf{p}}$  is no longer a torsion point in  $\text{Pic}^0(C)$  (this is precisely the situation considered by Ogus [Og76] in order to produce a counterexample to the formal principle for infinitesimal neighborhoods). In this situation, we may approximate  $\mathbf{p}$  by a sequence of configurations  $\mathbf{p}_m \subset C$  such that the corresponding line bundle  $L_{\mathbf{p}_m}$  is a torsion point of order  $m$  (just move a little bit  $p_9$  and take a suitable  $p_{9,m} \in C$  close to  $p_9$ ). The sequence of fibrations  $X_{\mathbf{p}_m} \rightarrow \mathbb{P}^1$  does not yield a fibration  $X_{\mathbf{p}} \rightarrow \mathbb{P}^1$  in the limit, but we believe that there might exist instead a holomorphic foliation on  $X_{\mathbf{p}}$ . In this foliation,  $C$  would be a closed leaf, and the generic leaf would be nonclosed and of conformal type  $\mathbb{C}$  (or possibly  $\mathbb{C}^*$ ). If indeed the foliation exists and admits a smooth invariant transversal volume form, then  $-K_{X_{\mathbf{p}}}$  would still be hermitian semipositive. We are thus led to the following question.

**QUESTION.** *Let  $X$  be compact Kähler manifold with  $-K_X$  nef and  $X$  rationally connected. Is then  $-K_X$  automatically hermitian semipositive? In particular, is it always the case that  $\mathbb{P}^2$  blown-up in 9 points of a nonsingular cubic curve has a semipositive anticanonical bundle?*

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