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We have been informed by Vo Van Tan on an omission in Theorem (5.5) of our paper [K-P]. In fact, Enoki constructed in [E] compact complex surfaces $S_{n,\alpha,t}$ depending on parameters $n \in \mathbb{N}$, $\alpha \in \mathbb{C}$, $0 < |\alpha| < 1$ and $t \in \mathbb{C}^n$, with second Betti number $b_2(S_{n,\alpha,t}) = n$, admitting an effective divisor $D = D_{n,\alpha,t}$ with n irreducible components such that $D^2 = 0$. He proved that any compact complex surface S of class VII_0 with $b_2(S) = n > 0$ having a divisor $D \neq 0$ with $D^2 = 0$, is isomorphic to some $S_{n,\alpha,t}$ and $D = rD_{n,\alpha,t}$.

Moreover, $S_{n,\alpha,t} \backslash D_{n,\alpha,t}$ is an affine $\mathbb{C}$-bundle over an elliptic curve which is a line bundle if $t = 0$. This affine bundle can be compactified to a ruled surface over an elliptic curve.

It is easily checked that $S_{n,\alpha,t} \backslash D_{n,\alpha,t}$ is Stein for $t \neq 0$ (for instance by [V] p. 4, 5). Taking this into account, Theorem (5.1) of [K-P] formulates now:

(5.1) THEOREM. *Let X be a compact complex surface and $C \subset X$ an irreducible curve with $C^2 = 0$ such that $X \backslash C$ is Stein. Then one of the following statements holds*:

(i) *X is algebraic,*
(ii) *X is a Hopf surface of algebraic dimension* 0 *with exactly one curve,*
(iii) *X is isomorphic to some $S_{1,\alpha,t}$ with $t \neq 0$ (and C is rational by* [E]).

In the same spirit, we have to add all surfaces $S_{n,\alpha,t}$, $t \neq 0$, in Theorem (5.5) of [K-P]. This does not cause any trouble for the applications in Section 5 of loc. cit., since the divisor D there contains always an elliptic curve which is wrong for $D = D_{n,\alpha,t} \subset S_{n,\alpha,t}$ by Enoki's paper.

References

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