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Errata to the papers: Connections between $B_{2,\chi}$ for even quadratic Dirichlet characters χ and class numbers of appropriate imaginary quadratic fields, Parts I and II

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Due to a printer's error, 74 pairs of parentheses in Kronecker symbols were omitted, which completely changed the sense of the theorems of both papers. Here are their correct versions.

Part I:

THEOREM 1. *Let for $k = 0, 1, 2$ and 3*

$$s_k = \sum_{l \in \{kD/8, (k+1)D/8\}} \left(\frac{D}{l}\right) l.$$

Then for $D \neq 5$:

$$(i) \quad k_2(D) = \frac{16}{45} \left(2 \left(\frac{D}{2} \right) - 7 \right) (s_0 + s_1) - \frac{2}{45} \left(2 \left(\frac{D}{2} \right) - 7 \right) D h(-4D),$$

$$(ii) \quad k_2(D) = -\frac{32}{75} \left(\left(\frac{D}{2} \right) + 4 \right) (s_0 + s_2) \\ + \frac{2}{75} \left(\left(\frac{D}{2} \right) + 4 \right) D \left(-\left(\left(\frac{D}{2} \right) + 2 \right) h(-4D) + 2h(-8D) \right),$$

$$(iii) \quad k_2(8D) = -32(s_1 + s_2) - 2D \left(2 \left(\frac{D}{2} \right) h(-4D) - h(-8D) \right),$$

$$(iv) \quad k_2(8D) + \left(\left(\frac{D}{2} \right) - 34 \right) k_2(D) = 64s_0 - 2D \left(\left(\frac{D}{2} \right) h(-4D) + h(-8D) \right), \\ k_2(8D) + 3 \left(3 \left(\frac{D}{2} \right) - 2 \right) k_2(D)$$

$$\begin{aligned}
&= -64s_1 - 2D \left(\left(\left(\frac{D}{2} \right) - 4 \right) h(-4D) + h(-8D) \right), \\
k_2(8D) - 3 \left(3 \left(\frac{D}{2} \right) - 2 \right) k_2(D) \\
&= -64s_2 - 2D \left(\left(3 \left(\frac{D}{2} \right) + 4 \right) h(-4D) - 3h(-8D) \right), \\
k_2(8D) + 15 \left(\left(\frac{D}{2} \right) - 2 \right) k_2(D) &= 64s_3 - 6D \left(\left(\frac{D}{2} \right) h(-4D) - h(-8D) \right).
\end{aligned}$$

COROLLARY 1. *Let φ denote Euler's totient function.*

(i) $k_2(D) \equiv 2h(-4D) + 2\varphi(D) + \varepsilon \pmod{32}$,

where $\varepsilon = 0$ unless $D = p \equiv -3 \pmod{8}$ a prime or $D = pq$, where $p \equiv q \not\equiv 1 \pmod{8}$ or $p \equiv q + 4 \equiv 3 \pmod{8}$, p, q -primes. In these cases $\varepsilon = 16$ if $p \equiv q \equiv -3 \pmod{8}$, $\varepsilon = -8$ if $p \equiv q \equiv -1 \pmod{8}$ and $\varepsilon = 8$ otherwise.

(ii) $k_2(D) \equiv 6h(-4D) - 4 \left(2 - \left(\frac{D}{2} \right) \right) h(-8D) \pmod{32}$,

(iii) $k_2(D) \equiv -2 \left(2 - \left(\frac{D}{2} \right) \right) \left(2h(-4D) - \left(\frac{D}{2} \right) h(-8D) \right) \pmod{32}$,

(iv) $k_2(8D) + \left(\left(\frac{D}{2} \right) - 34 \right) k_2(D)$

$$\begin{aligned}
&\equiv -2 \left(2 \left(\frac{D}{2} \right) - 1 \right) \left(\left(\frac{D}{2} \right) h(-4D) + h(-8D) \right) \pmod{64}, \\
k_2(8D) + 3 \left(3 \left(\frac{D}{2} \right) - 2 \right) k_2(D) \\
&\equiv -2 \left(2 \left(\frac{D}{2} \right) - 1 \right) \left(\left(\left(\frac{D}{2} \right) - 4 \right) h(-4D) + h(-8D) \right) \pmod{64}, \\
k_2(8D) - 3 \left(3 \left(\frac{D}{2} \right) - 2 \right) k_2(D) \\
&\equiv -2 \left(2 \left(\frac{D}{2} \right) - 1 \right) \left(\left(3 \left(\frac{D}{2} \right) + 4 \right) h(-4D) - 3h(-8D) \right) \pmod{64}, \\
k_2(8D) + 15 \left(\left(\frac{D}{2} \right) - 2 \right) k_2(D) \\
&\equiv -6 \left(2 \left(\frac{D}{2} \right) - 1 \right) \left(\left(\frac{D}{2} \right) h(-4D) - h(-8D) \right) \pmod{64}.
\end{aligned}$$

(v) *If $D = p = 8t + 1$ or $8t - 3$ a prime then:*

$$k_2(D) \equiv 2h(-4D) + 16t \pmod{32},$$

$$k_2(D) \equiv 32\alpha + 2\beta \left(- \left(2 + \left(\frac{D}{2} \right) \right) h(-4D) + 2h(-8D) \right) \pmod{64},$$

where $\alpha = 1$ if $p \equiv -3 \pmod{16}$ and $\alpha = 0$ otherwise, and $\beta = -1, -3$, resp. 5 if $p \equiv 1 \pmod{8}$, $p \equiv 5 \pmod{16}$, resp. $p \equiv -3 \pmod{16}$,

$$k_2(8D) \equiv 32\alpha + 2\beta \left(2 \left(\frac{D}{2} \right) h(-4D) - h(-8D) \right) \pmod{64},$$

where $\alpha = 0$ if $p \equiv 1 \pmod{16}$ and $\alpha = 1$ otherwise, and $\beta = -1, -3$, resp. 5 if $p \equiv 1 \pmod{8}$, $p \equiv -3 \pmod{16}$, resp. $p \equiv 5 \pmod{16}$.

THEOREM 2. *Let for $k = 0, 1, 2$ and 3*

$$s_k = \sum_{l \in [k\Delta/8, (k+1)\Delta/8)} \left(\frac{-\Delta}{l} \right) l.$$

Then for $\Delta \neq 3$:

$$(i) \quad k_2(4\Delta) = 16(s_0 + s_1) - 2\Delta \left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) \text{ (see [5], too),}$$

$$(ii) \quad k_2(4\Delta) = 32 \left(\frac{-\Delta}{2} \right) (s_0 + s_3) \\ + 2\Delta \left(\frac{-\Delta}{2} \right) \left(7 \left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) + 2h(-8\Delta) \right),$$

$$(iii) \quad k_2(8\Delta) = 32(s_0 - s_3) - 2\Delta \left(6 \left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) + h(-8\Delta) \right),$$

$$(iv) \quad k_2(8\Delta) + \left(\frac{-\Delta}{2} \right) k_2(4\Delta) = 64s_0 + 2\Delta \left(\left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) + h(-8\Delta) \right),$$

$$k_2(8\Delta) + \left(\left(\frac{-\Delta}{2} \right) - 4 \right) k_2(4\Delta) \\ = -64s_1 + 2\Delta \left(5 \left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) + h(-8\Delta) \right),$$

$$k_2(8\Delta) - \left(\left(\frac{-\Delta}{2} \right) + 4 \right) k_2(4\Delta) \\ = 64s_2 + 2\Delta \left(7 \left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) - 3h(-8\Delta) \right),$$

$$k_2(8\Delta) - \left(\frac{-\Delta}{2} \right) k_2(4\Delta) \\ = -64s_3 - 2\Delta \left(13 \left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) + 3h(-8\Delta) \right).$$

COROLLARY 1.

$$(i) \quad k_2(4\Delta) \equiv -6h(-\Delta) \left(\left(\frac{-\Delta}{2} \right) - 1 \right) + 2\varphi(\Delta) + \varepsilon \pmod{32},$$

where $\varepsilon = 0$ unless $\Delta = p \equiv 3 \pmod{4}$ a prime or $\Delta = pq$, where $p \equiv q + 2 \equiv -1 \pmod{8}$, p, q -primes, or $\Delta = pqr$, where $p \equiv q \equiv r \equiv -1, 3 \pmod{8}$, or $p \equiv q \equiv -1$, resp. $3 \pmod{8}$ and $r \equiv 3$, resp. $-1 \pmod{8}$, p, q, r -primes.

In these cases $\varepsilon = 4$ if $\Delta = p \equiv -1 \pmod{8}$, $\varepsilon = -4$ if $\Delta = p \equiv 3 \pmod{8}$ and $\varepsilon = 16$ otherwise.

$$(ii) \quad k_2(4\Delta) \equiv 6 \left(\frac{-\Delta}{2} \right) \left(7 \left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) + 2h(-8\Delta) \right) \pmod{32},$$

$$k_2(4\Delta) \equiv -4h(-8\Delta) \pmod{32}, \quad \text{if } \Delta \equiv -1 \pmod{8}, \text{ in particular,}$$

$$(iii) \quad k_2(8\Delta) \equiv 2 \left(1 - \left(\frac{-\Delta}{2} \right) \right) \left(6 \left(1 - \left(\frac{-\Delta}{2} \right) \right) h(-\Delta) - h(-8\Delta) \right) \pmod{32},$$

$$k_2(8\Delta) \equiv 2h(-8\Delta) \pmod{32}, \quad \text{if } \Delta \equiv -1 \pmod{8}, \text{ in particular,}$$

$$(iv) \quad k_2(8\Delta) + \left(\frac{-\Delta}{2} \right) k_2(4\Delta)$$

$$\equiv -2 \left(2 \left(\frac{-\Delta}{2} \right) - 1 \right) \left(\left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) + h(-8\Delta) \right) \pmod{64},$$

$$k_2(8\Delta) + \left(\left(\frac{-\Delta}{2} \right) - 4 \right) k_2(4\Delta)$$

$$\equiv -2 \left(2 \left(\frac{-\Delta}{2} \right) - 1 \right) \left(5 \left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) + h(-8\Delta) \right) \pmod{64},$$

$$k_2(8\Delta) - \left(\left(\frac{-\Delta}{2} \right) + 4 \right) k_2(4\Delta)$$

$$\equiv -2 \left(2 \left(\frac{-\Delta}{2} \right) - 1 \right) \left(7 \left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) - 3h(-8\Delta) \right) \pmod{64},$$

$$k_2(8\Delta) - \left(\frac{-\Delta}{2} \right) k_2(4\Delta)$$

$$\equiv 2 \left(2 \left(\frac{-\Delta}{2} \right) - 1 \right) \left(13 \left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) + 3h(-8\Delta) \right) \pmod{64}.$$

(v) If $\Delta = p = 8t - 1$ or $8t + 3$ a prime then:

$$k_2(4\Delta) \equiv -6h(-\Delta) \left(\left(\frac{-\Delta}{2} \right) - 1 \right) + 16t \pmod{32},$$

$$k_2(4\Delta) \equiv 32\alpha + 2\beta \left(\frac{-\Delta}{2} \right) \left(7 \left(\left(\frac{-\Delta}{2} \right) - 1 \right) h(-\Delta) + 2h(-8\Delta) \right) \pmod{64},$$

$$k_2(8\Delta) \equiv 32\alpha + 2\beta \left(13 \left(1 - \left(\frac{-\Delta}{2} \right) \right) h(-\Delta) + h(-8D) \right) \pmod{64},$$

where $\alpha = 1$ if $p \equiv 7 \pmod{16}$ and $\alpha = 0$ otherwise, and $\beta = -1, 3$, resp. 11 if $p \equiv -1 \pmod{8}$, $p \equiv 3 \pmod{16}$, resp. $p \equiv 11 \pmod{16}$.

Part II:

LEMMA 1 ([5], [1]). We have:

$$T_1 = \begin{cases} \frac{1}{4} \left(\frac{e}{2} \right) h(-4e) + \frac{1}{4} h(-8e), & \text{if } e > 0, \\ \frac{1}{4} \left(5 - \left(\frac{e}{2} \right) \right) h(e) - \frac{1}{4} h(8e) - \lambda(e), & \text{if } e < 0, \end{cases}$$

$$T_2 = \begin{cases} \frac{1}{4} \left(2 - \left(\frac{e}{2} \right) \right) h(-4e) - \frac{1}{4} h(-8e), & \text{if } e > 0, \\ \frac{1}{4} \left(-1 + \left(\frac{e}{2} \right) \right) h(e) + \frac{1}{4} h(8e) + \lambda(e), & \text{if } e < 0, \end{cases}$$

$$T_3 = \begin{cases} \frac{1}{4} \left(-2 - \left(\frac{e}{2} \right) \right) h(-4e) + \frac{1}{4} h(-8e), & \text{if } e > 0, \\ \frac{3}{4} \left(1 - \left(\frac{e}{2} \right) \right) h(e) + \frac{1}{4} h(8e) - \lambda(e), & \text{if } e < 0, \end{cases}$$

$$T_4 = \begin{cases} \frac{1}{4} \left(\frac{e}{2} \right) h(-4e) - \frac{1}{4} h(-8e), & \text{if } e > 0, \\ \frac{3}{4} \left(1 - \left(\frac{e}{2} \right) \right) h(e) - \frac{1}{4} h(8e) - \lambda(e), & \text{if } e < 0, \end{cases}$$

where $\lambda(e) = 1$, if $e = -3$, and $\lambda(e) = 0$, otherwise.

Moreover we have for $k = 5, 6, 7, 8$

$$T_k = \left(\frac{e}{-1} \right) T_{9-k}.$$

LEMMA 2 ([8]). *We have:*

$$S_1 = \begin{cases} \frac{1}{64} k_2(8e) - \frac{1}{64} \left(34 - \left(\frac{e}{2} \right) \right) k_2(e) \\ \quad + \frac{1}{32} e \left(\left(\frac{e}{2} \right) h(-4e) + h(-8e) \right) + 7\omega(e), & \text{if } e > 0, \\ \frac{1}{64} k_2(-8e) + \frac{1}{64} \left(\frac{e}{2} \right) k_2(-4e) \\ \quad - \frac{1}{32} e \left(\left(1 - \left(\frac{e}{2} \right) \right) h(e) - h(8e) \right) - \nu(e), & \text{if } e < 0, \end{cases}$$

$$S_2 = \begin{cases} -\frac{1}{64} k_2(8e) + \frac{3}{64} \left(2 - 3 \left(\frac{e}{2} \right) \right) k_2(e) \\ \quad + \frac{1}{32} e \left(\left(4 - \left(\frac{e}{2} \right) \right) h(-4e) - h(-8e) \right) - 3\omega(e), & \text{if } e > 0, \\ -\frac{1}{64} k_2(-8e) + \frac{1}{64} \left(4 - \left(\frac{e}{2} \right) \right) k_2(-4e) \\ \quad - \frac{1}{32} e \left(5 \left(-1 + \left(\frac{e}{2} \right) \right) h(e) + h(8e) \right) + 5\nu(e), & \text{if } e < 0, \end{cases}$$

$$S_3 = \begin{cases} -\frac{1}{64} k_2(8e) - \frac{3}{64} \left(2 - 3 \left(\frac{e}{2} \right) \right) k_2(e) \\ \quad + \frac{1}{32} e \left(\left(-4 - \left(\frac{e}{2} \right) \right) h(-4e) + 3h(-8e) \right) + 3\omega(e), & \text{if } e > 0, \\ \frac{1}{64} k_2(-8e) - \frac{1}{64} \left(4 + \left(\frac{e}{2} \right) \right) k_2(-4e) \\ \quad - \frac{1}{32} e \left(7 \left(1 - \left(\frac{e}{2} \right) \right) h(e) + 3h(8e) \right) - 7\nu(e), & \text{if } e < 0, \end{cases}$$

$$S_4 = \begin{cases} \frac{1}{64} k_2(8e) - \frac{15}{64} \left(2 - \left(\frac{e}{2} \right) \right) k_2(e) \\ \quad + \frac{1}{32} e \left(3 \left(\frac{e}{2} \right) h(-4e) - 3h(-8e) \right) + 9\omega(e), & \text{if } e > 0, \\ -\frac{1}{64} k_2(-8e) + \frac{1}{64} \left(\frac{e}{2} \right) k_2(-4e) \\ \quad - \frac{1}{32} e \left(13 \left(1 - \left(\frac{e}{2} \right) \right) h(e) - 3h(8e) \right) - 13\nu(e), & \text{if } e < 0, \end{cases}$$

where $\omega(e) = \frac{1}{4}$, if $e = 5$, $\omega(e) = 0$, otherwise, and $\nu(e) = \frac{1}{8}$, if $e = -3$, $\nu(e) = 0$, otherwise.

Moreover we have for $k = 5, 6, 7, 8$

$$S_k = eT_{9-k} - \left(\frac{e}{-1}\right)S_{9-k}.$$