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THE TORSION PROBLEM OF H.-J. REIFFEN AND U. VETTER

Erich Platte

1. Introduction

All rings are considered to be commutative with 1, all modules (and algebras) are unitary. For the theory of finite differential modules we refer the reader to the joint work [12] of G. Scheja and U. Storch, and also to the monography [11].

Let k be a valued field of characteristic zero and let A be a reduced and equidimensional local analytic k-algebra with finite differential module $\mathrm{D}_k(A)$. In 1966 H.-J. Reiffen and U. Vetter posed the problem (see [10], Einleitung, and especially §4, Satz 3 and Satz 4): Does always (with $k = \mathbb{C}$) the minimal number of generators of the finite differential module $\mathrm{D}_k(A)$ of A *not decrease*, after its torsion $\mathrm{t}(\mathrm{D}_k(A))$ has been divided out? By the Lemma of Nakayama this is equivalent to saying $\mathrm{t}(\mathrm{D}_k(A)) \subseteq \mathfrak{m}_A \mathrm{D}_k(A)$, and they proved this inclusion for hypersurfaces, see [10, Satz 4]. G. Scheja has given this pure algebraic description $\mathrm{t}(\mathrm{D}_k(A)) \subseteq \mathfrak{m}_A \mathrm{D}_k(A)$ of the torsion problem of H.-J. Reiffen and U. Vetter, see [11, p. 157], and also D. Denneberg in [1] is referring to this problem, when stating that there are not known any counterexamples, and additionally listing the condition $\mathrm{t}(\mathrm{D}_k(A)) \subseteq \mathfrak{m}_A \mathrm{D}_k(A)$ in the hypothesis of his Theorem (1.3) in [1], see also the following remarks there. The problem has a positive answer in the following cases: (a) for hypersurfaces, see [10, Satz 4] and [11, p. 157 and Satz (9.11)], (b) for the homogeneous case, see [11, p. 157], and also (c) for invariants of regular rings by finite groups, see [5, p. 9], and [7, Proposition (3.1), part 2]. The last case (c) of invariant rings can be used to improve the necessary condition given by K.-I. Watanabe and V. Kac in [3,15] for an invariant ring to be a complete intersection, see second part of §8 in [8]. In this paper we will show that the torsion problem, in general, has a negative answer.

2. The construction of a class of counterexamples

(2.1) Lemma: *Let k be a valued algebraically closed field and let μ, $\nu \geq 2$ be arbitrary natural numbers which are coprime, If f resp. g are any* *functions in the one-dimensional convergent power series ring $B = k\langle X\rangle$ of value μ resp. of value ν and if μ or ν are not divisible by the characteristic of the field k, then*

(i) *$B = k\langle X\rangle$ is the normalization of the hypersurface $k\langle f, g\rangle$.*

(ii) *The conductor from $B = k\langle X\rangle$ to $k\langle f, g\rangle$ has the value*

$$\mu \cdot \nu - \mu - \nu + 1 = (\mu - 1)\cdot(\nu - 1).$$

PROOF: We may assume $\mu \neq 0$ in k, and that the function f has the form $f = X^{\mu}$ with a suitable regular parameter X of B. Then the extension $P := k\langle X^{\mu}\rangle \to k\langle X\rangle = B$ is galois. If ζ is a primitive μ^{th}-root of unity in k and if $\tau \in \operatorname{Aut}_k(B)$ is defined by

$$\tau(X) = \zeta \cdot X,$$

then the cyclic group $\mathbb{Z}_{\mu} = \{\tau^0, \tau^1, \ldots, \tau^{\mu-1}\}$ of order μ acts on $B = k\langle X\rangle$ with the invariant ring $P = k\langle X^{\mu}\rangle$.

To prove assertion (i), we write g as a power series in X:

$$g = g_{\nu} X^{\nu} + g_{\nu+1} X^{\nu+1} + g_{\nu+2} X^{\nu+2} + \ldots +$$

with coefficients $g_i \in k$, $g_{\nu} \neq 0$, and we have to show that the function g has maximal degree μ over $P = k\langle X^{\mu}\rangle$: If τ^j is any automorphism of the Galois group $\mathbb{Z}_{\mu}$ with

$$\tau^j(g) = g,$$

then

$$g_{\nu} = g_{\nu} \cdot \zeta^{j\cdot\nu} \quad \text{with } 0 \leq j < \mu.$$

Therefore the number $j \cdot \nu$ is a multiple of μ, which is only possible, if $j = 0$, since μ and ν are coprime and $\mu > j$. Thus the identity $\tau^0 \in \mathbb{Z}_{\mu}$ is the only automorphism in $\mathbb{Z}_{\mu}$ which leaves the function g fixed. Therefore assertion (i) is clear.

To prove assertion (ii), let

$$\mathrm{G} = Z^{\mu} + a_1 Z^{\mu-1} + a_2 Z^{\mu-2} + \ldots + a_{\mu-1} Z + a_{\mu}$$

be the minimal polynomial of g over $P = k\langle X^{\mu}\rangle$ with uniquely determined coefficients $a_i \in k\langle X^{\mu}\rangle$. The polynomial

$$\prod_{\sigma \in \mathbb{Z}_{\mu}} [Z - \sigma(g)]$$

has leading coefficient 1, is of degree μ, has invariant coefficients, and has root g. Thus we necessarily have

$$\prod_{i=0}^{\mu-1}\left[Z-\tau^{i}(g)\right]=\mathrm{G}=Z^{\mu}+a_1Z^{\mu-1}+a_2Z^{\mu-2}+\ldots+a_{\mu-1}Z+a_{\mu}$$

Multiplying out the product we see (by induction) that the coefficients a_i, $1\leq i\leq\mu$, are of the following form: a_i is a certain sum of certain products of elements in $\{\tau^0(g),\ldots,\tau^{\mu-1}(g)\}$, where each such product has exactly i factors. Since $\mathbb{Z}_{\mu}$ acts linearly, and since g has value ν, the value of a_i (if $a_i\neq 0$) is therefore $\geq i\cdot\nu$, and we assert that for $1\leq i<\mu$ the value of a_i is *strictly greater* than $i\cdot\nu$ (this means: in the sum which defines the a_i, the sum of the terms of lowest degree vanishes). Let us assume $a_i\neq 0$ and that the value of a_i is equal to $i\cdot\nu$ for some i with $1\leq i<\mu$. Because of $a_i\in\mathfrak{m}_P\subseteq P=k\langle X^{\mu}\rangle$ there exists a natural number $l\geq 1$ with

$$l\cdot\mu=i\cdot\nu\quad\text{for some } i \text{ with } 1\leq i<\mu,$$

which, again, is impossible, since μ and ν are coprime and $\mu>i\geq 1$. Now we can compute the value of

$$\mathrm{G}'(g)=\mu\cdot g^{\mu-1}+(\mu-1)\cdot a_1g^{\mu-2}+(\mu-2)\cdot a_2g^{\mu-3}+\ldots+a_{\mu-1}.$$

By what we have seen above the value of $a_i\cdot g^{\mu-i-1}$ (in case $a_i\neq 0$) is strictly greater than $i\cdot\nu+(\mu-i-1)\cdot\nu=\mu\cdot\nu-\nu$, and the value of $\mu g^{\mu-1}$ is exactly $(\mu-1)\cdot\nu=\mu\cdot\nu-\nu$. Thus the value of $\mathrm{G}'(g)$ is exactly $\mu\cdot\nu-\nu$. Now the module of regular differential forms $\mathrm{R}^1_k(H)$ of $H=k\langle f,\ g\rangle$ in the sense of [4, §4] has the form

$$\mathrm{R}^1_k(H)=\frac{H}{\mathrm{G}'(g)}d(X^{\mu})=H\cdot\frac{\mu\cdot X^{\mu-1}}{\mathrm{G}'(g)}dX$$

and therefore, by a classical result in [14, p. 80] concerning singular curves, the conductor of H has the value

$$-\left[(\mu-1)-(\mu\cdot\nu-\nu)\right]=\mu\cdot\nu-\mu-\nu+1=(\mu-1)\cdot(\nu-1),$$

which proves assertion (ii). - The last conclusion follows also from [6, Satz 2, part 3] in any dimension.

We now prove:

(2.2) PROPOSITION (*Negative answer to the torsion problem of H.-J. Reiffen and U. Vetter*, 1966): *Let k be a valued algebraically closed field of characteristic zero and let $n\geq 1$ and $\mu\geq 3$ be arbitrary natural numbers.*

Then there exist local analytic Maculay domains A over k of Krull dimension n, of multiplicity μ and of embedding dimension $n+2$, such that

$$\mathrm{t}(\mathrm{D}_k(A)) \not\subseteq \mathfrak{m}_A \mathrm{D}_k(A).$$

Moreover, A can be taken a finite subring of a convergent power series ring over k in n variables.

PROOF: Let us first assume that for $n = \dim A = 1$ the counterexamples A already have been constructed. Let $B = k\langle X\rangle$ be the finite normalization of A. Then $\tilde{A} := A\langle Y_4, \ldots, Y_{n+2}\rangle \subseteq B\langle Y_4, \ldots, Y_{n+2}\rangle =: \tilde{B}$ has Krull dimension n, the same multiplicity μ and embedding dimension $n+2$. Furthermore, if x_1, x_2, x_3 minimally generate the maximal ideal $\mathfrak{m}_A$ of A, and hence $\mathrm{d}x_1$, $\mathrm{d}x_2$, $\mathrm{d}x_3$ minimally generate $\mathrm{D}_k(A)$ such that

$$\mathrm{d}x_1 - \alpha_2 \mathrm{d}x_2 - \alpha_3 \mathrm{d}x_3$$

is a torsion element in $\mathrm{D}_k(A)$ for suitable α_2, $\alpha_3 \in A$, then the image of this differential in $\mathrm{D}_k(\tilde{A})$ is also a torsion element, with $\mathrm{d}x_1$, $\mathrm{d}x_2$, $\mathrm{d}x_3$, $\mathrm{d}Y_4, \ldots, \mathrm{d}Y_{n+2}$ being a minimal system of generators of $\mathrm{D}_k(\tilde{A})$. So the torsion problem has a negative answer for $\tilde{A}$, too.

Thus we may restrict ourselves to the case $n = \dim A = 1$. Let again $B = k\langle X\rangle$ be the convergent power series ring in one variable over k. *For any fixed multiplicity number $\mu \geq 3$ we consider all pairs of natural numbers*

$$(\mu, \nu),$$

where $\nu > \mu$ is coprime to μ, except the two pairs

$$(\mu, \nu) = (3, 4) \quad \textit{and} \quad (\mu, \nu) = (3, 5).$$

Then there exists a correction number $\kappa > 0$(!) with

$$\mu + \nu + \kappa = \mu \cdot \nu - \mu - \nu.$$

(Observe that for $\mu = 2$ the correction number $\kappa > 0$ does not exist.). Let $h \in k\langle X\rangle = B$ be *any* power series of value $\nu + \kappa$ and let

$$g := X^\nu + h \quad (\text{for example: } h = r \cdot X^{\nu+\kappa},\ r \in k,\ r \neq 0),$$

and, finally, the hypersurface

$$H := k\langle X^\mu, g\rangle = k\langle X^\mu, X^\nu + h\rangle.$$

By (2.1) the conductor of H has the value

$$\mathrm{c} := \mu \cdot \nu - \mu + 1 = \mu + \nu + \kappa + 1.$$

We adjoin the *conductor less one* X^{c-1} and have a *strict algebra extension*

$$\varphi\colon H = k\langle X^{\mu},\ X^{\nu}+h\rangle$$
$$\to A := H[X^{c-1}] = k\langle X^{\mu},\ X^{\nu}+h,\ X^{\mu\cdot\nu-\mu-\nu}\rangle.$$

Because of (2.1) an element of the conductor $X^{c}\cdot B$ cannot be part of a minimal system of generators of $\mathfrak{m}_H$, i.e. $X^{c}\cdot B \subseteq \mathfrak{m}_H^2$. Therefore the embedding dimension of A is exactly 3, as can easily be proved.

With these notations we state and prove the following surprising

(2.3) Very phenomenon

The canonical induced homomorphism

$$\overline{\mathrm{D}\varphi}\colon \mathrm{D}_k(H)/\mathrm{t}(\mathrm{D}_k(H)) \xrightarrow{\sim} \mathrm{D}_k(A)/\mathrm{t}(\mathrm{D}_k(A))$$

is an isomorphism.

Proof: Observe that, since $\mathrm{D}_k(B)$ is (torsion-)free, the kernels of the canonical homomorphisms

$$\mathrm{D}_k(H) \to \mathrm{D}_k(B) \quad \text{and} \quad \mathrm{D}_k(A) \to \mathrm{D}_k(B)$$

are exactly the torsion submodules of $\mathrm{D}_k(H)$ and $\mathrm{D}_k(A)$. Let $h = \sum_{\iota=\nu+\kappa}^{\infty} h_\iota X^\iota$, with $h_\iota \in k$, $h_{\nu+\kappa} \neq 0$. By (2.1) the hypersurface

$$k\langle X^{\mu},\ X^{\nu} + \sum_{\iota=\nu+\kappa}^{c-1} h_\iota X^\iota\rangle$$

is equal to $H = k\langle X^{\mu},\ X^{\nu}+h\rangle$, since they have both the same conductor $X^{c}\cdot B$, and thus both contain the power series $\sum_{i=c}^{\infty} h_i X^\iota$. Therefore we may assume that $h = \sum_{\iota=\nu+\kappa}^{c-1} h_\iota X^\iota$, i.e. a polynomial. The following differential

$$\mu\cdot X^{\mu}\mathrm{d}(X^{\nu}+h) - \nu\cdot(X^{\nu}+h)\mathrm{d}(X^{\mu}) \in \mathrm{D}_k(H)$$

which is defined in $\mathrm{D}_k(H)$, is evidently of value $>\mu+\nu$, and has in $\mathrm{D}_k(B)$ the form

$$\mathrm{d}\left(\sum_{i=\nu+\kappa}^{c-1} \frac{\mu}{\mu+i}\cdot(i-\nu)\cdot h_\iota X^{\mu+\iota}\right) \quad \text{in } \mathrm{D}_k(B)$$

with a polynomial of value exactly $\mu+\nu+\kappa = c-1$, because $\kappa > 0$ and

$h_{\nu+\kappa} \neq 0$. Since the polynomial

$$\sum_{i=\nu+\kappa+1}^{c-1} \frac{\mu}{\mu+i} \cdot (i-\nu) \cdot h_i X^{\mu+i}$$

has value $\geq c$, it is an element in H, and hence its differential has a preimage in $D_k(H)$. Therefore the differential

$$\begin{aligned} d(X^{\mu+\nu+\kappa}) &= d(X^{\mu\cdot\nu-\mu-\nu}) \\ &= d(X^{c-1}) \in D_k(A)/t(D_k(A))[\subseteq D_k(B)] \end{aligned}$$

has a preimage in $D_k(H)$, too. Because of $X^{c-1} \cdot \mathfrak{m}_B \subseteq \mathfrak{m}_H$ we have a decomposition

$$A = k \oplus \mathfrak{m}_H \oplus k \cdot X^{c-1}.$$

Therefore *every* differential $u\mathrm{d}v \in D_k(A)/t(D_k(A))$, with u, $v \in A$, has a preimage in $D_k(H)$: the case $u \in H$ and $v = X^{c-1}$ follows from our construction; if $u = v = X^{c-1}$, then $X^{c-1}d(X^{c-1}) = 2^{-1}d(X^{2c-2})$ has a preimage in $D_k(H)$, and if $u = X^{c-1}$ and $v \in \mathfrak{m}_H$, then by the product rule we have

$$u\mathrm{d}v = d(v \cdot X^{c-1}) - v \cdot d(X^{c-1}) \quad \text{in} \quad D_k(A)/t(D_k(A)),$$

with $v \cdot X^{c-1} \in H$, and therefore the differential $u\mathrm{d}v = X^{c-1}\mathrm{d}v$ has a preimage in $D_k(H)$, too. Thus Proposition (2.2) together with the very phenomenon (2.3) has been proved completely.

(2.4) Concrete examples

Let the tripel (μ, ν, κ) be the same as in the proof of (2.2) and (2.3) and let $r \neq 0$ be an arbitrary fixed constant in k with $h := r \cdot X^{\nu+\kappa}$, and

$$A = k\langle X^\mu,\ X^\nu + r \cdot X^{\nu+\kappa},\ X^{\mu\cdot\nu-\mu-\nu}\rangle \supsetneq k\langle X^\mu,\ X^\nu + r \cdot X^{\nu+\kappa}\rangle = H.$$

Then the following differential, which is defined in $D_k(A)$,

$$d(X^{\mu\cdot\nu-\mu-\nu}) + \frac{\mu+\nu+\kappa}{r\cdot\kappa} \cdot \frac{\nu}{\mu} \cdot (X^\nu + r \cdot X^{\nu+\kappa}) \cdot d(X^\mu)$$

$$-\frac{\mu+\nu+\kappa}{r\cdot\kappa} \cdot X^\mu d(X^\nu + r \cdot X^{\nu+\kappa})$$

is a torsion element in $D_k(A)$. In the sense of our construction the smallest possible counterexample to the torsion problem is given by

$(\mu, \nu, \kappa) = (3, 7, 1)$. If $r := -2$, then $A = k\langle X^3, X^7 - 2X^8, X^{11}\rangle$, and a torsion element in $\mathrm{D}_k(A)$ is given by:

$$\mathrm{d}(X^{11}) = \tfrac{77}{6}(X^7 - 2 \cdot X^8)\mathrm{d}(X^3) + \tfrac{11}{2}X^3\mathrm{d}(X^7 - 2 \cdot X^8),$$

and for multiplicity $\mu = 4$ an example is given by $(\mu, \nu, \kappa) = (4, 5, 2)$, $r := -\frac{1}{2}$, $A = k\langle X^4, X^5 - \frac{1}{2}X^7, X^{11}\rangle$, with torsion element in $\mathrm{D}_k(A)$:

$$\mathrm{d}(X^{11}) - \tfrac{55}{4}\left(X^5 - \tfrac{1}{2} \cdot X^7\right)\mathrm{d}(X^4) + 11X^4\mathrm{d}\left(X^5 - \tfrac{1}{2} \cdot X^7\right).$$

(2.5) Remark: The notations are again the same as in (2.2) and (2.3) with the tripel (μ, ν, κ) of natural numbers and with the power series h of value $\nu + \kappa$, and let

$$\varphi\colon H = k\langle X^\mu, X^\nu + h\rangle \to A = k\langle X^\mu, X^\nu + h, X^{\mu\cdot\nu-\mu-\nu}\rangle$$

be the canonical extension. Let

$$\rho\colon k\langle U, V, Z\rangle \to A$$

be a surjective k-algebra homomorphism with

$$U \mapsto x_1 := X^\mu, \quad V \mapsto x_2 := X^\nu + h, \quad Z \mapsto X^{c-1}$$

and $\rho(k\langle U, V\rangle) = H$. Let $\mathrm{G}(U, V) \in k\langle U, V\rangle$ be sent to the minimal polynomial $\mathrm{G} = \mathrm{G}(V)$ of $X^\nu + h$ over $P = k\langle X^\mu\rangle$, i.e. $\mathrm{G}(X^\mu, V) = \mathrm{G}$. Then $H = k\langle U, V\rangle/\mathrm{G}(U, V)\cdot k\langle U, V\rangle$. Thus a system of generators of the kernel of

$$\rho\colon k\langle U, V\rangle\langle Z\rangle \to A$$

is given by $\mathrm{G}(U, V)$ *plus* the preimages of a system of generators of the kernel of the canonical homomorphism

$$\tilde{\rho}\colon H[Z] \to A$$

with $Z \mapsto X^{c-1}$. The three canonical relations

$$Z^2 - X^{2c-2}, \quad x_1 Z - x_1 \cdot X^{c-1}, \quad x_2 Z - x_2 \cdot X^{c-1}$$

in Kern $\tilde{\rho} \subseteq H[Z]$ generate the kernel of $\tilde{\rho}$: This is a consequence of the remainder theorem (division by $Z^2 - X^{2c-2}$) and the fact that for a linear relation

$$\alpha Z - \beta, \quad \alpha, \beta \in H,$$

α cannot be a unit (since $X^{c-1} \notin H$), i.e. α is a linear combination of x_1,

x_2, and of the fact that constant relations $\neq 0$ in H cannot occur, since the extension $H \to H[X^{c-1}] = A$ is injective. Thus the kernel of the surjective k-algebra homomorphism ρ: $k\langle U, V, Z\rangle \to A$ is generated by *four* elements (the preimages of the minimal polynomial G and the three canonical relations).

As remarked in the Introduction, H.-J. Reiffen and U. Vetter proved the conjecture for hypersurfaces in [10, Staz 4]. But G. Scheja in [11, p. 157] weakens the torsion problem by the following question: Is the torsion submodule of $\mathrm{D}_k(A)$ never (say in characteristic zero) a direct A-summand of $\mathrm{D}_k(A)$, except, of course, it is zero? - and he proves in [11, (9.10), (9.11)] that this problem *related to hypersurfaces* is not really weaker, and we believe that this conjecture is the correct one. If it could be proved, it would have interesting applications in differential calculus in algebra, for instance a proof of the purity of branch locus theorem for complete intersections of A. Grothendieck in [2]: Let C be normal (and smooth in codimension ≤ 1) and a complete intersection, and if $C \to A$ is *any* normal, say Macaulay, (module) finite algebra extension which is unramified in codimension ≤ 2, then

$$\mathrm{D}_C(A) \simeq \mathrm{t}(\mathrm{D}_k(A)) \subseteq \mathrm{D}_k(A)$$

is a direct A-summand of $\mathrm{D}_k(A)$, see [8, Satz (6.1)]. Therefore, if the weakened torsion problem of G. Scheja is true, it follows $\mathrm{D}_C(A) = 0$, and - as desired and proved by A. Grothendieck - the extension $C \to A$ is unramified everywhere, and hence, in general, $C = A$. Unfortunately, a proof of the weakened torsion conjecture seems to be methodically remote.

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