RUISHI KUWABARA

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Compositio Mathematica, tome 50, nº 1 (1983), p. 93-94
<http://www.numdam.org/item?id=CM_1983__50_1_93_0>

Compositio Mathematica 50 (1983) 93–94

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Ruishi Kuwabara

The proof of Lemma 3.3: (1) given in the paper in Vol. 47 [p. 201] is incorrect. We here give a complete proof of the lemma.

Define a differential operator δ_g^k: $S_{k+1} \to S_k$ by

$$\left(\delta_g^k a\right)^{i_1 \ldots i_k} = -(k+1)\nabla_\rho a^{\rho i_1 \ldots i_k},$$

∇ being the covariant differentiation with respect to g. Then, δ_g^k is the formally adjoint operator of $\hat{\nabla}_g^k$ with respect to the inner products in S_k's naturally defined by g. Set

$$D_g^k = \frac{1}{k+1}\delta_g^k \hat{\nabla}_g^k .$$

Then D_g^k is a non-negative, self-adjoint, elliptic differential operator of order 2, and the equation $D_g^k a = 0$ is equivalent to $\hat{\nabla}_g^k a = 0$ (see [2]).

Next, let us introduce various norms on the space of tensor fields on M. A (fixed) C^∞ Riemannian metric g_0 naturally defines a norm, $|\cdot|$, on each fibre of the tensor bundle over M. Various global norms for a tensor field T are defined by

$$|T|_k = \max_{0 \leqslant r \leqslant k} \sup_{x \in M} \Big\{ |\underbrace{\nabla \ldots \nabla}_{r} T(x)| \Big\},$$

$$\|T\|_k^2 = \sum_{r=0}^{k} \left(\int_M |\underbrace{\nabla \ldots \nabla}_{r} T|^2 dV_{g_0} \right),$$

for $k = 0, 1, 2, \ldots$, where ∇ is the covariant differentiation with respect to g_0.

Using these notations, we have for every $a \in S_k$,

$$\|D_g^k a - D_{g_0}^k a\|_0 \leqslant C_1 |g - g_0|_1 \|a\|_2 \qquad (\text{when } |g - g_0|_1 < 1), \tag{1}$$

C_1 being a constant, because D_g^k is a second order differential operator whose coefficients consist of g and its first derivatives. On the other hand, since $D^k_{g_0}$ is an elliptic operator of order 2, there is a constant C_2 such that

$$\|a\|_2 \leqslant C_2\left(\|a\|_0 + \|D^k_{g_0}a\|_0\right), \tag{2}$$

for every $a \in S_k$.

Now we prove that $\mathfrak{N}_k = \{g \in \mathfrak{R};\ (D^k_g)^{-1}(0) = \{0\}\}$ is an open subset of $\mathfrak{R}$. Suppose g_0 belongs to $\mathfrak{N}_k$. Noting that D^k_g has a discrete spectrum consisting of non-negative real eigenvalues, we have

$$\|D^k_{g_0}a\|_0 \geqslant \lambda\|a\|_0 \quad (\lambda > 0), \tag{3}$$

for every $a(\neq 0) \in S_k$, where λ is the least eigenvalue. We show g_0 is an interior point of $\mathfrak{N}_k$. If the contrary holds, there are sequences $\{g_n\}_{n=1}^\infty$ in $\mathfrak{R}$ and $\{a_n\}_{n=1}^\infty$ in S_k such that $D^k_{g_n}a_n = 0$, $\|a_n\|_0 = 1$, and $g_n \to g_0$ with respect to the C^∞ topology (i.e. $|g_n - g_0|_k \to 0$ for every $k \geqslant 0$) as $n \to \infty$. Using (1) and (2), we have

$$\begin{aligned}\|D^k_{g_0}a_n\|_0 &= \|D^k_{g_0}a_n - D^k_{g_n}a_n\|_0 \leqslant C_1|g_0 - g_n|_1\|a_n\|_2\\ &\leqslant C_1C_2|g_0 - g_n|_1\left(\|a_n\|_0 + \|D^k_{g_0}a_n\|_0\right)\\ &= C_1C_2|g_0 - g_n|_1\left(1 + \|D^k_{g_0}a_n\|_0\right).\end{aligned}$$

Hence, for sufficiently large n,

$$\|D^k_{g_0}a_n\|_0 \leqslant \frac{C_1C_2|g_0 - g_n|_1}{1 - C_1C_2|g_0 - g_n|_1}.$$

Therefore, we get $\|D^k_{g_0}a_n\|_0 \to 0$ as $n \to \infty$. This contradicts (3).

References

[1] R. Kuwabara: On isospectral deformations of Riemannian metrics. II. *Comp. Math. 47* (1982) 195–205.

[2] C. Barbance: Sur les tenseurs symétriques. *C.R. Acad. Sc. Paris 276* (1973) 387–389.

(Oblatum 21-XII-1982)

Department of Mathematics
College of General Education
The University of Tokushima
Minami-Josanjima-cho
Tokushima 770, Japan