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THE DIMENSIONS OF COMPLEMENTED HILBERTIAN SUBSPACES OF UNIFORMLY CONVEX BANACH LATTICES

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For X a given Banach space Dvoretzky's Theorem [1] implies that every finite dimensional $E \subset X$ contains a Hilbertian subspace F . In this paper we are interested in spaces X for which the F 's can always be chosen to be uniformly complemented in X , and especially in obtaining estimates for $\dim F$ in terms of $\dim E$. It is clearly necessary to suppose that X doesn't contain ℓ_1^n 's uniformly. For Banach lattices X Johnson and Tzafriri [8] have shown that the last condition is also sufficient. The novelty of the results presented here is the estimates for $\dim F$ in terms of $\dim E$, which are quite sharp. The main technique used in the proofs is the version of Dvoretzky's Theorem proven by Figiel, Lindenstrauss and Milman in [2]; for properly chosen ellipsoids the Levy means involved there can be estimated using the properties of p -summing operators defined on the space X .

This paper was submitted in another place in 1978, and so has been delayed in appearing. Since that time the results presented here have been considerably strengthened: Figiel and Tomczak-Jaegermann [21] extend these results to uniformly convex and k -convex spaces; Benyamini and Gordon [20] consider random factorizations of maps more general than the identity on ℓ_2^n ; Pisier's theorem [22] a space not containing ℓ_1^n 's must be k -convex shows all the results mentioned carry over to B -convex spaces.

The notation and terminology used here is for the most part standard. We only recall the definitions used in the statements of theorems.

A Banach lattice L is q -concave if there is a constant $A > 0$ with

$$\left[\sum_{i \leq n} \|x_i\|^q \right]^{1/q} \leq A \left\| \left[\sum_{i \leq n} |x_i|^q \right]^{1/q} \right\|$$

for all $x_1, x_2, \dots, x_n \in L$. Similarly L is p -convex if there is a constant

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$B > 0$ with

$$\left\| \left[\sum_{i \leq n} |x_i|^p \right]^{1/p} \right\| \leq B \left[\sum_{i \leq n} \|x_i\|^p \right]^{1/p}$$

for all $x_1, x_2, \dots, x_n \in L$. In each case we write $K_q(L)$ and $K^p(L)$ for the best constants A and B appearing in the inequalities. The basic facts about q -concave and p -convex lattices may be found in [9], [13] and [12].

For $2 \leq s < \infty$ and E a given space we take the s -cotype constant of E , $\alpha_s(E)$, to be the smallest $\alpha > 0$ such that

$$\left[\sum_{i \leq n} \|x_i\|^s \right]^{1/s} \leq \alpha \left[\int_0^1 \left\| \sum_{i \leq n} r_i(t) x_i \right\|^s dt \right]^{1/s}$$

for any $x_1, x_2, \dots, x_n \in E$. Here $r_1, r_2, \dots, r_n, \dots$ are the Rademacher functions on $[0, 1]$. It is obvious that $\alpha_s(\ell_s^N) = 1$, $2 \leq s < \infty$.

The Banach-Mazur distance between isomorphic spaces E and F is

$$d(E, F) = \inf\{\|u\| \|u^{-1}\| : u: E \rightarrow F \text{ an isomorphism}\}.$$

Let X be a fixed space and $\lambda \geq 1$. For $E \subset X$ a finite dimensional subspace we define $c_\lambda(E)$ to be the maximum of the dimensions of those $F \subset E$ for which

- (i) $d(F, \ell_2^{\dim F}) \leq 2$, and
- (ii) there is a projection of X onto F of norm at most λ .

In the terminology of Pelczynski and Rosenthal [16] X is called *locally π -Euclidean* if there is a constant $\lambda \geq 1$ and a function f on the natural numbers such that $c_\lambda(E) \geq n$ whenever $\dim E \geq f(n)$.

Finally for $1 < p < \infty$, p' is the conjugate of p ($1/p + 1/p' = 1$).

THEOREM 1: *Let X be a space which is a subspace of quotient of a p -convex and q -concave Banach lattice L , $1 < p \leq 2 \leq q < \infty$. There is a $\lambda \geq 1$ so that, for $E \subset X$ any n dimensional subspace and $s \in [2, q]$,*

$$c_\lambda(E) \geq \lambda^{-1} \min\{n^{2/p'}, \alpha_s(E)^{-2} n^{2/s}\}.$$

Before giving the proof we point out some instances of the theorem.

(a) The hypothesis of the theorem implies that X is cotype q [13], and so for some constant $a > 0$ and $\alpha = \min(2/p', 2/q)$,

$$c_\lambda(E) \geq a(\dim E)^\alpha \quad (*)$$

whenever $X \subset E$ is finite dimensional. In particular X is locally π -Euclidean. This result is also stated in [8], though no estimate for $c_\lambda(E)$ is given.

(b) The lattice $L = L_p(\mu)$ is both p -convex and p -concave, $1 < p < \infty$, and consequently $(*)$ holds with $\alpha = \min(2/p', 2/p)$. For $2 \leq p < \infty$ this is well-known, and follows from the results of [2] and [14]. Taking $E = \ell_p^n$ and using the results of [2] shows that this lower bound for $c_\lambda(E)$ is best possible for L an L_p -space.

(c) In case $2 \leq s \leq q$ and $d(E, \ell_s^n) \leq n^{1/s-1/p'}$

$$\alpha_s(E)^{-2} n^{2/s} \geq d(E, \ell_s^n)^{-2} n^{2/s} \geq n^{2/p'},$$

and hence $(*)$ is true with $\alpha = 2/p'$. For E a 2-isomorph of ℓ_s^n , $2 \leq s \leq \min(p', q)$, this gives a lower bound for $c_\lambda(E)$ depending only on the convexity of L . For Hilbertian subspaces of $L_p(\mu)$ -spaces $1 < p < 2$ this lower estimate cannot be improved; by [2] ℓ_p^n , $1 < p < 2$, contains a Hilbert subspace of dimension $c_1 n$, but no complemented Hilbert subspaces of dimension greater than $c_2 n^{2/p'}$. For L a p -convex lattice ($1 < p \leq 2$) with some non-trivial concavity, every n dimensional Hilbert subspace is $c_3 n^{1/p-1/2}$ -complemented [11].

Below X , L , p and q have the same meaning as in the statement of Theorem 1. The proof is preceded by three short lemmas, the first mentioned by Pisier in [18].

The lattice structure enters into the proof only through Lemma 1.

LEMMA 1: If $u: X \rightarrow G$ is q' -integral then u' is p' -summing and

$$\pi_{p'}(u') \leq K^p(L) K_q(L) i_{q'}(u).$$

PROOF: It is enough to show that for $u: L \rightarrow G$ q' -summing,

$$i'_p(u') \leq K_p(L) K^q(L) \pi'_q(u).$$

By Proposition 3.1 of [11] u' maps the closed unit ball of G' into an order bounded set and

$$\left\| \sup_{\|x'\| \leq 1} |u'(x')| \right\| \leq K_q(L) \pi'_q(u).$$

Since L' is p' -concave with $K'_p(L') = K^p(L)$ [9], the same proposition gives

$$i_{p'}(u') \leq K^p(L) \left\| \sup_{\|x'\| \leq 1} |u'(x')| \right\|.$$

LEMMA 2: If $2 \leq s < \infty$, H is an n dimensional Hilbert space and $u: H \rightarrow G$ any map, there is a subspace $A \subset H$ with $\dim A \geq n/2$ and

$$\|u|_A\| \leq (2/n)^{1/s} \pi_{(2,s)}(u).$$

PROOF: If the conclusion fails inductively choose m vectors $x_1, x_2, \dots, x_m \in H$ to satisfy

$$\|x_i\| = 1, \quad \|u(x_i)\| > (2/n)^{1/s} \pi_{(2,s)}(u)$$

and

$$x_k \in [x_1, x_2, \dots, x_{k-1}]^\perp, \quad 1 \leq k \leq m.$$

It is clearly possible to choose $m = [n/2 + 1] \geq n/2$ such vectors. But since the x_i 's are orthonormal

$$\pi_{(2,s)}(u) \geq \left[\sum_{i \leq m} \|u(x_i)\|^s \right]^{1/s} > m^{1/s} (2/n)^{1/s} \pi_{(2,s)}(u),$$

a contradiction.

For G a finite dimensional space and $\|\cdot\|_2$ a Hilbertian norm on G , G_2 denotes G under $\|\cdot\|_2$.

LEMMA 3: Let $E \subset X$ be any n dimensional subspace. There is a Hilbertian norm $\|\cdot\|_2$ on E and an operator $v: X \rightarrow E_2$ such that, if $u: E_2 \rightarrow X$ is the formal inclusion, then $vu = 1_E$ and $\pi_q(u) = i'_q(v) = n^{1/2}$.

PROOF: By Theorem 1.1 of [10] there is an isomorphism $w: \mathcal{L}_2^n \rightarrow E$ so that $\pi_q(w) = 1$ and $i'_q(w^{-1}) = n$. Define $\|\cdot\|_2$ on E by $\|x\|_2 = n^{-1/2} \|w^{-1}(x)\|$. Clearly $\pi_q(u) = i'_q(u^{-1}) = n^{1/2}$. For v take any map $v: X \rightarrow E_2$ with $v|E = u^{-1}$ and $i'_q(v) = i'_q(u^{-1})$ (such an extension exists by the defining factorization of q' -integral maps).

PROOF OF THEOREM 1: Given $E \subset X$ of dimension n , choose $\|\cdot\|_2$, u and v as in Lemma 3. We claim there is a constant $a > 0$ (depending only on q and L) and a subspace $B \subset E$ with $\dim B \geq n/4$ such that, if $u_1 = u|B_2$ and v_1 is v followed by the orthogonal projection of E_2 onto B_2 , then

$$v_1 u_1 = 1_B, \tag{1}$$

$$\pi_q(u_1) \leq n^{1/2}, \tag{2}$$

$$\pi_{p'}(v'_1) \leq a n^{1/2}, \tag{3}$$

$$\|u_1\| \leq a \alpha_s(E) n^{1/2-1/s}, \tag{4}$$

and

$$\|v_1\| \leq a n^{1/p-1/2}. \tag{5}$$

Trivially, $\pi_{(2,q)}(u) \leq \pi_q(u)$. For $2 \leq s < q$ the proof of a result of Maurey ([15], Proposition 74, p. 90) implies

$$\pi_{(2,s)}(u) \leq c(q) \alpha_s(E) \pi_q(u) \leq c(q) \alpha_s(E) n^{1/2},$$

where $c(q)$ is Khintchin's constant. By Lemma 2 there is an $A \subset E$ with $\dim A \geq n/2$ and

$$\|u|_A\| \leq (2/n)^{1/s} \pi_{(2,s)}(u) \leq c_2 \alpha_s(E) n^{1/2-1/s}.$$

By Lemma 1

$$\pi_{(2,p')}(v') \leq \pi_{p'}(v') \leq c_L i_{q'}(v) = C_L n^{1/2}$$

and thus, again by Lemma 2, there is a $B \subset E$ with $\dim B \geq (\dim A)/2 \geq n/4$ and

$$\|v'|_B\| \leq (4/n)^{1/p'} \pi_{(2,p')}(v') \leq c_4 n^{1/p-1/2}.$$

Properties (1), (2) and (3) follow immediately from the corresponding properties of u and v (and Lemma 1).

The remainder of the proof now follows using the results and techniques of [2]. Let $S \subset B_2$ be the unit sphere $\| \cdot \|_2 = 1$ and dm be the normalized, rotational invariant measure on S . Recall that the Levy mean of a continuous real valued function f on S is the number M_f such that

$$m(x \in S: f(x) \geq M_f) = m(x \in S: f(x) \leq M_f).$$

Let M be the Levy mean of $x \rightarrow \|u_1(x)\| = \|x\|$ on S and $M^\#$ be the Levy mean of $x \rightarrow \|v'_1(x)\|$ on S (of course $B'_2 = B_2$ naturally). Equality (1) implies that for $x \in S$,

$$1 = \langle u_1(x), v'_1(x) \rangle \leq \|u_1(x)\| \|v'_1(x)\|,$$

and consequently

$$1 \leq MM^\#. \quad (6)$$

We now claim that there is a constant $b > 0$, depending only on p, q and L , such that

$$M \leq b \quad \text{and} \quad M^\# \leq b. \quad (7)$$

To prove the first let $a(q)$ be the constant satisfying

$$\|z\|_2 = a(q) \left[\int |(x, z)|^q m(dx) \right]^{1/q}, \quad z \in B_2;$$

$a(q)$ is the q -summing norm of the identity on B_2 and $n/4 \leq \dim B$, so $a(q) \geq c_5 n^{1/2}$ for some constant c_5 depending only on q (cf. [4]). By Pietsch's integral representation theorem [17] there is a probability measure μ on S with

$$\|u_1(x)\| \leq \pi_q(u_1) \left[\int |(x, z)|^q \mu(dz) \right]^{1/q}, \quad x \in B_2.$$

Thus

$$\begin{aligned} M^q &\leq 2 \int \|u_1(x)\|^q m(dx) \\ &\leq 2 \pi_q(u_1)^q \int \int |(x, z)|^q m(dx) \mu(dz) \\ &= 2 \pi_q(u_1)^q a(q)^{-q} \\ &\leq 2 n^{q/2} c_5^{-q} n^{-q/2}, \end{aligned}$$

the last by (2). The inequality $M^\# \leq b$ follows similarly from (3).

By Theorem 2.6 of [2] (and the remarks following) there is an absolute constant $c > 0$ and an $F \subset E$ with

$$\| \cdot \| \text{ 2-equivalent to } M \| \cdot \|_2 \text{ on } F, \quad (8)$$

$$\text{the norm } x \rightarrow \|v'_1(x)\| \text{ 2-equivalent to } M^\# \| \cdot \|_2 \text{ on } F, \text{ and} \quad (9)$$

$$\dim F \geq cn \min\{\|u_1\|^{-1} M, \|v_1\|^{-1} M^\#\}^2. \quad (10)$$

By (6) and (7) M and $M^\#$ are at least b^{-1} so, using (4) and (5),

$$\dim F \geq c_6 \min\{\alpha_s(E)^{-2} n^{2/s}, n^{2/p'}\}.$$

Finally, let $w: B_2 \rightarrow F_2$ be the orthogonal projection. Since $\|v'_1(x)\| \leq 2b\|x\|_2$ for $x \in F$, the projection wv_1 has norm at most $2b$ as an operator from X into F_2 . But $\|y\| \leq 2b\|y\|_2$ for $y \in F$, so $\|wv_1\| \leq 4b^2$ as an operator from X into F . This concludes the proof. $\square$

A review of the proof of Theorem 1 shows that, once the Hilbert norm $\| \cdot \|_2$ and the operators u, v have been chosen, the key inequalities are the upper estimates for M and $M^\#$ given in terms of $\pi_q(u)$ and $\pi_{p'}(v)$. Such estimates are available in several other instances.

Given $1 \leq p \leq \infty$ a space X contains $\mathcal{L}_p^n$'s uniformly if there is a sequence $(E_n)_{n \geq 1}$ of finite dimensional subspaces of X with $\sup_n d(E_n, \mathcal{L}_p^n) < \infty$. If in addition there are projections $u_n: X \rightarrow E_n$ with $\sup_n \|u_n\| < \infty$, then X contains uniformly complemented $\mathcal{L}_p^n$'s.

THEOREM 2: *Let L be a Banach lattice not containing ℓ_∞^n 's uniformly, and let $X \subset L$. Then either*

(a) *X contains uniformly complemented ℓ_1^n 's,*

or

(b) *X is locally π -Euclidean.*

In the second case there are positive constants λ and α such that

$$c_\lambda(E) \geq \lambda^{-1}(\dim E)^\alpha$$

for all finite dimensional $E \subset X$.

THEOREM 3: *There is an absolute constant $c > 0$ with the following property. If E is an n dimensional space with a monotone symmetric basis, there is an m dimensional $F \subset E$ and a projection $w: E \rightarrow F$ with*

$$(a) \ d(F, \ell_2^m) \leq 2,$$

$$(b) \ m \geq c^{-1} d(E, \ell_2^n)^{-2} n,$$

and

$$(c) \ \|w\| \leq c \log n.$$

Again the first part of Theorem 2 is stated without proof in [8]. The conclusion of Theorem 3 is of interest only in case $d(E, \ell_2^n)^{-2}n$ is substantially larger than $(\log n)^2$; by John's Theorem [7] $d(E, \ell_2^n) \leq n^{1/2}$ for every n dimensional space, and every m dimensional $F \subset E$ is at least $M^{1/2}$ -complemented (of [3]).

PROOF OF THEOREM 2: Assume that X' doesn't contain ℓ_∞^n 's uniformly. The arguments of Pisier in [18] show that there is a constant $c > 0$ and indices p and q , $1 < p \leq 2 \leq q < \infty$, so that $\pi_p(v') \leq ci'_q(v)$ for every q' -integral map on X . Once this is established as a substitute for Lemma 1, the proof can proceed exactly as before.

PROOF OF THEOREM 3: Let $(e_i)_{i \leq n}$ be a monotone symmetric basis for E , set

$$\|x\|_2 = \left[\sum_{i \leq n} |x_i|^2 \right]^{1/2} \quad \text{for } x = \sum_{1 \leq i} x_i e_i$$

and write $u: E_2 \rightarrow E$, $v: E \rightarrow E_2$ for the formal identities. Every map g of

the form $g(e_i) = \varepsilon_i e_{\pi(i)}$, with $|\varepsilon_i| = 1$ for each i and π a permutation of $\{1, 2, \dots, n\}$, is an isometry of both E and E_2 ; further the only maps $E_2 \rightarrow E$ which commute with all such g are scalar multiples of u . By an averaging argument (cf. [5], Lemma 5.2) $\alpha(u)\alpha^*(v) = n$ for every Banach ideal norm α . Thus we may assume, normalizing $\|\cdot\|_2$ if necessary, that

$$i_\infty(u) = n^{1/2} \quad \text{and} \quad \pi_1(v) = n^{1/2}.$$

Let a and b be the best constants satisfying

$$a^{-1}\|x\|_2 \leq \|x\| \leq b\|x\|_2, \quad x \in E.$$

Another averaging argument shows

$$ab = d(E, \ell_2^n).$$

M and $M^\#$ are defined as in the proof of Theorem 1. By that proof, for any $q \geq 2$,

$$M \leq 2^{1/q} \pi_q(\ell_2^n)^{-1} \pi_q(u) \quad \text{and} \quad M^\# \leq 2^{1/q} \pi_q(\ell_2^n)^{-1} \pi_q(v'),$$

where $\pi_q(\ell_2^n)$ denotes the q -summing norm of the identity on ℓ_2^n . Using the expression given in [4] for $\pi_q(\ell_2^n)$ and Stirling's formula there is an absolute constant $a > 0$ such that $\pi_q(\ell_2^n)^{-1} \leq a(q/n)^{1/2}$ for all $q \geq 2$.

Any map w into an n dimensional space satisfies $\pi_q(w) \leq n^{1/q} i_\infty(w)$ (cf. [11], Corollary 1.7). Consequently, combining inequalities yields

$$\begin{aligned} M &\leq 2^{1/q} a(q/n)^{1/2} \pi_q(u) \\ &\leq 2an^{1/q} q^{1/2} n^{-1/2} i_\infty(u) \\ &= 2an^{1/q} q^{1/2} \end{aligned}$$

and similarly

$$\begin{aligned} M^\# &\leq 2an^{1/q} q^{1/2} i_\infty(v') n^{-1/2} \\ &= 2an^{1/q} q^{1/2} \gamma_1(v) n^{-1/2} \\ &\leq 2an^{1/q} q^{1/2} \pi_1(v) n^{-1/2} \\ &= 2an^{1/q} q^{1/2}, \end{aligned}$$

the last inequality by [6], Lemma 3.3, since E has a monotone uncondi-

tional basis. Now taking $q = \log n$,

$$MM^\# \leq c_1 \log n$$

for some absolute constant c_1 . Trivially, $M \geq a^{-1}$ and $M^\# \geq b^{-1}$. Using the method of Figiel-Lindenstrauss-Milman as in the proof of Theorem 1 produces an m dimensional $F \subset E$, which is 2-isomorphic to ℓ_2^m , $c_2 \log n$ complemented in E and with

$$m \geq c_2 n \min\{b^{-1}M, a^{-1}M^\#\}^2 \geq c_2 nd(E, \ell_2^n)^{-2}. \quad \square$$

There are a number of natural questions about complemented Hilbert subspaces. Let X be a space with some Rademacher type. Is there a constant $\lambda \geq 1$, depending on X , with

$$c_\lambda(E) \geq \lambda^{-1} d(E, \ell_2^n)^{-2} n$$

for all n dimensional $E \subset X$? For X a p -convex and q -concave lattice it is known [11] that $d(E, \ell_2^n) \leq n^{1/p - q/q}$ for $E \subset X$ having dimension n . For such lattices Theorem 1 gives an apparently stronger result, although in this case it is likely that the correct distance estimate is

$$d(E, \ell_2^n) \leq c_L \max\{n^{1/p - 1/2}, n^{1/2 - 1/q}\}.$$

The lattice structure enters into the proofs of our results only through the inequality

$$\pi_s(v') \leq ci_r(v) \quad (\#)$$

for operators on X . For X the Schatten p -trace class of operators on ℓ_2 , Pisier [18] has shown that $(\#)$ fails for every non-trivial pair $1 < r \leq 2 \leq s < \infty$. We know of no non-trivial lower estimates for $c_\lambda(E)$ if $E \subset C_p$, $1 < p < 2$, although sharp upper estimates for $d(E, \ell_2^n)$ are available [19]. Finally, we know of no space X on which $(\#)$ is true which is not a subspace of a quotient of a Banach lattice having some Rademacher type.

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