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SOME REMARKS ON SYMMETRIC BASIC SEQUENCES IN L_1

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Summary

Every subspace of L_1 with an unconditional basis is isomorphic to a complemented subspace of a subspace of L_1 with a symmetric basis.

1. Introduction

The purpose of this paper is to show that, in spite of the quite simple representation of subspaces of L_1 with symmetric bases, given in [2], such spaces can be very complicated from the point of view of the structure of their complemented subspaces.

THE MAIN THEOREM: *Let $(x_i)_{i=1}^{\infty}$ be an unconditional basic sequence in L_p , $1 \leq p \leq 2$. There exists a symmetric basic sequence $(s_i)_{i=1}^{\infty}$ in L_p such that $(x_i)_{i=1}^{\infty}$ is equivalent to a block basis with constant coefficients of $(s_i)_{i=1}^{\infty}$. In particular $[x_i]_{i=1}^{\infty}$ is isomorphic to a complemented subspace of $[s_i]_{i=1}^{\infty}$.*

Theorems of such a nature were previously discovered by Lindenstrauss [8], Szankowski [15] and Davis [4]. The proof of the main theorem, given in §2, uses the technique of Davis' proof as well as some results concerning the space X_p of Rosenthal [12].

Section 3 is devoted mainly to an alternative proof of the following reduction (cf. Dacunha-Castelle and Krivine [3]): every subspace of L_1 contains some l_p if and only if every subspace of L_1 with a symmetric basis contains one.

Notions which are not explained here can be found in [9] or [10].

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2. The main result

We begin with two known lemmas:

LEMMA 1 (Rosenthal [12]): *Let $1 < p < 2$. There exists a constant K_p such that if $m \geq 1$ is a real number and $(y_{i,m})_{i=1}^{\infty}$ is a sequence of independent, identically distributed, symmetric random variables (on $[0, 1]$) each taking three values in such a manner that $\|y_{i,m}\|_{L_p} = m^{-1}$ and $\|y_{i,m}\|_{L_2} = m$, then:*

$$K_p^{-1} \left\| \sum_{i=1}^{\infty} c_i y_{i,m} \right\|_{L_p} \leq \inf \left\{ \left(\sum_{i=1}^{\infty} |a_i|^2 + \left(\sum_{i=1}^{\infty} |b_i|^p \right)^{2/p} \right)^{1/2}; \forall i \geq 1, \right. \\ \left. m^{-1} a_i + m b_i = c_i \right\} \leq K_p \left\| \sum_{i=1}^{\infty} c_i y_{i,m} \right\|_{L_p}$$

PROOF: Let $q = p/(p-1)$ and let $z_{i,m} = m^{-2} y_{i,m}$, $i = 1, 2, \dots$. It is easily checked that $\|z_{i,m}\|_{L_2} = m^{-1}$ and $\|z_{i,m}\|_{L_q} = m$, $i = 1, 2, \dots$. Thus, by the main result of [12]:

$$A_p^{-1} \left\| \sum_{i=1}^{\infty} c_i z_{i,m} \right\|_{L_q} \leq \max \left\{ m^{-1} \left(\sum_{i=1}^{\infty} |c_i|^2 \right)^{1/2}, m \left(\sum_{i=1}^{\infty} |c_i|^q \right)^{1/q} \right\} \leq A_p \left\| \sum_{i=1}^{\infty} c_i z_{i,m} \right\|_{L_q}$$

and $P: L_q \rightarrow L_q$ defined by:

$$Pf = \sum_{i=1}^{\infty} \left(\int_0^1 f \frac{z_{i,m}}{\|z_{i,m}\|_{L_2}} \right) z_{i,m} = \sum_{i=1}^{\infty} \left(\int_0^1 f y_{i,m} \right) z_{i,m}$$

is a projection of norm $\|P\| \leq A_p$, where A_p depends only on p . It follows that $(y_{i,m})_{i=1}^{\infty}$ is equivalent, with constant depending on p only, to sequence of the functionals biorthogonal to the unit vector basis in the space

$$X_{q,m} = \left\{ (c_i)_{i=1}^{\infty}; \|(c_i)_{i=1}^{\infty}\| = \max \left\{ m^{-1} \left(\sum_{i=1}^{\infty} |c_i|^2 \right)^{1/2}, m \left(\sum_{i=1}^{\infty} |c_i|^q \right)^{1/q} \right\} < \infty \right\}$$

and the result follows. ■

Let $1 \leq p < 2$, $m \geq 1$ and define a norm on ℓ_2 by

$$\|x\|_m = \inf \{ (\|y\|_{\ell_2}^2 + \|z\|_{\ell_p}^2)^{1/2}; m^{-1}y + mz = x \}.$$

The unit vectors $(e_i)_{i=1}^{\infty}$ in ℓ_2 clearly form a 1-symmetric basis for

$(\ell_2, |\cdot|_m)$. We define

$$\lambda_m(n) = \left| \sum_{i=1}^n e_i \right|_m \quad n = 1, 2, \dots$$

In the following lemma we collect some properties of the $\lambda_m(n)$'s. For a proof see [4] or [10, lemma 3.b.3 and the proof of proposition 3.b.4].

LEMMA 2: (1) $\lambda_m(n) = (m^{-2}n^{-1} + m^2n^{-2/p})^{-1/2}$ for every integer n and every $m \geq 1$.

(2) For a fixed n :

$$\max_{m \geq 1} \lambda_m(n) = 2^{-1/2} n^{(1/2+1/p)(1/2)}$$

and the maximum is attained at $m = m(n) = n^{(1/p-1/2)(1/2)}$

(3) For any sequence of positive numbers $(\epsilon_i)_{i=1}^\infty$ there exists a subsequence $(n_i)_{i=1}^\infty$ of the integers such that, putting $m_i = m(n_i)$, $i = 1, 2, \dots$, we get:

$$\sum_{j=1, j \neq i}^\infty \lambda_{m_i}(n_j) / \lambda_{m_j}(n_j) < \epsilon_k$$

and

$$\sum_{i=1}^\infty \frac{1}{m_i} \leq 1$$

We are ready now for the proof of the main theorem. The proof is written for the case $p = 1$ only, the case $1 < p < 2$ requires only minor changes (and the case $p = 2$ is trivial).

PROOF OF THE MAIN THEOREM: Let $(x_k)_{k=1}^\infty$ be a normalized unconditional basic sequence in L_1 and let $1 < p < 2$. $(\epsilon_i)_{i=1}^\infty$ will denote a sequence of positive numbers to be specified later and $(n_i)_{i=1}^\infty$ will denote subsequence of the integers with the properties stated in Lemma 2.3.

Let $(y_{i,k})_{i=1, k=1}^\infty$ be a double sequence of independent, symmetric, three-valued random variables on $[0, 1]$ such that

$$\|y_{i,k}\|_{L_p} = m_k^{-1} \text{ and } \|y_{i,k}\|_{L_2} = m_k \quad i, k = 1, 2, \dots$$

By Lemma 1:

$$(1) \quad K_p^{-1} |(c_i)_{i=1}^\infty|_{m_k} \leq \left\| \sum_{i=1}^\infty c_i y_{i,k} \right\| \leq K_p |(c_i)_{i=1}^\infty|_{m_k}$$

for all scalars $(c_i)_{i=1}^\infty$, where K_p depends only on p .

Define $s_i \in L_p([0, 1]^2)$, $i = 1, 2, \dots$ by:

$$s_i(r, t) = \sum_{k=1}^{\infty} y_{i,k}(r)x_k(t)$$

Then:

$$\|s_i\|_{L_1(L_p)} = \int \left(\int |s_i(r, t)|^p dr \right)^{1/p} dt \leq \sum_{k=1}^{\infty} \|y_{i,k}\|_{L_p} \leq 1$$

and $s_i(r, t)$, $i = 1, 2, \dots$ clearly constitutes a 1-symmetric basic sequence in $L_1(L_p)$.

Choose a sequence $(\Gamma_\ell)_{\ell=1}^\infty$ of disjoint subsets of the integers with $\bar{\Gamma}_\ell = n_\ell$ (\bar{A} denotes the cardinality of the set A), and put

$$w_\ell = \sum_{i \in \Gamma_\ell} s_i / \lambda_{m_\ell}(n_\ell) \quad \ell = 1, 2, \dots$$

$(w_\ell)_{\ell=1}^\infty$ is a block basis with constant coefficients of $(s_i)_{i=1}^\infty$; we are going to show that, if the ϵ_i 's are chosen properly, $(w_\ell)_{\ell=1}^\infty$ is equivalent to $(x_\ell)_{\ell=1}^\infty$.

Let

$$v_\ell(n, t) = \sum_{i \in \Gamma_\ell} y_{i,\ell}(r) \cdot x_\ell(t) / \lambda_{m_\ell}(n_\ell) \quad \ell = 1, 2, \dots$$

and note that, by the triangle inequality, (1) and Lemma 2:

$$\begin{aligned} (2) \quad \left\| \sum_{\ell=1}^{\infty} a_\ell w_\ell \right\| &= \left\| \sum_{\ell=1}^{\infty} a_\ell \sum_{k=1}^{\infty} \sum_{i \in \Gamma_\ell} y_{i,k}(r)x_k(t) / \lambda_{m_\ell}(n_\ell) \right\|_{L_1(L_p)} \\ &\leq \left\| \sum_{\ell=1}^{\infty} a_\ell v_\ell(n, t) \right\|_{L_1(L_p)} \\ &\quad + \sum_{\ell=1}^{\infty} |a_\ell| \sum_{k \neq \ell} \left\| \sum_{i \in \Gamma_\ell} y_{i,k} \right\|_{L_p} / \lambda_{m_\ell}(n_\ell) \\ &\leq \left\| \sum_{\ell=1}^{\infty} a_\ell v_\ell(r, t) \right\|_{L_1(L_p)} \\ &\quad + K_p \sum_{\ell=1}^{\infty} |a_\ell| \sum_{k \neq \ell} \lambda_{m_k}(n_\ell) / \lambda_{m_\ell}(n_\ell) \\ &\leq \left\| \sum_{\ell=1}^{\infty} a_\ell v_\ell(r, t) \right\|_{L_1(L_p)} + K_p \left(\sum_{\ell=1}^{\infty} \epsilon_\ell \right) \max_{i \leq \ell < \infty} |a_\ell| \end{aligned}$$

for all scalars $(a_\ell)_{\ell=1}^\infty$. And similarly:

$$(3) \quad \left\| \sum_{\ell=1}^{\infty} a_\ell w_\ell \right\| \geq \left\| \sum_{\ell=1}^{\infty} a_\ell v_\ell(r, t) \right\|_{L_1(L_p)} - K_p \left(\sum_{i=1}^{\infty} \epsilon_i \right) \max_{1 \leq \ell < \infty} |a_\ell|$$

Now

$$\left(\sum_{i \in \Gamma_\ell} y_{i,\ell} \right)_{\ell=1}^\infty$$

is a sequence of symmetric independent random variables and thus forms a 1-unconditional basic sequence in L_p [cf. 12]; it follows that $(v_\ell)_{\ell=1}^\infty$ is a 1-unconditional basic sequence in $L_1(L_p)$, thus

$$(4) \quad \left\| \sum a_\ell v_\ell(r, t) \right\|_{L_1(L_p)} \geq \max_{1 \leq \ell < \infty} |a_\ell| \cdot \min_{1 \leq \ell < \infty} \|v_\ell(r, t)\|_{L_1(L_p)} \geq K_p^{-1} \max_{1 \leq \ell < \infty} |a_\ell|$$

Hence, if the ϵ_i 's satisfy

$$\sum_{i=1}^{\infty} \epsilon_i < \frac{1}{2} K_p^{-2}$$

we get from (2), (3) and (4) that

$$\frac{1}{2} \left\| \sum a_\ell v_\ell \right\| \leq \left\| \sum a_\ell w_\ell \right\| \leq \frac{3}{2} \left\| \sum a_\ell v_\ell \right\|$$

for all choices of scalars $(a_\ell)_{\ell=1}^\infty$, so it is enough to prove that $(v_\ell)_{\ell=1}^\infty$ is equivalent to $(x_\ell)_{\ell=1}^\infty$.

The sequence

$$\left(\sum_{i \in \Gamma_\ell} y_{i,\ell} / \lambda_{m_\ell}(n_\ell) \right)_{\ell=1}^\infty$$

is, as was already mentioned, a bounded away from zero 1-unconditional orthogonal sequence and, for $\ell = 1, 2, \dots$

$$\left\| \sum_{i \in \Gamma_\ell} y_{i,\ell} / \lambda_{m_\ell}(n_\ell) \right\|_{L_2} = \|y_{1,\ell}\|_{L_2} \cdot n_\ell^{1/2} / \lambda_{m_\ell}(n_\ell) = m_\ell \cdot n_\ell^{1/2} / \lambda_{m_\ell}(n_\ell) = 2^{1/2}$$

by Lemma 2.

It follows easily that

$$\left(\sum_{i \in I_\ell} y_{i,\ell} \lambda_{m_\ell}(n_\ell) \right)_{\ell=1}^\infty$$

is equivalent, in L_p , to the unit vector basis of ℓ_2 and thus there exists a constant K_p^1 such that

$$(K_p^1)^{-1} \left\| \left(\sum_{\ell=1}^\infty a_\ell^2 x_\ell^2 \right)^{1/2} \right\|_{L_1} \leq \left\| \sum_{\ell=1}^\infty a_\ell v_\ell(r, t) \right\|_{L_1(L_p)} \leq K_p^1 \left\| \left(\sum_{\ell=1}^\infty a_\ell^2 x_\ell^2 \right)^{1/2} \right\|_{L_1}$$

Finally, by the unconditionality of $(x_\ell)_{\ell=1}^\infty$ and by Khinchine's inequality, the expression

$$\left\| \left(\sum_{\ell=1}^\infty a_\ell^2 x_\ell^2 \right)^{1/2} \right\|_{L_1} \text{ is equivalent to } \left\| \sum_{\ell=1}^\infty a_\ell x_\ell \right\|_{L_1}. \quad \blacksquare$$

CONCLUSION: *Let $1 \leq p \leq 2$ there exists a 1-symmetric sequence $(s_i)_{i=1}^\infty$ in L_p such that every unconditional basic sequence $(x_i)_{i=1}^\infty$ in L_p is equivalent, with constant depending only on the unconditionality constant of $(x_i)_{i=1}^\infty$, to a block basis with constant coefficients of $(s_i)_{i=1}^\infty$.*

PROOF: By [14] there exists, in L_p , a 1-unconditional basic sequence such that any other unconditional basic sequence in L_p is equivalent, with constant depending on the unconditionality constant only, to a subsequence of it. Apply the main theorem to this universal basic sequence. \blacksquare

3. Dacunha-Castelle and Krivine's reduction

This section is devoted to the proof of the following proposition.

PROPOSITION 3: *If every subspace of L_1 with a symmetric basis contains an isomorphic copy of ℓ_p for some $1 \leq p \leq 2$ then every subspace of L_1 contains such a subspace.*

We begin by recalling two definitions.

DEFINITION 1 ([6], [7]): An unconditional basis $(x_i)_{i=1}^\infty$ is said to be q -concave ($1 \leq q < \infty$) if there exists a constant $c > 0$ such that for

every N and every N elements $y_n = \sum_{i=1}^{\infty} a_{i,n} x_i$, $n = 1, \dots, N$ in $[x_i]_{i=1}^{\infty}$ we have:

$$\left\| \sum_{i=1}^{\infty} \left(\sum_{n=1}^N |a_{i,n}|^q \right)^{1/q} x_i \right\| \geq c \left(\sum_{n=1}^N \|y_n\|^q \right)^{1/q}$$

DEFINITION 2 ([5]): Given $1 \leq p \leq 2$, an 1-unconditional normalized basis $(x_i)_{i=1}^{\infty}$ and a sequence $m_n \nearrow \infty$ with $\sum_{n=1}^{\infty} m_n^{-1} < \infty$, let $Y = Y(\ell_2, \ell_p, (x_i)_{i=1}^{\infty}, (m_n)_{n=1}^{\infty})$ be the space of all sequences of scalars $\alpha \in \ell_2$ such that

$$\|\alpha\|_Y = \left\| \sum_{n=1}^{\infty} |\alpha|_{m_n} x_n \right\|_X < \infty$$

($|\cdot|_{m_n}$ is defined in the preceding section). $(e_i)_{i=1}^{\infty}$ will denote the unit vectors in this space: note that they form a 1-symmetric basis for Y .

PROPOSITION 4: Let $q < 2$ and let $(x_i)_{i=1}^{\infty}$ be a q -concave 1-unconditional basic sequence in L_1 , let $q < p < 2$, and let $(m_n)_{n=1}^{\infty}$ and $(s_i)_{i=1}^{\infty}$ be as in the proof of the main theorem. Then $(s_i)_{i=1}^{\infty}$ is equivalent to $(e_i)_{i=1}^{\infty}$.

PROOF: We use the same notation as in the proof of the main theorem. By Khinchine's inequality and the triangle inequality in $\ell_{2/p}$ we get that:

$$\begin{aligned} \left\| \sum_{i=1}^{\infty} a_i s_i \right\|_{L_1(L_p)} &= \int \left(\int \left| \sum_{k=1}^{\infty} \sum_{i=1}^{\infty} a_i y_{i,k}(r) x_k(t) \right|^p dr \right)^{1/p} dt \\ &\geq A_p \int \left(\int \left(\sum_{k=1}^{\infty} \left| \sum_{i=1}^{\infty} a_i y_{i,k}(r) x_k(t) \right|^2 \right)^{p/2} dr \right)^{1/2} dt \\ &\geq A_p \int \left(\sum_{k=1}^{\infty} \left(\int \left| \sum_{i=1}^{\infty} a_i y_{i,k}(r) \right|^p dr \right)^{2/p} |x_k(t)|^2 \right)^{1/2} dt \\ &\geq A_p K_p^{-1} \|(a_i)_{i=1}^{\infty}\|_Y \end{aligned}$$

A_p denotes here the Khinchine's constant of L_p : note that we have used the fact that for a fixed t , $(\sum_{i=1}^{\infty} a_i y_{i,k}(r) x_k(t))_{k=1}^{\infty}$ constitutes a 1-unconditional basic sequence in L_p .

For the other side inequality we need the fact [cf. 1] that there exists an isometry (into) $T: L_p \rightarrow L_1$ and a constant C such that

$$C\|f\|_{L_p} = C\|Tf\|_{L_1} \geq \|Tf\|_{L_q} \geq \|Tf\|_{L_1} = \|f\|_{L_p}$$

for all $f \in L_p$. Using this fact and the q -concavity of $(x_i)_{i=1}^\infty$ (with constant c , say) we get that,

$$\begin{aligned}
 \left\| \sum_{i=1}^{\infty} a_i s_i \right\|_{L_1(L_p)} &= \iint \left| \sum_{k=1}^{\infty} \sum_{i=1}^{\infty} a_i (Ty_{i,k})(r) x_k(t) \right| dt dr \\
 &\leq \left(\iint \left\| \sum_{k=1}^{\infty} \sum_{i=1}^{\infty} a_i (Ty_{i,k})(r) x_k \right\|_{L_1}^q dr \right)^{1/q} \\
 &\leq c^{-1} \left\| \sum_{k=1}^{\infty} \left(\int \left| \sum_{i=1}^{\infty} a_i Ty_{i,k}(r) \right|^q dr \right)^{1/q} x_k \right\|_{L_1} \\
 &\leq c^{-1} C \left\| \sum_{k=1}^{\infty} \left\| \sum_{i=1}^{\infty} a_i y_{i,k} \right\|_{L_p} x_k \right\|_{L_1} \\
 &\leq c^{-1} CK_p \|(a_i)_{i=1}^\infty\|_Y. \quad \blacksquare
 \end{aligned}$$

We shall need also the following standard.

LEMMA 5: *Let U be an infinite dimensional subspace of $Y = Y(\ell_2, \ell_p, (x_i)_{i=1}^\infty, (m_n)_{n=1}^\infty)$. Then U contains a subspace which embeds isomorphically in $[x_i]_{i=1}^\infty$.*

PROOF: Using standard arguments (compare for example the proof of proposition 3.b.4 in [10]) the proof reduces to showing that the identity map from Y to ℓ_2 is strictly singular.

Now any infinite dimensional subspace of ℓ_2 contains a norm one vector with arbitrarily small ℓ_∞ norm. Fix n and let x be a norm one vector in ℓ_2 such that $\|x\|_{\ell_\infty} \leq m_n^{-2p/(2-p)}$. Let y and z be elements of ℓ_2 such that

$$|x|_{m_n} \geq (\|y\|_2^2 + \|z\|_p^2)^{1/2} - 1 \quad \text{and} \quad m_n^{-1}y + m_n z = x$$

We may clearly assume also that $\|m_n z\|_{\ell_\infty} \leq \|x\|_{\ell_\infty}$. If $\|m_n^{-1}y\|_{\ell_2} \geq \frac{1}{2}$ then $\|x\|_Y \geq |x|_{m_n} \geq m_n/2 - 1$. If $\|m_n^{-1}y\|_{\ell_2} < \frac{1}{2}$ then $\|m_n z\|_{\ell_2} \geq \frac{1}{2}$ and we get:

$$\frac{1}{2} \leq \|m_n z\|_{\ell_2} \leq \|m_n z\|_{\ell_p}^{p/2} \|m_n z\|_{\ell_\infty}^{(2-p)/2}$$

and

$$\|m_n z\|_{\ell_p} \geq m_n^2 / 2^{2/p}$$

so that

$$\|x\|_Y \geq |x|_{m_n} \geq m_n / 2^{2/p} - 1$$

Hence every infinite dimensional subspace of Y contains vectors with norm one in ℓ_2 and with arbitrarily big norm in Y . ■

PROOF OF PROPOSITION 3: Assume that every subspace of L_1 with a symmetric basis contains some ℓ_p . Notice first that if $(x_i)_{i=1}^\infty$ is a 1-unconditional basic sequence in L_1 which is q -concave for some $q < 2$ then, by the main theorem, Proposition 4, Lemma 5 and the fact [cf. 11] that every infinite dimensional subspace of ℓ_p contains an isomorph of ℓ_p , $[x_i]_{i=1}^\infty$ contains ℓ_p isomorphically for some p (necessarily $1 \leq p \leq q$).

Now let X be a subspace of L_1 . If X does not contain ℓ_1 then by Rosenthal's theorem [13] X is isomorphic to a subspace of L_r for some $1 < r \leq 2$ and since L_r has an unconditional basis the image of X in L_r contains an unconditional basic sequence $(y_i)_{i=1}^\infty$. It is clearly enough to show that $[y_i]_{i=1}^\infty$ contains some ℓ_p .

Let $(g_i)_{i=1}^\infty$ be a sequence in L_1 isometrically equivalent to the unit vector basis of $\ell_{2/r}$ (for instance a sequence of L_1 -norm one, independent $2/r$ stable random variables), and define, for $i = 1, 2, \dots$ and $0 \leq s, t \leq 1$

$$x_i(s, t) = g_i(s)|y_i(t)|^r$$

Then $(x_i)_{i=1}^\infty$ is 1-unconditional in $L_1((0, 1)^2)$ and by the triangle inequality in $\ell_{2/r}$

$$\begin{aligned} \left\| \sum_{i=1}^\infty \left(\sum_{j=1}^\infty |a_{i,j}|^{2/r} \right)^{r/2} x_i \right\| &= \int_0^1 \left(\sum_{j=1}^\infty \sum_{i=1}^\infty |a_{i,j}|^{2/r} |y_i(t)|^2 \right)^{r/2} dt \\ &\geq \left(\sum_{j=1}^\infty \left(\int_0^1 \left(\sum_{i=1}^\infty |a_{i,j}|^{2/r} |y_i(t)|^2 \right)^{r/2} dt \right)^{2/r} \right)^{r/2} \\ &= \left(\sum_{j=1}^\infty \left\| \sum_{i=1}^\infty a_{i,j} x_i \right\|^{2/r} \right)^{r/2} \end{aligned}$$

i.e. $(x_i)_{i=1}^\infty$ is $2/r$ concave. Thus, it follows from the first part of the proof that $[x_i]_{i=1}^\infty$ contains a subspace isomorphic to ℓ_p for some $1 \leq p \leq 2/r$. Passing to a smaller subspace we may assume that there exist disjoint blocks

$$b_n = \sum_{i \in E_n} a_i x_i \qquad n = 1, 2, \dots$$

of $(x_i)_{i=1}^\infty$ which are equivalent to the usual basis of ℓ_p . It is easily

checked that the blocks

$$c_n = \sum_{i \in E_n} |a_i|^{1/r} y_i \quad n = 1, 2, \dots$$

of $(y_i)_{i=1}^{\infty}$ are equivalent to the usual basis of ℓ_p . ■

REFERENCES

- [1] J. BRETAGNOLLE, D. DACUNHA-CASTELLE and J.L. KRIVINE: Lois stable et espaces L^p , *Ann. Inst. Henri Poincaré, série B*, 2 (1966) 231–263.
- [2] D. DACUNHA-CASTELLE: “Variables aléatoires échangeable et espaces d’Orlicz.” Séminaire Maurey-Schwartz, École Polytechnique (1974–1975).
- [3] D. DACUNHA-CASTELLE and J.L. KRIVINE: Sous-espaces de L^1 , *Israel J. Math.*, 26 (1977) 320–351.
- [4] W.J. DAVIS: Embedding spaces with unconditional bases, *Israel J. Math.*, 20 (1975) 189–191.
- [5] W.J. DAVIS, T. FIGIEL, W.B. JOHNSON and A. PELCZYNSKI: Factoring weakly compact operators, *J. Funct. Anal.*, 17 (1974) 311–327.
- [6] E. DUBINSKY, A. PELCZYNSKI and H.P. ROSENTHAL: On Banach spaces X for which $\pi(\mathcal{L}_x, X) = B(\mathcal{L}_x, X)$, *Studia Math.*, 44 (1972) 617–648.
- [7] T. FIGIEL and W.B. JOHNSON: A uniformly convex Banach space which contains no ℓ_p , *Compositio Math.*, 29 (1974) 179–190.
- [8] J. LINDENSTRAUSS: A remark on symmetric bases, *Israel J. Math.*, 13 (1972) 317–320.
- [9] J. LINDENSTRAUSS and L. TZAFRIRI: Classical Banach spaces. *Lecture Notes in Mathematics* 338. Springer, Berlin etc. (1973).
- [10] J. LINDENSTRAUSS and L. TZAFRIRI: Classical Banach Spaces, sequence, spaces. Springer, Berlin.
- [11] A. PELCZYNSKI: Projections in certain Banach spaces, *Studia Math.*, 19 (1960) 209–228.
- [12] H.P. ROSENTHAL: On the subspaces of L^p ($p > 2$) spanned by sequences of independent random variables, *Israel J. Math.*, 8 (1970) 273–303.
- [13] H.P. ROSENTHAL: On subspaces of L^p , *Ann. Math.*, 97 (1973) 344–373.
- [14] G. SCHECHTMAN: On Pelczynski’s paper “Universal bases”, *Israel J. Math.*, 22 (1975) 181–184.
- [15] A. SZANKOWSKI: Embedding Banach spaces with unconditional bases into spaces with symmetric bases, *Israel J. Math.*, 15 (1973) 53–59.

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