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QUOTIENT RINGS OF GROUP RINGS

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1. Introduction

In this paper we continue the study of the maximal quotient ring of a group algebra, initiated by Formanek [3]. We begin by fixing some notation. Throughout, G will denote a fixed group, K a field, and KG the group algebra of G over K. In addition, we write

$$\Delta(G) = \{x \in G : |G : C_G(x)| < \infty\},$$

so that $\Delta(G)$ is the FC-subgroup of G, and

$$\Delta^+(G) = \{x \in \Delta G : |x| < \infty\}.$$

Thus $\Delta(G)$ and $\Delta^+(G)$ are characteristic subgroups of G; see [8] Lemma 19.3 for details.

If R is a ring, (with 1), we shall denote the maximal right quotient ring of R by $Q_{max}(R)$, and the classical right quotient ring of R by $Q_{cl}(R)$, whenever this latter ring exists. For basic properties of maximal right quotient rings, we refer to [6]. The centre of the ring R will be denoted by $C(R)$.

The study of quotient rings of arbitrary group algebras was begun by M. Smith [9], who showed that if KG is a semiprime group algebra for which $Q_{cl}(KG)$ exists, then

$$C(Q_{cl}(KG)) = Q_{cl}(C(KG)). \tag{1}$$

In [7], Passman demonstrated that (1) remains true without the assumption that KG is semiprime. In [3], Formanek showed that, assuming once more that KG is semiprime, an identification analogous to (1) can be made for maximal right quotient rings.

In this paper, we extend Formanek's work by showing that if we replace the assumption that KG is semiprime by the hypothesis that $\Delta^+(G)$ is finite, the main result of [3] remains true. That is, we prove

THEOREM A: *Let KG be a group algebra, and suppose that $\Delta^+(G)$ is finite. Then $Q_{max}(C(KG))$ may be identified with the centre of $Q_{max}(KG)$.*

The identification described in Theorem A is "natural" in the sense that it corresponds to the identification obtained in the semiprime case by Formanek, and it will be explicitly described in the course of the proof of the theorem.

There are two main ideas involved in the proof of Theorem A. The first was observed by Formanek, who proved in [3], Theorem 7, that if KG is any group algebra, $C(Q_{max}(KG))$ is a subring of $Q_{max}(K\Delta(G))$. The second is a result from [2], also obtained by Horn, [4], which states that if $\Delta^+(G)$ is finite, $Q_{cl}(K\Delta(G))$ exists and is a QF-ring, and so in particular is equal to $Q_{max}(K\Delta(G))$. On combining these two facts, we obtain, as an intermediate stage in the proof of Theorem A:

PROPOSITION 8: *If $\Delta^+(G)$ is finite,*

$$C(Q_{max}(KG)) = Q_{cl}(C(KG)).$$

Theorem A is deduced from Proposition 8 by examining the structure of $Q_{cl}(C(KG))$.

2. Basic results

In this section we restate, in forms adapted to our purpose, some results from [2], [3] and [7]. We also assemble some facts about quotient rings. We shall for the most part use the formulation of the maximal right quotient ring of a ring in terms of homomorphisms defined on dense right ideals, as introduced by Utumi; see [6], §4.3. In brief, the maximal right quotient ring Q of a ring R may be viewed as the set of pairs (D, f), where D is a dense right ideal of R and f is an R-homomorphism from D to R, and where two such pairs (D_1, f_1) and (D_2, f_2) are identified if $\mathrm{f}_1(\mathrm{d}) = \mathrm{f}_2(\mathrm{d})$ for all $\mathrm{d} \in D_1 \cap D_2$.

Let R be a subring of a ring S. Then S is called *a partial right quotient ring of R* if, for every $0 \neq s \in S$,

$$s^{-1}R = \{r \in R : sr \in R\}$$

is a dense right ideal of R, and $s(s^{-1}R) \neq 0$.

We shall denote the set of regular elements of the ring R by $\mathscr{C}_R(0)$. We require the following elementary facts about quotient rings and dense right ideals.

LEMMA 1: *Let Q be the maximal right quotient ring of a ring R.*

(a) *An element α of Q lies in $C(Q) \Leftrightarrow \alpha$ commutes with every element of R.*

(b) *If S is a partial right quotient ring of R, the identity map on R can be uniquely extended to a monomorphism of S_R into Q_R, and this map is a ring homomorphism.*

(c) *Let T be a multiplicatively closed set of regular elements of R, and suppose that R satisfies the right Ore condition with respect to T. Then the over-ring of R obtained by inverting the elements of T is a partial right quotient ring of R.*

(d) *If D and E are dense right ideals of R, so is $D \cap E$.*

(e) *If D is a dense right ideal of R, and S is a partial right quotient ring of R, then DS is a dense right ideal of S.*

(f) *If $I \lhd R$, I is a dense right ideal of $R \Leftrightarrow$ the left annihilator of I is zero.*

PROOF: (a) $\Rightarrow$ *Clear.*

$\Leftarrow$ Let I_R denote the injective hull of R_R, and put $H = Hom_R(I_R, I_R)$. Then $Q = Hom_H({}_HI, {}_HI)$, (see [6], §4.3), and if α commutes with every element of R, $\alpha \in H$. Thus α commutes with every element of Q.

(b) See [6], §4.3, Proposition 8.

(c) This is immediate from the definition.

(d) See [6], §4.3, Lemma 3.

(e) By [6], §4.3, Proposition 4, we must show that if $0 \neq s_1 \in S$, $s_2 \in S$, then there exists $s \in S$ such that $s_1 s \neq 0$ and $s_2 s \in DS$. By (b), we can consider S as a subring of Q, and by definition, $s_1^{-1}R$ is a dense right ideal of R. By [6], §4.3, Lemma 2, $s_2^{-1}D$ is a dense right ideal of R, so by (d), $s_1^{-1}R \cap s_2^{-1}D$ is a dense right ideal of R. It follows from the proof of [6], §4.3, Proposition 5 that

$$s_1(s_1^{-1}R \cap s_2^{-1}D) \neq 0,$$

since $S \subseteq Q$. If we choose $s \in (s_1^{-1}R \cap s_2^{-1}D)$ such that $s_1 s \neq 0$, then $s_2 s \in D$, and the proof is complete.

(f) See [6], §4.3, Corollary to Proposition 4.

LEMMA 2 [3, Theorem 1]: *If H is a subnormal subgroup of G, and D is a dense right ideal of KH, then DG is a dense right ideal of KG.*

Using Lemma 2, Formanek showed in [3], Theorem 2 that, when H is subnormal in G, $Q_{max}(KH)$ may be viewed as a certain subring of $Q_{max}(KG)$. This is done by identifying the element (D, f) of $Q_{max}(KH)$ with the element $(DG, \bar{\mathrm{f}})$ of $Q_{max}(KG)$, where

$$\bar{\mathrm{f}} : DG \to KG : \sum_{i \in I} \alpha_i \gamma_i \to \sum_{i \in I} \mathrm{f}(\alpha_i) g_i,$$

where $\{g_i : \mathrm{i} \in I\}$ is a right transversal to H in G. Formanek shows that $\bar{\mathrm{f}}$ is well-defined. Note that DG is a dense right ideal of KG by Lemma 2.

THEOREM 3 [3, Theorem 7]: *$C(Q_{max}(KG))$ is a subring of $Q_{max}(K\Delta(G))$.*

In considering the above result, it is important to bear in mind the nature of the embedding of $C(Q_{max}(KG))$ in $Q_{max}(K\Delta(G))$. Formanek shows that if $\alpha \in C(Q_{max}(KG))$, then we can represent α as $(D_1 G, \mathrm{f}_1)$, where D_1 is a G-invariant dense ideal of $K\Delta(G)$, (so that $D_1 G$ is an ideal of KG), $\mathrm{f}_1(D_1) \subseteq K\Delta(G)$, and f_1 is a bimodule homomorphism. Theorem 3 follows from this fact via the embedding of $Q_{max}(K\Delta(G))$ in $Q_{max}(KG)$ described above.

The results of this paper will follow from an examination of $Q_{max}(K\Delta(G))$, under the assumption that $\Delta^+(G)$ is finite. Crucial to this approach is the following special case of [2], Theorem B.

THEOREM 4: *If $\Delta^+(G)$ is finite,*

$$Q_{max}(K\Delta(G)) = Q_{cl}(K\Delta(G)). \tag{2}$$

PROOF: By [8], Lemma 19.3, $\Delta(G)/\Delta^+(G)$ is a torsion-free abelian group, so $\Delta(G)$ is contained in the class $\mathfrak{B}$ defined in [2].

In terms of the representation of $Q_{max}(K\Delta(G))$ by means of dense right ideals, (2) can be expressed as follows. The element ac^{-1} of $Q_{cl}(K\Delta(G))$ corresponds to the element $(cK\Delta(G),\ [ac^{-1}])$ of

$Q_{max}(K\Delta(G))$, where

$$[ac^{-1}]: cK\Delta(G) \to K\Delta(G): c\beta \to a\beta;$$

the identity (2) implies that every element of $Q_{max}(K\Delta(G))$ can be so represented. If we now embed $Q_{max}(K\Delta(G))$ in $Q_{max}(KG)$ as described above, we see that each element of $Q_{max}(K\Delta(G))$ can be represented by a pair $(cKG, [ac^{-1}])$, where a, $c \in K\Delta(G)$, $c \in \mathscr{C}_{KG}(0)$, and

$$[ac^{-1}]: cKG \to KG: c\beta \to a\beta. \tag{3}$$

Theorem 4 allows us to apply in the present context the results of Passman [7] on the structure of the centre of the classical right quotient ring of a group algebra. In fact, an examination of the proof of the main result of [7] shows that Passman actually obtains the following result.

THEOREM 5 [7, p. 224]: *Let T be a multiplicatively closed set of regular elements of the group algebra KG, such that KG satisfies the right Ore condition with respect to T. Suppose that*

$$\mathscr{C}_{KG}(0) \cap C(KG) \subseteq T.$$

If $\alpha \in C(Q)$, where Q denotes the partial right quotient ring of KG obtained by inverting the elements of T, then there exist elements a, $c \in C(KG)$, $c \in \mathscr{C}_{KG}(0)$, such that $\alpha = ac^{-1}$.

An important ingredient of Passman's proof of the above result is Lemma 2 of [7], which states that an element of $C(KG)$ is regular in $C(KG)$ if and only if it is regular in KG. In fact, Passman proves slightly more than this, and it is this stronger form of his result that we shall need.

LEMMA 6 [7, Lemma 2]: *Let α be a central element of KG, and put $H = \langle supp\ \alpha \rangle$. Then α is regular in KG if and only if α is a regular element of the ring $C(KG) \cap KH$.*

3. Proof of Theorem A

Theorem A will be proved by showing that when $\Delta^+(G)$ is finite, the rings with which we are concerned are actually classical quotient rings; in particular we shall show that $C(Q_{max}(KG))$ may be identified with $Q_{cl}(C(KG))$. Without assuming $\Delta^+(G)$ to be finite, we have

LEMMA 7: *$Q_{cl}(C(KG))$ is a subring of $C(Q_{max}(KG))$.*

PROOF: If $a \in C(KG)$ and $c \in \mathscr{C}_{C(KG)}(0)$, it follows from Lemma 6 that cKG is a two-sided ideal of KG with zero left annihilator – in particular, cKG is a dense right ideal of KG, by Lemma 1(f). Thus $(cKG, [ac^{-1}])$ represents, in our notation, an element of $Q_{max}(KG)$,, and it is easy to check that this affords an embedding of $Q_{cl}(C(KG))$ in $Q_{max}(KG)$. Now under the representation of $Q_{max}(KG)$ by homomorphisms on dense right ideals, KG embeds in $Q_{max}(KG)$ via the map

$$KG \to \mathrm{End}(KG \mid KG) : r \to \psi_r,$$

where

$$\psi_r : KG \to KG : \gamma \to r\gamma.$$

It follows that, with a and c as above, the element $(cKG, [ac^{-1}])$ of $Q_{max}(KG)$ commutes with every element of KG, and so lies in the centre of $Q_{max}(KG)$, by Lemma 1(a).

PROPOSITION 8: *If $\Delta^+(G)$ is finite,*

$$C(Q_{max}(KG)) = Q_{cl}(C(KG)).$$

PROOF: We have to prove that the embedding described in Lemma 7 is actually onto when $\Delta^+(G)$ is finite. By Theorems 3 and 4, and in particular the identity (3), each element α of $C(Q_{max}(KG))$ can be represented by a pair $(cKG, [ac^{-1}])$, where $a, c \in K\Delta(G)$, $c \in \mathscr{C}_{KG}(0)$, and

$$[ac^{-1}] : cKG \to KG : c\beta \to a\beta.$$

If r is any element of KG, then since $\alpha \in C(Q_{max}(KG))$, we have

$$[ac^{-1}r]d = [rac^{-1}]d \tag{4}$$

for all $d \in D$, where D, the intersection of the domains of the maps $[ac^{-1}r]$ and $[rac^{-1}]$, is a dense right ideal of KG, by Lemma 1(d). Now

$K\Delta(G)$ has a classical right quotient ring, by Theorem 4, so the subset $\mathscr{C}_{K\Delta(G)}(0)$ of $\mathscr{C}_{KG}(0)$ is a right Ore set in KG, by [10], Lemma 2.6. Thus by Lemma 1(c) we can form the partial right quotient ring Q of KG obtained by inverting the elements of $\mathscr{C}_{K\Delta(G)}(0)$. By Lemma 1(b), Q is a subring of $Q_{max}(KG)$, and by (4),

$$(ac^{-1}r - rac^{-1})D = 0,$$

where we view the multiplication as taking place in Q. However, by Lemma 1(e), DQ is a dense right ideal of Q, and so

$$ac^{-1}r = rac^{-1},$$

for all $r \in KG$.

We now apply Theorem 5, taking $T = \mathscr{C}_{K\Delta(G)}(0)$, so that the partial quotient ring Q is as defined above, to deduce that there exist b, $d \in C(KG)$, $d \in \mathscr{C}_{KG}(0)$, such that

$$bd^{-1} = ac^{-1}.$$

Thus the embedding defined in Lemma 7 is onto, and the proof is complete.

Theorem A will follow from the above result if we can show that

$$Q_{cl}(C(KG)) = Q_{max}(C(KG)) \tag{5}$$

under the assumption that $\Delta^{+}(G)$ is finite. Since

$$Q_{cl}(C(KG)) \subseteq Q_{max}(C(KG)),$$

by Lemma 1(c) and (b), to prove (5) it is enough to show that

$$Q_{max}[Q_{cl}(C(KG))] = Q_{cl}(C(KG));$$

see [6], §4.3, Proposition 2. This we shall do in Lemma 10, but first we must prove that $Q_{cl}(C(KG))$ is 'locally Artinian'. This follows from the next result and Lemma 6.

LEMMA 9: *Let H be a finitely generated normal subgroup of $\Delta(G)$.*

Then

$$Q_{cl}(C(KG) \cap KH)$$

is Artinian.

PROOF: Since H is a finitely generated FC-group, H contains a central subgroup of finite index. Since this subgroup is also finitely generated, H contains a torsion-free central subgroup Z, normal in G, such that $|H:Z| < \infty$. Since Z is torsion-free abelian, KZ is a domain. Let $\alpha \in KZ\backslash\{0\}$; then if

$$\alpha = \alpha_1, \alpha_2, \ldots, \alpha_n$$

denote the finitely many G-conjugates of α, which all lie in KZ as $Z \triangleleft G$, we have

$$0 \neq \alpha\alpha_2 \ldots \alpha_n \in C(KG) \cap KZ, \tag{6}$$

since KZ is commutative. Put $R = C(KG) \cap KZ$, so that $R^* = R\backslash\{0\}$ is a right divisor set of regular elements of KG. It follows from (6) that F, the partial quotient ring of KZ formed by inverting the elements of R^*, is a field, the quotient field of KZ. Furthermore, F contains the subfield

$$L = Q_{cl}(R) \subseteq Q_{cl}(C(KG) \cap KH).$$

We claim that $|F:L| < \infty$. Since Z is a finitely generated subgroup of $\Delta(G)$, and Z is normal in G, it follows that $C_G(Z)$ is a normal subgroup of finite index in G, so that $\bar{G} = G/C_G(Z)$ acts faithfully as a group of automorphisms of Z, and so of KZ. Notice that $\bar{G}$ fixes R, so that $\bar{G}$ acts on F as a group of field automorphisms, with fixed field L. To see this, observe that if $\beta\delta^{-1} \in F$ is fixed by all elements of G, where $\delta \in R$, then $\beta \in R$ and so $\beta\delta^{-1} \in L$. Hence by Galois Theory, [1], Theorem 14, $|F:L| = |\bar{G}| < \infty$.

Since $|H:Z| < \infty$, $KH \otimes_{KZ} F$ is a finite dimensional algebra over F, and so is an Artinian subring of $Q_{cl}(KH)$, so that

$$Q_{cl}(KH) = KH \otimes_{KZ} F.$$

Thus, since $|F:L| < \infty$, $Q_{cl}(KH)$ is a finite dimensional algebra over L. By Lemma 6,

$$L \subseteq Q_{cl}(C(KG) \cap KH) \subseteq Q_{cl}(KH),$$

and we deduce that $Q_{cl}(C(KG) \cap KH)$ is a finite dimensional L-algebra, as required.

REMARK: The argument to show that $|F:L| < \infty$ in the above proof is adapted from the proof of [8], Theorem 6.5.

LEMMA 10: *If $\Delta^+(G)$ is finite,*

$$Q_{cl}(C(KG)) = Q_{max}(C(KG)).$$

PROOF: As observed above, it is enough to prove that $Q_{cl}(C(KG))$ is its own maximal quotient ring. This will be accomplished if we can show that $Q_{cl}(C(KG))$ is the only dense ideal of $Q_{cl}(C(KG))$. We must prove, therefore, that every proper ideal of $Q_{cl}(C(KG))$ has non-zero annihilator.

We show first that:–

$$Q_{cl}(C(KG))/N[Q_{cl}(C(KG))] \text{ is regular.} \tag{7}$$

We shall denote the above ring by R. Take $a, c \in C(KG)$, $c \in \mathscr{C}_{KG}(0)$, such that $\alpha = ac^{-1} \in Q_{cl}(C(KG)) \backslash N[Q_{cl}(C(KG))]$, and put $H = \langle supp\ a, supp\ c \rangle$, so that H is a finitely generated normal subgroup of $\Delta(G)$. By Lemma 9.

$$Q_{cl}(C(KG) \cap KH)/N[Q_{cl}(C(KG) \cap KH)]$$

is Artinian. Note that

$$N[Q_{cl}(C(KG))] \cap Q_{cl}(C(KG) \cap KH) = N[Q_{cl}(C(KG) \cap KH)],$$

since $Q_{cl}(C(KG) \cap KH) \subseteq Q_{cl}(C(KG))$, by Lemma 6, and since $Q_{cl}(C(KG))$ is commutative. Since $\alpha \in Q_{cl}(C(KG) \cap KH)$, there exists $\beta \in Q_{cl}(C(KG) \cap KH)$ such that

$$\alpha - \alpha\beta\alpha \in N[Q_{cl}(C(KG) \cap KH)] \subseteq N[Q_{cl}(C(KG))].$$

Thus (7) follows, since $\beta \in Q_{cl}(C(KG))$.

By [5], Theorem 2.1, R is either Artinian, or R contains an infinite set of pairwise orthogonal idempotents. If the latter is the case, then by lifting over $N[Q_{cl}(C(KG))]$ we deduce that $Q_{cl}(C(KG))$ must have an infinite set of orthogonal idempotents. As previously noted, since $\Delta^+(G)$ is finite Theorem A of [2] implies that $Q_{cl}(K\Delta(G))$ exists and is

Artinian. Now $C(KG)$ is a subring of $K\Delta(G)$, and indeed it follows from Lemma 6 that $Q_{cl}(C(KG))$ is a subring of $Q_{cl}(K\Delta(G))$. In particular, therefore, $Q_{cl}(C(KG))$ must satisfy the descending chain condition on annihilator ideals. Since this precludes the existence of an infinite set of orthogonal idempotents in $Q_{cl}(C(KG))$, we deduce that R is Artinian, say

$$R = R\bar{e}_1 \oplus R\bar{e}_2 \oplus \ldots \oplus R\bar{e}_n,$$

where $\bar{e}_i^2 = \bar{e}_i \in R$, and $R\bar{e}_i$ is a field, $1 \leq i \leq n$.

Now

$$N[Q_{cl}(C(KG))] = N(C(KG))Q_{cl}(C(KG)),$$

and $N(C(KG)) \subseteq N(KG)$. Since $\Delta^+(G)$ is finite, $N(KG)$ is nilpotent, by [8], Theorem 20.3, and so $N[Q_{cl}(C(KG))]$ is also nilpotent. Lifting $\{\bar{e}_1, \bar{e}_2, \ldots, \bar{e}_n\}$ to a complete set of orthogonal idempotents $\{e_1, e_2, \ldots, e_n\}$ of $Q_{cl}(C(KG))$, we conclude that

$$Q_{cl}(C(KG)) = \sum_{i=1}^{n} {}^{\oplus} Q_{cl}(C(KG))e_i,$$

where for $i = 1, \ldots, n$,

$$\frac{Q_{cl}(C(KG))e_i}{N[Q_{cl}(C(KG))e_i]} \cong R\bar{e}_i,$$

which is a field, and $N[Q_{cl}(C(KG))e_i]$ is nilpotent, so that $Q_{cl}(C(KG))e_i$ is a local ring. It is now clear that $Q_{cl}(C(KG))$ is the only dense ideal of $Q_{cl}(C(KG))$, thus proving the lemma.

For convenience, we summarise the main steps in the proof of Theorem A.

PROOF OF THEOREM A:

We are given that $\Delta^+(G)$ is finite, and we have to show that

$$C(Q_{max}(KG)) \cong Q_{max}(C(KG)).$$

By Proposition 8,

$$C(Q_{max}(KG)) \cong Q_{cl}(C(KG)),$$

and by Lemma 10,

$$Q_{cl}(C(KG)) = Q_{max}(C(KG)).$$

The theorem follows.

REMARKS: (i) In view of the proof of Lemma 10, it is natural to ask whether $Q_{cl}(C(KG))$ is actually Artinian when $\Delta^+(G)$ is finite. I do not know the answer to this question. Clearly it would be sufficient to show that $N[Q_{cl}(C(KG))]$ is a finitely generated ideal of $Q_{cl}(C(KG))$, but although $N(KG)$ is a finitely generated right ideal of KG when $\Delta^+(G)$ is finite, by [8], Theorem 20.2, it is not clear that the desired result can be deduced from this fact.

(ii) Since, when $\Delta^+(G)$ is finite, $Q_{cl}(C(KG))$ is its own maximal quotient ring, one might suspect that this ring is always self-injective. This is the case when G is abelian, for example, by [2], Theorem B, and when KG is semiprime. However it is certainly not the case in general. For example, if we take

$$G = \langle a, b : a^4 = b^4 = 1,\ a^2 = b^2,\ b^{-1}ab = a^{-1}\rangle,$$

the quaternion group of order eight, and K is the field of two elements, then $C(KG)$ is Artinian, and so

$$Q_{cl}(C(KG)) = C(KG).$$

Now $C(KG)$ contains the ideals

$$I = \{0, (a + a^3)\},$$
$$J = \left\{0, \sum_{g \in G} g\right\},$$

and since the augmentation ideal A of KG is nilpotent, and $KG/A \cong K$, KG has no non-trivial idempotents. Thus if $C(KG)$ is a self-injective ring, it must be an essential extension of both I and J. This is clearly impossible.

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