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CORRECTION TO "CANONICAL DIVISORS AND THE
ADDITIVITY OF THE KODAIRA DIMENSION FOR
MORPHISMS OF RELATIVE DIMENSION ONE"**

Eckart Viehweg

Trying to make the application of duality theory as easy as possible, I made a silly mistake in Corollary 5.3, claiming, that the singularities of $V'$, $W'$ and $V_s$ are Gorenstein. However (as the participants of the Conference on Algebraic Geometry in Avignon pointed out to me) it is only true, that they are Cohen–Macaulay and that some power of the canonical sheaves are invertible. This is easily shown for every quotient singularity and since $\pi_s$ is a flat Gorenstein morphism, for $V_s$ too. You have to argue slightly differently to prove:

\[(6.7) \quad K(\omega_{V/W}, V_s) \leq K(\omega_{V/W}, V).\]

Using the notation of 6.7 and the arguments used in 6.5, we get an isomorphism $f^*\omega_V \cong \omega_V$, and hence a morphism $f^*\omega_V \to \omega_V$. We can apply 6.1 iii) to $h_1, h, g$ and to the flat morphism $\pi_s$. Choose $r \in \mathbb{N}$, such that the $r$-th power of every canonical sheaf is invertible. The trace map of $h_2$ gives a morphism $h_2^*: \omega_{V/W} \to \omega_{V/W}$ and therefore $h_2^*(\omega_{V/W})^\otimes r \to h_2^*\omega_{V/W}^r$. Using the exact sequence

\[0 \to \epsilon \to h_2^*(\omega_{V/W})^\otimes r \to \omega_{V/W}^r \to 0\]

we get an injection $\omega_{V/W}^r \to h_2^*\omega_{V/W}^r$ (everything is isomorphic on an open set!). Putting both parts together we have $f^*\omega_V^r \to h_2^*\omega_{V/W}^r \otimes \pi_s^*\omega_{V/W}^r$.

If we tensor both sides with $\left(\pi_s^*\omega_{V/W}^r\right)^{-1}$ we obtain the injection $f^*\omega_{V/W}^r \to h^*\omega_{V/W}^r$.