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A BANACH SPACE WITH A SYMMETRIC BASIS WHICH CONTAINS NO ℓ_p OR c_0, AND ALL ITS SYMMETRIC BASIC SEQUENCES ARE EQUIVALENT

Z. Altshuler*

Abstract

A Banach space having the properties described in the title of this paper is constructed.

In this paper we investigate the symmetric basic sequences in a Banach space with a symmetric basis. It is well known that the unit vector basis in the spaces c_0 and $\ell_p(1 \leq p < \infty)$, is a symmetric basis, and every symmetric basic sequence in each of these spaces is equivalent to it. A natural question is whether there exists any other Banach space X, with a symmetric basis $\{e_n\}_{n=1}^{\infty}$, which has the same property. Let us recall that by [1], it turns out that if, in addition to the assumption that every symmetric basic sequence in X is equivalent to $\{e_n\}_{n=1}^{\infty}$, we know that the same holds in X^*, the dual of X, with respect to $\{f_n\}_{n=1}^{\infty}$, the sequence of the biorthogonal functionals associated to $\{e_n\}_{n=1}^{\infty}$, then $\{e_n\}_{n=1}^{\infty}$ is equivalent to the unit vector basis of c_0 or ℓ_p, for some $1 \leq p < \infty$.

We answer the question raised above affimatively by proving the following

THEOREM: *There exists a Banach space X, with a symmetric basis $\{e_n\}_{n=1}^{\infty}$, such that all symmetric basic sequences in X are equivalent to each other, and X is not isomorphic to c_0 or ℓ_p, for any $1 \leq p < \infty$.*

* This is part of the author's Ph.D. thesis, prepared at the Hebrew University of Jerusalem under the supervision of Professor L. Tzafriri. The author wishes to thank Professor Tzafriri for his guidance.

Clearly a Banach space having the properties described in the theorem above contains no subspace isomorphic to c_0 or ℓ_p ($1 \leq p < \infty$). A natural candidate for such an example is the space constructed by Figiel and Johnson [2], which has a symmetric basis and no subspace of which is isomorphic to c_0 or ℓ_p. Our example is obtained by a modification of their construction. Before passing to the proof of the theorem we need some definitions and notations.

DEFINITION: *Let X be a Banach space with a symmetric basis $\{e_n\}_{n=1}^\infty$. Let N_i $i = 1, 2, \ldots$ be subsets of the set of natural numbers N, so that $\bar{\bar{N_i}} = \bar{\bar{N}}$ for every i, $N = \cup_{i=1}^\infty N_i$ and $N_i \cap N_j = \emptyset$ for all $i \neq j$. For any $0 \neq \alpha = \Sigma_i \alpha_i e_i \in X$ put $u_i^{(\alpha)} = \Sigma_{j=1}^\infty \alpha_j e_{i,j}$ where $N_i = \{i, j\}_{j=1}^\infty$ for $i = 1, 2, \ldots$. The sequence $\{u_i^{(\alpha)}\}_{i=1}^\infty$ is called a basic sequence generated by α.*

Clearly for any $\alpha \in X$ $\{u_i^{(\alpha)}\}_{i=1}^\infty$ is a symmetric basic sequence in X.

If $\{u_n\}_{n=1}^\infty$ and $\{v_n\}_{n=1}^\infty$ are basic sequences in Banach spaces X, respectively Y, we say that $\{u_n\}_{n=1}^\infty$ and $\{v_n\}_{n=1}^\infty$ are equivalent when a series $\Sigma_i \alpha_i u_i$ converges if and only if $\Sigma_i \alpha_i v_i$ converges. We write in this case $\{u_n\} \sim \{v_n\}$. We say that a basic sequence $\{u_n\}_{n=1}^\infty$ is bounded if there exists an $M > 0$ such that $M^{-1} < \inf_n \|u_n\| \leq \sup_n \|u_n\| < M$.

The first example of a Banach space which contains no ℓ_p or c_0 is due to Tsirelson [4]. Figiel and Johnson [2], described the dual of this space, which will be denoted by T, and showed that T contains no subsymmetric basic sequence. We also recall that the unit vector basis $\{x_n\}_{n=1}^\infty$ of T is an unconditional basis.

We are ready now to construct our example. First we define a sequence of norms, $|\cdot|_n$, on c_0, by

$$(1) \qquad |\alpha|_n = \sup_j \left[\sum_{i=1}^j \hat{\alpha}_i \omega_i / (2^n + 2^{-n} s_j) \right] \quad \text{where } \alpha = \{\alpha_i\}_{i=1}^\infty \in c_0$$

$\{\hat{\alpha}_i\}_{i=1}^\infty$ is the rearrangement in non-increasing order of $\{|\alpha_i|\}_{i=1}^\infty$, $\omega_i = i^{-1}$, and $s_j = \Sigma_{i=1}^j \omega_i$. Notice that since

$$(2) \qquad 2^{-n-1} \sup_j |\alpha_j| \leq |\alpha|_n \leq \left(\sup_j |\alpha_j| \right) \cdot s_j / (2^n + 2^{-n} s_j) \leq 2^n \sup_j |\alpha_j|$$

we have that for all $n = 1, 2, \ldots$, $|\cdot|_n$ is equivalent to the sup norm on c_0. We put now $Y = \{y \in c_0; \|\Sigma_n |y|_n x_n\|_T < \infty\}$ where $\{x_n\}_{n=1}^\infty$ is the unit vector basis of T. The space Y is a subspace (called the diagonal) of the direct sum $Z = (\Sigma_{n=1}^\infty \oplus (c_0, |\cdot|_n))_T$. Since for any unit vector e_j, $j = 1, 2, \ldots$ we get $|e_j|_n = (2^n + 2^{-n})^{-1}$, we deduce that the sequence of unit vectors $\{e_j\}_{j=1}^{\infty}$ belong to Y, and they clearly form a symmetric basis, with symmetric constant 1. We also remark that we may assume, without loss of generality, that every symmetric basic sequence in Y is equivalent to a symmetric block basic sequence of $\{e_n\}_{n=1}^{\infty}$. So in order to prove the theorem it suffices to check the block bases of $\{e_n\}_{n=1}^{\infty}$.

LEMMA 1: *Let $y_m = \Sigma_{i=p_m+1}^{p_{m+1}} \alpha_i e_i$ be a normalized block basis in Y. If $\lim_{i\to\infty} \alpha_i = 0$ then there exists a subsequence $\{y_{m_j}\}_{j=1}^{\infty}$ of $\{y_m\}_{m=1}^{\infty}$ which is equivalent to a block basis of $\{x_i\}_{i=1}^{\infty}$, the unit vector basis of T.*

PROOF: For fixed m and N we have by (2) that

$$\sum_{n=1}^{N-1} |y_m|_n \leq \sum_{n=1}^{N-1} 2^n \cdot \max\{|\alpha_i|; p_m < i \leq p_{m+1}\}.$$

Therefore we can construct inductively two increasing sequences of integers $\{m_j\}_{j=1}^{\infty}$, and $\{N_j\}_{j=1}^{\infty}$ such that $\|\Sigma_{n=N_j}^{\infty} |y_{m_j}|_n x_n\|_T < 2^{-j-1}$ for all $j \geq 1$ and $\|\Sigma_{n=1}^{N_{j-1}-1} |y_{m_j}|_n x_n\|_T < 2^{-j-1}$ for all $j > 1$. The block basis $\{y_{m_j}\}_{j=1}^{\infty}$ can be identified with the basic sequence $\{\hat{y}_{m_j}\}_{j=1}^{\infty}$ in Z where $\hat{y}_{m_j} = (y_{m_j}, y_{m_j}, \ldots, y_{m_j}, \ldots) \in Z$ $j = 1, 2, \ldots$. Put $v_j = (0, 0, \ldots, 0, \overset{N_{j-1}}{y_{m_j}}, \overset{N_{j-1}+1}{y_{m_j}}, \ldots, y_{y_{mj}}^{N_j-1}, 0, 0, \ldots) \in Z$ $j = 1, 2, \ldots$ and notice that for each j,

$$\|\hat{y}_{m_j} - v_j\|_Z = \left\| \sum_{n=1}^{N_{j-1}-1} |y_{m_j}|_n x_n + \sum_{n=N_j}^{\infty} |y_{m_j}|_n x_n \right\|_T < 2^{-j}.$$

Hence the basic sequence $\{y_{m_j}\}_{j=1}^{\infty}$ in Y is equivalent to $\{\hat{v}_j\}_{j=1}^{\infty}$ which, in turn, is equivalent to the block basis $z_j = \Sigma_{n=N_{j-1}}^{N_j-1} |y_{m_j}|_n x_n$ $j = 1, 2, \ldots$ of $\{x_n\}_{n=1}^{\infty}$.

We can already state some consequences of Lemma 1.

PROPOSITION 1: *Let Y and $\{e_i\}_{i=1}^{\infty}$ be as above. Then the following assertions are true*:

(i) *There is no symmetric block basis $y_m = \Sigma_{i=p_m+1}^{p_{m+1}} \alpha_i e_i$ $m = 1, 2, \ldots$ of $\{e_i\}_{i=1}^{\infty}$ such that the coefficients $\{\alpha_i\}_{i=1}^{\infty}$ tend to zero.*

(ii) *Y contains no subspace isomorphic to c_0 or ℓ_p for any $1 \leq p < \infty$.*

PROOF: The first assertion follows from Lemma 1 and the fact that T contains no subsymmetric basic sequence. To prove the second assertion we assume first that there is a block basis $\{u_j\}_{j=1}^{\infty}$ of $\{e_i\}_{i=1}^{\infty}$ which is equivalent to the unit vector basis of ℓ_p, for some $p \geq 1$. Since $\|\Sigma_{j=1}^{n} u_j\|_Y \to \infty$ as $n \to \infty$ it is easy to construct a block basis $\{v_m\}_{m=1}^{\infty}$ of $\{u_j\}_{j=1}^{\infty}$ with coefficients, in the expansion with respect to

$\{e_i\}_{i=1}^{\infty}$, tending to zero. The proof of this case can be then completed by using (i).

Suppose now that there is a block basis $\{u_j\}_{j=1}^{\infty}$ of $\{e_i\}_{i=1}^{\infty}$ which is equivalent to the unit vector basis of c_0. If the coefficients of the y_j's form a sequence tending to zero, then we complete the proof of (ii) by (i). Otherwise, it follows easily that $\{e_i\}_{i=1}^{\infty}$ itself is equivalent to the unit vector basis of c_0, hence for all $k = 1, 2, \ldots$ $\|\Sigma_{i=1}^{k} e_i\|_Y \leq M$, for some $M > 0$. On the other hand for any $k \geq 4$ we pick an integer $n = n(k)$ such that $s_k/2 < 2^{2n} \leq 2s_k$. For these values of k and $n = n(k)$ we have

$$\left\| \sum_{i=1}^{k} e_i \right\|_Y \geq \left| \sum_{i=1}^{k} e_i \right|_n = s_k/(2^n + 2^{-n}s_k) \geq \sqrt{s_k}/6 = \left(\sum_{i=1}^{k} i^{-1} \right)^{1/2} \Big/ 6 \to \infty \quad \text{as } k \to \infty.$$

We consider now block bases generated by one vector in Y.

Lemma 2: *Every block basis $\{u_i^{(\alpha)}\}_{i=1}^{\infty}$ of $\{e_j\}_{j=1}^{\infty}$ generated by a vector $\alpha \in Y$, is equivalent to $\{e_i\}_{i=1}^{\infty}$.*

Proof: Let $\{u_i^{(\alpha)}\}_{i=1}^{\infty}$ be a block basis generated by a vector $0 \neq \alpha = \Sigma_i \alpha_i e_i \in Y$. Then, for any $\beta = \Sigma_i \beta_i e_i \in Y$, we have $\|\Sigma_i \beta_i u_i^{(\alpha)}\| \geq (\sup_j |\alpha_j|)\|\Sigma_i \beta_i e_i\|$, so in order to prove that $\{u_i^{(\alpha)}\} \sim \{e_i\}$ we have to show that $\Sigma_i \beta_i u_i^{(\alpha)}$ converges, for any $\beta \in Y$. We first observe that it is enough to prove this for $\beta = \alpha$ i.e. to show that $\Sigma_i \alpha_i u_i^{(\alpha)}$ is a convergent series for any $0 \neq \alpha = \Sigma_i \alpha_i e_i \in Y$. Indeed, if this is true for any $\alpha = \Sigma_i \alpha_i e_i \in Y$ then for any $\beta = \Sigma_i \beta_i e_i \in Y$ we would get that $\Sigma_i (\alpha_i + \beta_i) u_i^{(\alpha+\beta)}$, and therefore also that $\Sigma_i \beta_i u_i^{(\alpha)}$, is a convergent series. Fix $\alpha = \{\alpha_i\}_{i=1}^{\infty} \in c_0$ with $1 \geq \alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_i \geq \cdots \geq 0$, and notice that in order to check whether $\Sigma_i \alpha_i u_i^{(\alpha)}$ converges in Y we have to compute the $|\cdot|_n$—norms of the double sequence $\{\alpha_i \alpha_j\}_{i,j=1}^{\infty}$, which is the expansion of $\Sigma_i \alpha_i u_i^{(\alpha)}$ with respect to $\{e_i\}_{i=1}^{\infty}$. Let $\alpha(t)$ be a non-increasing function on $[1, \infty)$ such that $\alpha(i) = \alpha_i$ for all i. If, for some integer m, $i \cdot j = m$ then at least one of the integers i or j is greater than or equal to $m^{1/2}$, and therefore $\alpha_i \alpha_j \leq \alpha(m^{1/2})$. It follows that the non-increasing rearrangement of $\{\alpha_i \alpha_i\}_{i,j=1}^{\infty}$ (as a one indexed sequence) is majorated by the sequence $\beta = \{\beta_i\}_{i=1}^{\infty}$ whose explicit form is

$$\beta = (\overbrace{\alpha(1^{1/2})}^{\tau(1)\text{ times}}, \overbrace{\alpha(2^{1/2}), \alpha(2^{1/2})}^{\tau(2)\text{ times}}, \ldots \overbrace{\alpha(m^{1/2}), \ldots, \alpha(m^{1/2})}^{\tau(m)\text{ times}}, \ldots),$$

where $\tau(m)$ is the number of distinct divisors of m. Thus, for every n, we have $|\Sigma_i \alpha_i u_i^{(\alpha)}|_n \le |\beta|_n$.

For each integer m, let $\varphi(m)$ be the first place where $\alpha(m)$ appears in the sequence β. Then, for $\varphi(m) \le k < \varphi(m+1)$ we have

$$\left(\sum_{i=1}^{k} \beta_i i^{-1}\right) \Big/ (2^n + 2^{-n} s_k) \le \left(\sum_{j=1}^{m} \sum_{i=\varphi(j)}^{\varphi(j+1)-1} \beta_i i^{-1}\right) \Big/ (2^n + 2^{-n} s_k)$$
$$\le \left(\sum_{j=1}^{m} \beta_{\varphi(j)} \varphi(j)^{-1} (\varphi(j+1) - \varphi(j))\right) \Big/ (2^n + 2^{-n} s_{\varphi(m)})$$
$$\le \left(\sum_{j=1}^{m} \alpha_j \varphi(j)^{-1} (\varphi(j+1) - \varphi(j))\right) \Big/ (2^n + 2^{-n} s_{\varphi(m)})$$

Since $s_{\varphi(m)} \ge \log \varphi(m)$ we get that

$$(3) \quad |\beta|_n \le \sup_m \left[\sum_{j=1}^{m} \alpha_j \varphi(j)^{-1} (\varphi(j+1) - \varphi(j))\right] \Big/ (2^n + 2^{-n} \log \varphi(m)) \quad \text{for all } n.$$

To estimate further the norm of β we use the quite known fact (see e.g. [3, p. 118]) that $\Sigma_{i=1}^{k} \tau(i) = k \log k + (2\gamma - 1)k + 0(k^{1/2})$ where $\gamma = 0.57721\ldots$ is the Euler constant. Notice that $\varphi(1) = 1$, and for $j > 1$ we have $\varphi(j) = \Sigma_{i=1}^{j^2-1} \tau(i) = (j^2 - 1) \log (j^2 - 1) + (2\gamma - 1)(j^2 - 1) + 0(j)$, consequently

$$(4) \quad \varphi(j) \ge 1 + c_1 j^2 \cdot \log j \quad \text{for all } j \ge 1, \text{ and some constant } c_1 > 0.$$

We also have

$$(5) \quad \varphi(j+1) - \varphi(j) = \sum_{i=1}^{(j+1)^2-1} \tau(i) - \sum_{i=1}^{j^2-1} \tau(i)$$
$$\le (j^2 + 2j) \log (j^2 + 2j) - (j^2 - 1) \log (j^2 - 1) + (2\gamma - 1)(2j + 1) + 0(j)$$
$$\le c_2(1 + j \log j) \quad \text{for some constant } c_2 > 0.$$

Using the fact that s_m behaves asymptotically as $\log m$ and substituting (4) and (5) in (3) we deduce that

$$|\beta|_n < c_3 \sup_m \left(\left(\sum_{j=1}^{m} \alpha_j j^{-1}\right) \Big/ (2^n + 2^{-n} s_m)\right) = c_3 |\alpha|_n, \quad \text{for all } n \text{ and some } c_3 > 0.$$

Since $\alpha \in Y$ implies that $\alpha \in c_0$ we get that $\|\beta\|_Y \le c_3 \|\alpha\|_Y$, i.e. $\|\Sigma_i \alpha_i u_i^{(\alpha)}\|_Y \le c_3 \|\alpha\|_Y$ for all $\alpha \in Y$.

We are ready to give the proof of the theorem. Let $y_m = \Sigma_{i=p_m+1}^{p_{m+1}} \alpha_i e_i$ be a symmetric normalized block basic sequence in Y. We may assume without loss of generality that $\alpha_{p_m+1} \ge \alpha_{p_m+2} \ge \cdots \alpha_{p_{m+1}} \ge 0$, for all $m = 1, 2, \ldots$. If $\sup_m (p_{m+1} - p_m) < +\infty$ then clearly $\{y_m\} \sim \{e_m\}$, hence we may assume also that $\sup_m (p_{m+1} - p_m) = +\infty$.

Suppose now that for any $\epsilon > 0$ there exists an integer $N = N(\epsilon)$ such that $\|\Sigma_{i=p_m+N}^{p_{m+1}} \alpha_i e_i\| < \epsilon$ for all m with $p_{m+1} - p_m \geq N$. In this case $\{y_m\}_{m=1}^\infty$ is equivalent to a block basis generated by one vector and thus, by Lemma 2, $\{y_m\} \sim \{e_m\}$. Indeed, for any $\epsilon > 0$, let $u_m = \Sigma_{i=1}^{p_{m+1}-p_m} \alpha_{i+p_m} e_i$ and $u'_m = \Sigma_{i=1}^{N(\epsilon)} \alpha_{i+p_m} e_i$ $m = 1, 2, \ldots$. Using the fact that for all $m = 1, 2, \ldots$ u'_m have at most the first $N(\epsilon)$ coordinates distinct from zero, and $\|u_m - u'_m\| < \epsilon$, we deduce that $\{u_m\}_{m=1}^\infty$ is a relatively compact set in Y. Hence there exists a subsequence $\{u''_m\}_{m=1}^\infty$ of $\{u_m\}_{m=1}^\infty$ such that $\lim_{m\to\infty} u''_m = \beta = \Sigma_i \beta_i e_i \in Y$. Clearly, a subsequence $\{u_{m_j}\}_{j=1}^\infty$ of $\{u''_m\}$ can be chosen such that $\|u_{m_j} - \beta\| < 2^{-j}$, and since $\|y_m\|_Y = \|u_m\|_Y = 1$, for all m, we have $\|\beta\|_Y = 1$. Notice that $\{u_{m_j}\}_{j=1}^\infty$ is a "translation" of $\{y_{m_j}\}_{j=1}^\infty$. Hence $\{y_{m_j}\}_{j=1}^\infty$ is equivalent to a block basis generated by β.

We treat now the case when such an $N(\epsilon)$ does not exist for all $\epsilon > 0$. In this case there exists an $\epsilon > 0$ and an increasing sequence of integers $\{p_{m_j}\}_{j=1}^\infty$ such that $p_{m_j+1} - p_{m_j} > j$ and $\|\Sigma_{i=p_{m_j}+j+1}^{p_{m_j+1}} \alpha_i e_i\|_Y \geq \epsilon$ for all j. Put $v_j = \Sigma_{i=p_{m_j}+j+1}^{p_{m_j+1}} \alpha_i e_i$ and $u_j = \Sigma_{i=p_{m_j}+1}^{p_{m_j+1}} \alpha_i e_i$. Notice that $\alpha_{p_{m_j}+j} > c$ for some constant c and every j imply $1 \geq \|u_j\|_Y \geq c\|\Sigma_{i=1}^j e_i\|$ $j = 1, 2, \ldots$ i.e. $c = 0$. Thus, $\lim_{j\to\infty} \alpha_{p_{m_j}+j} = 0$ which means that $\{v_j\}_{j=1}^\infty$ is a bounded block basis of $\{e_i\}_{i=1}^\infty$ with coefficients tending to zero. By Lemma 1 (and passing to a subsequence if necessary) we can assume that $\{v_j\}_{j=1}^\infty$ is equivalent to a block basis $\{z_j\}_{j=1}^\infty$ of $\{x_n\}_{n=1}^\infty$, the unit vector basis of T. The definition of the norm in T implies the existence of a constant $A_1 > 0$ such that, for every k, we have $\|\Sigma_{i=k+1}^{2k} z_i\|_T \geq A_1 k$. It follows that for every integer k and some constant $A_2 > 0$ $\|\Sigma_{j=1}^{2k} y_{m_j}\|_Y \geq \|\Sigma_{j=1}^{2k} v_j\|_Y \geq A_2 \|\Sigma_{j=1}^{2k} z_j\|_T \geq A_1 A_2 k$. Since $\{y_{m_j}\}_{j=1}^\infty$ is a symmetric basic sequence we get that $\{y_{m_j}\}_{j=1}^\infty$, and therefore $\{y_m\}_{m=1}^\infty$, is equivalent to the unit vector basis of ℓ_1, contrary to Proposition 1. This completes the proof of the theorem.

Remark: One can check that the unit balls determined by the norms $|\cdot|_n$ $n = 1, 2, \ldots$, are the sets $2^{-n}B_0 + 2^n B_d$, where B_0 and B_d are the unit balls of c_0, respectively of the Lorentz space $d(i^{-1}, 1)$. (Recall that $d(i^{-1}, 1)$ is the space of all sequences $\{\alpha_i\}_{i=1}^\infty \in c_0$ such that $\|\alpha\|_d = \Sigma_{i=1}^\infty \hat{\alpha}_i i^{-1} < \infty$, where $\{\hat{\alpha}_i\}_{i=1}^\infty$ is the non-increasing rearrangement of $\{|\alpha_i|\}_{i=1}^\infty$). Similarly, it can be shown that the sequence of norms $\|\cdot\|_n$ $n = 1, 2, \ldots$ defined by Figiel and Johnson in [2] can be given explictly by the formulas

$$\|\alpha\|_n = \sup_j \left[\left(\sum_{i=1}^j \hat{\alpha}_i \right) \Big/ (2^n + 2^{-n} j) \right] \quad n = 1, 2, \ldots.$$

In this case it is not true any more that for any $\alpha = \{\alpha_i\}_{i=1}^{\infty} \in c_0$ we have $\|\{\alpha_i\}_{i=1}^{\infty}\|_n \leq c\|\{\alpha_i\alpha_j\}_{i,j=1}^{\infty}\|_n$ for all n and some $c > 0$. Indeed, for the sequence $\alpha_i = i^{-1/2}$ we can find constants A, $B > 0$ such that $\|\{i^{-1/2}\}_{i=1}^{\infty}\|_n \leq A$ but $\|\{(ij)^{-1/2}\}_{i,j=1}^{\infty}\|_n \geq Bn^{1/2}$ for all $n = 1, 2, \ldots$.

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