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A DUALITY THEOREM FOR DIVISORS ON CERTAIN ALGEBRAIC CURVES

by

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Notations and introduction

In this paper k is an algebraically closed field. An algebraic curve X over k will be an irreducible algebraic k-scheme of dimension one, which is complete and which has no imbedded components, (i.e. X is locally defined by a primary ideal). So X may be non-reduced. But moreover we suppose that the Zariski tangent space in the generic point has dimension one. That means: an open non-void part of X is imbeddable in a smooth surface. Note that this restriction is, in contrast with the condition that X is imbeddable in a smooth surface, rather mild. For instance the curve with affine part

$$\operatorname{Spec}(k[\xi_1, \xi_2, \xi_3]/(\xi_2^2-\xi_3\xi_1, \xi_2\xi_3, \xi_3^2))$$

is not imbeddable in a smooth surface, but outside $\xi_1 = 0$ it is. (For a more detailed description of multiple algebraic curves see [3] or [5]).

In the Riemann-Roch formula for reduced algebraic curves the speciality index is explained with the help of differentials ([6] IV, 11). The Riemann-Roch formula has also been proved in the non-reduced case (without restrictions) ([5], 3.3.4) however without a calculation for the speciality index. The duality theorem in this paper explains this index.

Of course the machinery of Grothendieck-Hartshorne [2] produces a duality, but still for the curves we consider it is a big job to compute the dualizing complex (i.e. the generalization of the canonical divisor). The fact that the case of curves is already quite complicated depends on the fact that such a curve is not locally a complete intersection. We construct, without using an imbedding, a sheaf on X, which gives a duality for divisors on X. Our corstruction is different from the one given by Grothendieck-Hartshorne. It comes out that there can exist several sheaves giving a duality for divisors.

1 The differentials on a curve; repartitions

Let X be an algebraic curve, and let R be the ring of rational functions on X, that is the local ring in the generic point of X. We denote by $\mathfrak{n}$ the ideal of nilpotent elements of R. Let R_0 be the field of rational functions on the reduced curve $X_{\text{red}} = X_0$, then $R_0 = R/\mathfrak{n}$. As R contains k, it follows $R \to R_0$ can be lifted, so R is an R_0-algebra (in a non-canonical way) and $R \to R_0$ is an R_0-morphism ([4] 31, 1). We fix such an R_0-structure on R. The dimension of the R_0-vectorspace R is called the multiplicity n of X; there is a composition sequence for $\mathfrak{n}$ of length n

$$\mathfrak{n} = \mathfrak{n}_1 \supset \mathfrak{n}_2 \cdots \supset \mathfrak{n}_n = \{0\} \text{ with } \mathfrak{n}_i/\mathfrak{n}_{i+1} \simeq R_0.$$

Let $u_0 \in R\backslash\mathfrak{n}_1$ and choose $u_i \in \mathfrak{n}_i\backslash\mathfrak{n}_{i+1}$ (for $i = 1, \cdots, n-1$), then $\{u_0, \cdots, u_{n-1}\}$ is a base for the R_0-vectorspace R.

The R-(resp. R_0-) module of differentials $\Omega_{R/k}$(resp. $\Omega_{R_0/k}$) is defined in the usual way. We find

$$\begin{array}{ccccc} \Omega_{R_0/k} & \hookrightarrow & R \otimes_{R_0} \Omega_{R_0/k} & \to & \Omega_{R/k} \\ \omega & \mapsto & 1 \otimes \omega & & \\ & & f \otimes \omega & \mapsto & f\omega \end{array}$$

DEFINITION: The R-module $R \otimes_{R_0} \Omega_{R_0/k}$ is called *the module of the special one-differentials on* X and will be denoted by U_X.

Note that $U_X \hookrightarrow \Omega_{R/k}$ but that in general $U_X \not\simeq \Omega_{R/k}$. In the rest of the paper we use U_X, we don't use $\Omega_{R/k}$. It comes out that $\Omega_{R/k}$ is in general "too big". As known ([6] II,7) $\Omega_{R_0/k}$ is a free R_0-module of rank one, generated by some dt with $t \in R_0\backslash k$. Therefore U_X is a free R-module of rank one generated by dt.

For a curve X, let (X) be the set of closed points of X. As usual we define a repartition on X to be a family $\{r_P\}_{P\in(X)}$, with $r_P \in R$ and such that for almost all $P \in (X)$ we have $r_P \in \mathcal{O}_P$ (the local ring in the point P). The R-module of repartitions is denoted by A. Let D be a divisor on X we define

$$A(D) = \{\{r_P\}_{P\in(X)} \in A | r_P \in g^{-1}\mathcal{O}_P, \text{with } [g]_P = D(P)\}$$

($[g]_P = D(P)$ means g defines D in the point P). There is no difficulty to generalize ([6], II prop 3).

$$H^1(X, \mathcal{L}(D)) \simeq A/(A(D)+R). \tag{1.1}$$

We write $J(D) = \operatorname{Hom}_k(A/(A(D)+R), k)$ and $J = \cup J(D)$ (the union is taken over all divisors D on X).

2 The residue

We choose $\{u_0, \cdots, u_{n-1}\}$ as before and it is fixed until (4.2); any $f \in R$ can be uniquely written as

$$f = \sum_{i=0}^{n-1} \alpha_i(f)u_i \text{ with } \alpha_i(f) \in R_0 .$$

The local ring of the reduced curve in the closed point P is denoted by $\mathcal{O}_{0,P}$. For $\omega \in U_X$ and $\omega = gdt$ we define

$$\mathrm{Res}_P^i(\omega) = \mathrm{Res}_{\mathcal{O}_{0,P}}^{R_0}(\alpha_i(g)\,dt) \quad i = 0, 1, \cdots, n-1.$$

The residue $\mathrm{Res}_{\mathcal{O}_{0,P}}^{R_0}(-)$ is the residue defined by Tate ([7], 3); in case P is a non-singular point this definition is the usual one ([6], II.7) and in [7] this definition is extended to singular P. This definition is independent of the choice of t, but it depends on the choice of $\{u_0, \cdots, u_{n-1}\}$.

DEFINITION: $\sum_{i=0}^{n-1} \mathrm{Res}_P^i(\omega)$ is called *the residue of ω in the closed point P of X*; it is denoted by $\mathrm{Res}_P(\omega)$.

Note that Tate ([7], 3) already defined $\mathrm{Res}_{\mathcal{O}_P}^{R}(-)$ but this residue doesn't give enough information about "nilpotent" differentials (also if we take elements of $\Omega_{R/k}$), (see also [1] VIII § 2).

(2.1) LEMMA: *If $\omega \in U_X$ and $\{r_P\} \in A$, then* $\mathrm{Res}_P(r_P\omega) = 0$ *for almost all $P \in (X)$.*

PROOF: a) Let $f \in R$ and $f = \sum_{i=0}^{n-1} \alpha_i(f)u_i$ then $\alpha_i(f) \in R_0$ so for almost all $P \in (X)$ we have $\alpha_i(f) \in \mathcal{O}_{0,P}$ for $i = 0, 1, \cdots, n-1$.

b) Because $\{r_P\}$ is a repartition; $r_P \in \mathcal{O}_P$ for almost all $P \in (X)$. It follows that for almost all P we have $\alpha_i(r_P) \in \mathcal{O}_{0,P}$ for $i = 0, 1, \cdots, n-1$.

c) Let $u_iu_j = \sum_{i=0}^{n-1} t_{ijk}u_k$ with $t_{ijk} \in R_0$ then for almost all P we have $t_{ijk} \in \mathcal{O}_{0,P}$.

From a) b) c) and the well-known fact that

$$\mathrm{Res}_{\mathcal{O}_{0,P}}^{R_0}(fdg) = 0 \text{ if } f, g \in \mathcal{O}_{0,P},$$

it follows that $\mathrm{Res}_P(r_P\omega) = 0$ for almost all P.

From this lemma it follows that we can define:

DEFINITIONS: For $\omega \in U_X$ and $r = \{r_P\} \in A$

$$\langle \omega, r \rangle = \sum_{P \in (X)} \mathrm{Res}_P(r_P\omega).$$

Let D be a divisor; we consider the constant sheaf $\mathscr{U}$ given by U_X on X and define the subsheaf $\mathscr{U}(D)$ of U by

$$\mathscr{U}(D)_P = \{\omega \in U_X | \forall f \in \mathcal{O}_P, \mathrm{Res}_P(fg^{-1}\omega) = 0, \text{ with } [g]_P = D(P)\}.$$

We write $U(D) = H^0(X, \mathscr{U}(D))$.

3 Properties of $\langle -, - \rangle$

1) If $r \in R$ and $\omega \in U_X$, then $\langle \omega, r \rangle = 0$.

PROOF. It suffices to prove this in the reduced case with $r = 1$. For a non-singular curve see ([7] corollary of (3.3) or [6] II Proposition 6). For a curve X with singularities, let Y be the normalization of X. Let $Q \in (X)$ be a singular point and let $S_Q \subset (Y)$ be the set of point which are mapped onto Q, we have

$$\mathcal{O}_{Y,(S)} := \bigcap_{P \in S_Q} \mathcal{O}_{Y,P} = (\mathcal{O}_{X,Q})' \text{ the normalization of } \mathcal{O}_{X,Q}.$$

The k-vector space $(\mathcal{O}_{X,Q})'/\mathcal{O}_{X,Q}$ is finite dimensional hence ([7] 2, R1)

$$\sum_{P \in S} \mathrm{Res}_P(\omega) = \mathrm{Res}^{R_0}_{\mathcal{O}_{Y,(S)}}(\omega) = \mathrm{Res}^{R_0}_{\mathcal{O}_{X,Q}}(\omega)$$

and it follows that

$$\sum_{Q \in (X)} \mathrm{Res}_Q(\omega) = \sum_{P \in (Y)} \mathrm{Res}_P(\omega) = 0.$$

2) If $r \in A(D)$ and $\omega \in U(D)$ then $\langle \omega, r \rangle = 0$.

3) If $t \in R$ then $\langle t\omega, r \rangle = \langle \omega, rt \rangle$.

(3.1) THEOREM: *For every divisor D on X*

$$\langle -, - \rangle : U(D) \times A/A(D) + R) \to k$$

is a perfect pairing, hence the map

$$\Theta_D : U(D) \to J(D)$$

defined by $\Theta_D(\omega) = \langle \omega, - \rangle$ is an isomorphism.

PROOF: First note that if $\omega \in U_X$, then there is always a divisor D with $\omega \in U(D)$ so we have an R_0-linear map

$$\Theta : U_X \to J$$

CLAIM: Let $\omega \in U_X$ with $\Theta(\omega) \in J(D)$, then $\omega \in U(D)$.

PROOF: Suppose $\omega \notin U(D)$ so we can find P and $f \in \mathcal{O}_P$ such that $\mathrm{Res}_P(fg^{-1}\omega) \neq 0$ with $[g]_P = D(P)$. The repartition $\{r_Q\}$ with $r_Q = 0$ for $Q \neq P$ and $r_P = fg^{-1}$ is an element of $A(D)$ but $\langle \omega, \{r_Q\} \rangle \neq 0$, contradiction.

So it remains to prove Θ is an isomorphism. We first prove Θ to be injective. Let $\Theta(\omega) = 0$, then, by the claim, $\omega \in U(D)$ for all divisors D. Suppose $\omega \neq 0$ then $\omega = f dt = \sum_{i=0}^{n-1} \alpha_i(f) u_i dt$ with t a uniformizing element in a non-singular point P of X_0 and $\alpha_i(f) \neq 0$ for some i. Let i be such that $\alpha_i(f) \neq 0$ and $i = 0$ or $\alpha_j(f) = 0$ for $0 \leqq j < i$. From the fact that the Zariski tangent space of the generic point has dimension one it follows that there exists a $v \in R$ with $vu_i \in \mathfrak{n}_{n-1}\backslash\{0\}$, then $vu_i = su_{n-1}$ with $s \in R_0\backslash\{0\}$. There exists a divisor D such that then element $\alpha(_i f)^{-1}\ s^{-1}t^{-1}v$ of R belongs to $g^{-1}\mathcal{O}_P$ with $[g]_P = D(P)$. So $\mathrm{Res}_P(\alpha_i(f)^{-1}s^{-1}t^{-1}v\omega) = \mathrm{Res}_P(u_{n-1}t^{-1}dt) = 1_k$ so $\omega \notin U(D)$ contradiction. The R-module U_X is free of rank one, so it is a free R_0-module of rank n. The following lemma says that J is an R_0-module of rank $\leqq n$. Once this is proved, it follows that Θ is an isomorphism.

(3.2) LEMMA: $\dim_{R_0} J \leqq n$.

PROOF: a) $n = 1$.

For a non-singular curve see ([6] II,6 prop 4). Suppose X has singularities, let $\pi : Y \to X$ be the normalization morphism. We denote by A_X resp. by A_Y the repartitions on X resp. on Y. We have $A_X \subset A_Y$ and a divisor D on X induces a divisor $\pi^{-1}D$ on Y so we have a commutative diagram with exact rows

$$\begin{array}{ccccccccc}
0 & \longrightarrow & A_Y(\pi^{-1}D)+R & \longrightarrow & A_Y & \longrightarrow & A_Y/(A_Y(\pi^{-1}D)+R) & \longrightarrow & 0 \\
 & & \uparrow & & \uparrow & & \uparrow \varphi(D) & & \\
0 & \longrightarrow & A_X(D)+R & \longrightarrow & A_X & \longrightarrow & A_X/(A_X(D)+R) & \longrightarrow & 0
\end{array}$$

In general φ isn't an injection but it is if

$$(A_Y(\pi^{-1}D)+R)+A_X = A_X(D)+R.$$

We know

$$(A_Y(\pi^{-1}D)+R) \cap A_X = A'_X(D)+R \text{ where } A'_X(D) = \{\{r_P\} \in A_X | r_P \in g^{-1}\mathcal{O}'_P\},$$

with $[g]_P = D(P)$ and $\mathcal{O}'_P$ the normalization of $\mathcal{O}_P$. So if $A'_X(D)/A_X(D)$ is generated over k by $r \in R$ with $\{r\} \in A'_X(D)$, then $A'_X(D)+R = A_X(D)+R$. But $A'_X(D)/A_X(D)$ is finite dimensional, so for some D' with $D' > D$ and $D'(P) = D(P)$ for all singular points, we have $A'_X(D')+R = A_X(D')+R$ and hence $\varphi(D')$ is injective. So $J_Y(\pi^{-1}D') \to J_X(D')$ is surjective. Such divisors on X form a final system for the divisors on X, so $J_Y \to J_X$ is surjective and hence $\dim_R J_X \leqq \dim_R J_Y \leqq 1$.

b) n arbitrary.

A repartition $\{r_P\}$ on X gives n repartitions

$$\{\alpha_i(r_P)\}_{0 \leqq i \leqq n-1} \text{ on } X_0.$$

In lemma (2.1) b) is proved that these are repartitions. Denote by A_0 the space of repartitions on X_0, we have an isomorphism of R_0-vector-spaces $\varphi : A \to (A_0)^n$. An element $\alpha \in J$ is a morphism $\beta : A \to k$ with $\beta(A(D)+R) = 0$ for some D. Let

$$\beta_i : A_0 \hookrightarrow (A_0)^n \to A \xrightarrow{\beta} k$$

where $A_0 \hookrightarrow (A_0)^n$ is the inclusion on the i-th place. If we prove that β_i is zero on some $A_0(D^i)+R_0$, then we have proved φ^{-1} gives an injection

$$J \hookrightarrow (J_0)^n = \{\cup \operatorname{Hom}_k(A_0/(A_0(D)+R_0), k)\}^n.$$

And hence $\dim_{R_0} J \leqq \dim_{R_0}(J_0)^n = n$. So it remains to prove that $\beta_i(A_0(D^i)+R_0) = 0$ for some D^i.

a) $\beta_i(R_0) = 0$ because $\{r\} \in R_0$ corresponds with $\{ru_i\} \in R$ and $\beta(R) = 0$.

b) We construct D^i. First we construct divisors B^i that wipe out the possible negative influence of u_i. Let B^i for $i = 0, 1, \cdots, n-1$ be negative divisors on X_0 such that for all P, i we have $g_i^{-1}u_i \in \mathcal{O}_P$ where $[g_i]_P = B^i(P)$. We can define a divisor C on X_0 such that for all P we have $C(P) = [h]_P$ with $h \in f^{-1}\mathcal{O}_P$ where $[f]_P = D(P)$ and h is an element of R_0 by $R_0 \hookrightarrow R$. Define D^i by $D^i = C+B^i$ then D^i clearly satisfies $\beta_i(A_0(D^i)) = 0$.

(3.3) Corollary: *For each divisor D on X we have:*

$$\dim_k H^1(X, \mathscr{L}(D)) = \dim_k U(D);$$

in particular $\dim_k U(D)$ *doesn't depend on the choice of* $\{u_0, \cdots, u_{n-1}\}$.

4 The Riemann-Roch theorem

Let $\omega \in U_X$ so $\omega = g\,dt = \sum_{i=0}^{n-1} \alpha_i(g)u_i dt$. Suppose $\alpha_0(g) \neq 0$ then we define a sheaf

$$\mathscr{L}(\omega) \text{ on } X \text{ by } \mathscr{L}(\omega)_P = \{t \in R | \forall a \in \mathcal{O}_P, \operatorname{Res}_P(at\omega) = 0\}.$$

If $\omega = g'dt = \sum_{i=0}^{n-1} \alpha_i(g')u_i dt$ is any differential with $\alpha_0(g') \neq 0$ then there exists a $f \in R^*$ with $\omega' = f\omega$. It is clear that multiplication by f gives an isomorphism between $\mathscr{L}(\omega)$ and $\mathscr{L}(\omega')$. We have even more, see (4.2) and the remark at the end of the section.

Let D be a divisor, let $[g_P]_P = D(P)$. We have

$$\omega'' \in U(D) \Leftrightarrow \{\forall P\ \forall a \in \mathcal{O}_P, \operatorname{Res}_P(ag_P^{-1}\omega'') = 0\} \Leftrightarrow$$
$$\{f\omega = \omega'' \text{ with } \forall P, f \in g_P\mathscr{L}(\omega)_P\}. \text{ So } \omega'' \in U(D) \Leftrightarrow$$

$$\omega'' = f\omega \text{ with } f \in H^0(X, \mathscr{L}(\omega) \otimes_{\mathcal{O}_X} \mathscr{L}(-D)).$$

It is clear that

$$\dim_k H^0(X, \mathscr{L}(\omega) \otimes_{\mathcal{O}_X} \mathscr{L}(-D))$$

doesn't depend on the choice of ω, we denote it by $\ell(S-D)$. We can formulate the Riemann-Roch theorem:

(4.1) THEOREM: *Let D be a divisor on X, let $\mathscr{L}(D)$ be its divisorial sheaf and let* $\pi = \dim_k H^1(X, \mathcal{O}_X)$,

$$\deg(D) = \dim_k H^0(X, \mathscr{L}(D)/\mathscr{L}(D) \cap \mathcal{O}_X) - \dim_k H^0(X, \mathcal{O}_X/\mathscr{L}(D) \cap \mathcal{O}_X)$$
$$r(D) = \dim_k H^0(X, \mathscr{L}(D)) - \dim_k H^0(X, \mathcal{O}_X)$$

then

$$r(D) - \ell(S-D) = \deg(D) - \pi.$$

PROOF. Follows immediately from ([5], 3.3.4) and corollary (3.3).

REMARK: Although the notation $\ell(S-D)$ may suggest that S is a divisor, we haven't proved it. In fact the following examples show that S is in general not a divisor.

EXAMPLES: a) Let an affine part of a curve X be defined by Spec$k$ $[U, V, W]/(U^2-VW, W^2, VW)$(so X isn't a complete intersection) and take $R_0 \hookrightarrow R$ defined by $U \mapsto U$. Let $u_0 = 1$, $u_1 = V$ and $u_2 = UW$ and $\omega = dU$. An easy computation shows that in the point P given by $U = 0$ we have $\mathscr{L}(\omega)_P = \{$the $\mathcal{O}_P$-ideal generated by U and $V\}$. So $\mathscr{L}(\omega)$ isn't locally free.

b) Let an affine part of a curve X be given by Spec $(k[U, V]/V^2)$ and let $k(U) \hookrightarrow k(U)[V]/V^2$ be given by $U \mapsto U + U^{-1}V$ (so $R_0 \hookrightarrow R$ doesn't define a section in the point $U = 0$). Let $u_0 = 1$ and $u_1 = V$ and let $\omega = dU$ then $\mathscr{L}(\omega)_{U=0} = (U^2, V)$. If we take $R_0 \hookrightarrow R$ given by $U \mapsto U$ we get a locally free $\mathscr{L}(\omega)$ hence $\mathscr{L}(\omega)$ depends on the choice of $R_0 \hookrightarrow R$.

(4.2) PROPOSITION: *The sheaf $\mathscr{L}(\omega)$ doesn't depend on the choice of* $\{u_0, \cdots, u_{n-1}\}$.

PROOF. To indicate the system $u = \{u_0, \cdots, u_{n-1}\}$ we use, we denote in the proof $\mathscr{L}(\omega)$ by $\mathscr{L}(\omega)_u$. We will give an isomorphism between $\mathscr{L}(\omega)_u$ and $\mathscr{L}(\omega)_{u'}$ where u' is obtained from u by a "basic" change, namely

a) $$u' = \{u_0, \cdots, u_i + u_{i+1}, u_{i+1}, \cdots, u_{n-1}\}.$$

b) $$u' = \{u_0, \cdots, u_{i-1}, fu_i, u_{i+1}, \cdots, u_{n-1}\}.$$

Note that combinations of a) and b) give all possible changes. Let $u_i u_j = \sum \alpha_{i,j,k} u_k$ with $\alpha_{i,j,k} \in R_0$. So

1) $\alpha_{i,j,k} \in R_0^*$ if $k = i+j$.

2) $\alpha_{i,j,k} = 0$ if $k < i+j$.

a) Define $\mathscr{L}(\omega)_u \xrightarrow{\sim} \mathscr{L}(\omega)_{u'}$ by multiplication by a particular function g. Before writting out g we tell the principle of the construction of g. The residue of an element $\sum \alpha_i u_i dt$ is in fact the residue in the reduced case of $(\sum \alpha_i)dt$. We construct a function g such that this sum remains the same. We start the construction for elements of the form $\alpha_{n-1}u_{n-1}dt$, then for $(\alpha_{n-2}u_{n-2}+\alpha_{n-1}u_{n-1})dt$ etc. We have $g = 1u_0+0u_1+\cdots 0u_{n-i-2}+\sum_{l=n-i-1}^{n-1}\beta_l u_l$ β_1 is defined by $\beta_1 = (\alpha_{l,n-l-1,n-1})^{-1}(1-\sum_{k=0}^{n-1}\gamma_k^1)$ with $\sum_{k=0}^{n-1}\gamma_k^l u_k' = u_{n-l-1}(1u_0+0u_1+ \cdot \cdot \cdot \; 0u_{n-i-2}+\sum_{m=ni-i-1}^{l-1}\beta_m u_m)$. Denote the residue in P computed with u by $\mathrm{Res}_P^u(-)$. It is easy to see that $\mathrm{Res}_P^u(at\omega) = \mathrm{Res}_P^{u'}(agt\omega)$ and hence

$$t \in (\mathscr{L}(\omega)_u)_P \Leftrightarrow gt \in (\mathscr{L}(\omega)_{u'})_P.$$

b) In this case we can use a similar construction.

So $\mathscr{L}(\omega)$ depends only on the choice of $R_0 \hookrightarrow R$ and example b) shows that different maps $R_0 \hookrightarrow R$ can give non-isomorphic sheaves (see also [5] page 3).

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