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## SIMPLICIAL MAPS WHICH STABILIZE TO NEAR-HOMEOMORPHISMS

by

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### 1. Introduction

Let  $\mathcal{U}$  be an open cover of a space  $Y$ . Maps  $f, g : X \rightarrow Y$  are  $\mathcal{U}$ -close if for each  $x$  in  $X$ ,  $f(x)$  and  $g(x)$  lie in some member of  $\mathcal{U}$ . A map  $f : X \rightarrow Y$  is a *near-homeomorphism* if it can be uniformly approximated by homeomorphisms – i.e., for every open cover  $\mathcal{U}$  of  $Y$  there exists a homeomorphism  $h : X \rightarrow Y$  such that  $f$  and  $h$  are  $\mathcal{U}$ -close. If  $f \times \text{id} : X \times Q \rightarrow Y \times Q$  is a near-homeomorphism, where  $Q = \prod_1^\infty [0, 1]_i$  is the Hilbert cube, then  $f$  stabilizes to a near-homeomorphism.

The recognition of (stable) near-homeomorphisms, and their application in inverse limit calculations (see below), play an important role in the recent proof by Schori and West [7] that  $2^I$  is homeomorphic to  $Q$ . It seems likely that techniques involving near-homeomorphisms will be useful in further investigations of hyperspaces.

Our main theorem (3.2) characterizes the stable near-homeomorphisms in the simplicial category as the surjections with compact and contractible point-inverses. The proof is by means of  $Q$ -factor decompositions, discussed in § 2.

Brown showed in [3] that if  $(X_i, f_i)$  is an inverse sequence such that each  $X_i$  is a copy of a compact metric space  $X$  and each  $f_i$  is a near-homeomorphism, then  $\text{Lim } (X_i, f_i)$  is homeomorphic to  $X$ . In § 4 we note some immediate applications using (3.2), and extend Brown's theorem to complete metric spaces.

### 2. $Q$ -factor decompositions

A space  $X$  is a  $Q$ -factor if  $X \times Q \simeq Q$ . Note that if  $X \times Y \simeq Q$ , then  $X \times Q \simeq X \times (X \times Y)^\infty \simeq (X \times Y)^\infty \simeq Q$ , and  $X$  is a  $Q$ -factor. Every  $Q$ -factor is a compact metric  $AR$ ; it is not known whether the converse is true. West [8] has shown that every compact contractible polyhedron is a  $Q$ -factor.

A closed subset  $A$  of  $X$  is a  $Z$ -set in  $X$  if for every nonempty open

homotopically trivial ( $n$ -connected for all  $n \geq 0$ ) subset  $U$  of  $X$ ,  $U \setminus A$  is nonempty and homotopically trivial.  $Z$ -sets were introduced by Anderson [2], who showed that every homeomorphism between  $Z$ -sets in  $Q$  extends to a homeomorphism of  $Q$ . The endslice  $W = \{0\} \times \prod_2^\infty [0, 1]_i \subset Q$  is a  $Z$ -set; in general, boundaries and collared sets are  $Z$ -sets. One useful technique for verifying the  $Z$ -set property is the following:

2.1. LEMMA (cf. [8], Lemma 2.2). *A closed subset  $A$  of a metric ANR,  $X$  is a  $Z$ -set in  $X$  if for each  $\varepsilon > 0$  there exists a map  $f : X \rightarrow X \setminus A$  with  $d(f, \text{id}) < \varepsilon$ .*

PROOF. Clearly  $A$  is nowhere dense. Let  $U$  be open and homotopically trivial, and  $g : S^n \rightarrow U \setminus A$  a map of the  $n$ -sphere. There exists an extension  $\bar{g} : C^{n+1} \rightarrow U$  of  $g$  to the  $(n+1)$ -cell. As a metric ANR,  $X$  is locally equiconnected, and therefore has the property that for every open cover  $\mathcal{V}$  there exists an open cover  $\mathcal{W}$  such that maps into  $X$  which are  $\mathcal{W}$ -close are  $\mathcal{V}$ -homotopic (paths of the homotopy lie in members of  $\mathcal{V}$ ) [6]. By the compactness of  $C^{n+1}$  there exists  $\varepsilon > 0$  such that for any map  $f : X \rightarrow X \setminus A$  with  $d(f, \text{id}) < \varepsilon$ ,  $f\bar{g}(C^{n+1}) \subset U \setminus A$  and  $g$  is homotopic to  $f \circ g$  in  $U \setminus A$ . This homotopy together with the map  $f \circ \bar{g}$  provides an extension  $\tilde{g} : C^{n+1} \rightarrow U \setminus A$  of  $g$ .

2.2. DEFINITION.  $\{X_\alpha\}$  is a  $Q$ -factor decomposition of a Hausdorff space  $X$  if:

- i)  $\{X_\alpha\}$  is a locally finite cover of  $X$  by  $Q$ -factors,
- ii)  $X_1, X_2 \in \{X_\alpha\}$  and  $X_1 \cap X_2 \neq \phi$  imply  $X_1 \cap X_2 \in \{X_\alpha\}$ ,
- iii)  $X_1, X_2 \in \{X_\alpha\}$  and  $X_1 \not\subseteq X_2$  imply  $X_1$  is a  $Z$ -set in  $X_2$ .

The spaces admitting  $Q$ -factor decompositions comprise a proper subclass of the class of locally compact metrizable ANR's, and include the locally compact polyhedra.

2.3. DEFINITION.  $Q$ -factor decompositions  $\{X_\alpha\}$  and  $\{Y_\alpha\}$  indexed by the same set are *similar* if  $X_1 \cap X_2 \neq \phi$  is equivalent to  $Y_1 \cap Y_2 \neq \phi$ .  $\{X_\alpha\}$  and  $\{Y_\alpha\}$  are *isomorphic* if  $X_1 \subset X_2$  is equivalent to  $Y_1 \subset Y_2$ .

Isomorphic decompositions are similar: if  $X_1 \cap X_2 \neq \phi$ , then  $X_1 \cap X_2 = X_3 \in \{X_\alpha\}$ ,  $X_3 \subset X_1$  and  $X_3 \subset X_2$ , therefore  $Y_3 \subset Y_1$  and  $Y_3 \subset Y_2$ , and  $Y_1 \cap Y_2 \supset Y_3 \neq \phi$ .

For any space  $X$ ,  $\tau^n : X \times Q \rightarrow I^n$  will denote the projection onto the first  $n$  factors of  $Q$ .

2.4. THEOREM. *Let  $\{X_\alpha\}$  and  $\{Y_\alpha\}$  be isomorphic  $Q$ -factor decompositions of  $X$  and  $Y$ , respectively, and let a function  $p : A \rightarrow \mathbb{Z}^+$  from the indexing set into the positive integers be given. Then there exists a homeomorphism  $H : X \times Q \rightarrow Y \times Q$  such that  $H(X_\alpha \times Q) = Y_\alpha \times Q$  and  $\tau^{p(\alpha)}/X_\alpha \times Q = \tau^{p(\alpha)}/Y_\alpha \times Q$  for each  $\alpha$ .*

PROOF. Since  $X_\alpha = X_\beta$  is equivalent to  $Y_\alpha = Y_\beta$ , and since  $\{X_\alpha|X_\alpha = X_\beta\}$  is a finite collection for each  $X_\beta$ , there is no loss of generality in assuming that the isomorphic decompositions  $\{X_\alpha\}$  and  $\{Y_\alpha\}$  are faithfully indexed – i.e.,  $X_\alpha = X_\beta$  only if  $\alpha = \beta$ . For any subcollection  $\{X_\alpha|\alpha \in B \subset A\}$  of  $\{X_\alpha\}$ , let  $\text{Min} \{X_\alpha|\alpha \in B\} = \{X_\alpha|\alpha \in B; \beta \in B \text{ with } X_\beta \subset X_\alpha \text{ implies } \alpha = \beta\}$ , the collection of minimal elements. Inductively define  $X^{(i)} = X^{(i-1)} \cup \text{Min} \{X_\alpha|X_\alpha \not\subset X^{(i-1)}\}$ ,  $i \geq 0$ , with  $X^{(-1)} = \phi$ . Then  $\{X_\alpha\} = \bigcup X^{(i)}$ ; similarly  $\{Y_\alpha\} = \bigcup Y^{(i)}$ . It is easily seen that  $X_\alpha \in X^{(i)}$  is equivalent to  $Y_\alpha \in Y^{(i)}$ . Since the indicator function  $p : A \rightarrow Z^+$  can be redefined by setting  $p'(\alpha) = \max \{p(\beta)|X_\alpha \subset X_\beta\}$ , we may assume that  $X_\alpha \subset X_\beta$  implies  $p(\beta) \leq p(\alpha)$ .

For each  $\alpha$ , let  $\mathcal{H}_\alpha$  denote the non-empty collection of homeomorphisms of  $X_\alpha \times Q$  onto  $Y_\alpha \times Q$  of the form  $h_\alpha = \tilde{h}_\alpha \times \text{id}_\alpha$ , where  $\tilde{h}_\alpha : X_\alpha \times \prod \{I_i|i > p(\alpha)\} \rightarrow Y_\alpha \times \prod \{I_i|i > p(\alpha)\}$  and  $\text{id}_\alpha$  is the identity map on  $I^{p(\alpha)} = \prod_1^{p(\alpha)} I_i$ . Suppose inductively that there exists a homeomorphism  $H_i : \bigcup \{X_\alpha|X_\alpha \in X^{(i)}\} \times Q \rightarrow \bigcup \{Y_\alpha|Y_\alpha \in Y^{(i)}\} \times Q$  such that  $H_i/X_\alpha \times Q$  is in  $\mathcal{H}_\alpha$  for each  $X_\alpha \in X^{(i)}$ . Consider  $X_\beta \in X^{(i+1)} \setminus X^{(i)} = \text{Min} \{X_\alpha|X_\alpha \not\subset X^{(i)}\}$ , and set  $\tilde{X}_\beta = \bigcup \{X_\alpha|X_\alpha \not\subset X_\beta\}$ . Then  $\tilde{X}_\beta$ , as a finite union of  $Z$ -sets, is a  $Z$ -set in  $X_\beta$  (it may be empty), and  $\tilde{X}_\beta = X_\beta \cap (\bigcup \{X_\alpha|X_\alpha \in X^{(i)}\})$ . Similarly for  $\tilde{Y}_\beta$ ; note that  $H_i(\tilde{X}_\beta \times Q) = \tilde{Y}_\beta \times Q$ . Since  $p(\beta) \leq p(\alpha)$  for each  $X_\alpha \subset X_\beta$ , an application of Anderson's homeomorphism extension theorem to  $X_\beta \times \prod \{I_i|i > p(\beta)\}$  and  $Y_\beta \times \prod \{I_i|i > p(\beta)\}$  shows there exists  $h_\beta \in \mathcal{H}_\beta$  such that  $h_\beta/\tilde{X}_\beta \times Q = H_i/\tilde{X}_\beta \times Q$ . For distinct elements  $X_\alpha$  and  $X_\beta$  of  $X^{(i+1)} \setminus X^{(i)}$ , either  $X_\alpha \cap X_\beta = \phi$  or  $X_\alpha \cap X_\beta \in X^{(i)}$ . Since  $\{X_\alpha\}$  is a locally finite closed cover of  $X$ , we may define  $H_{i+1} : \bigcup \{X_\alpha|X_\alpha \in X^{(i+1)}\} \times Q \rightarrow \bigcup \{Y_\alpha|Y_\alpha \in Y^{(i+1)}\} \times Q$  by requiring that  $H_{i+1}$  extend  $H_i$  and  $H_{i+1}/X_\beta \times Q = h_\beta$  for each  $X_\beta \in X^{(i+1)} \setminus X^{(i)}$ . Then  $H : X \times Q \rightarrow Y \times Q$  defined by  $H/X_\alpha \times Q = H_i/X_\alpha \times Q$  for  $X_\alpha \in X^{(i)}$ ,  $i \geq 0$ , is the desired homeomorphism.

In [5] we obtain an extension of (2.4) to similar  $Q$ -factor decompositions, in which the requirement  $H(X_\alpha \times Q) = Y_\alpha \times Q$  is replaced by  $H(X_\alpha \times Q) \subset \text{St}(Y_\alpha) \times Q$ . This result promises to be useful in recognizing stable near-homeomorphisms in situations where Theorem 3.2 (see below) does not apply.

### 3. Stable near-homeomorphisms

In this section we shall be dealing with simplicial maps between locally finite complexes. A map  $f : K \rightarrow L$  is *compact* or *contractible* if  $f^{-1}(x)$  is compact or contractible for each  $x$  in  $L$ .

3.1. LEMMA. *Let  $f : K \rightarrow L$  be a compact contractible simplicial surjection, and let  $\mathcal{U}$  be an open cover of  $L$ . Then there exist isomorphic  $Q$ -*

factor decompositions  $\{K_\alpha\}$  of  $K$  and  $\{L_\alpha\}$  of  $L$  such that  $\{L_\alpha\}$  refines  $\mathcal{U}$  and  $K_\alpha = f^{-1}(L_\alpha)$  for each  $\alpha$ .

PROOF. It is well-known that there exist subdivisions  $K_*$  of  $K$  and  $L_*$  of  $L$  such that  $f: K_* \rightarrow L_*$  is simplicial and the cover by vertex stars of  $L_*$  refines  $\mathcal{U}$ . For notational convenience assume that  $K = K_*$  and  $L = L_*$ . We show that the dual structures on  $K$  and  $L$  described by Cohen [4] are the desired  $Q$ -factor decompositions.

Let  $L'$  be the standard barycentric subdivision of  $L$ , and let  $K'$  be a barycentric subdivision of  $K$  chosen so that  $f: K' \rightarrow L'$  is simplicial. The barycenter of a simplex  $\sigma$  is denoted by  $\hat{\sigma}$ . If  $\sigma_0 \subset \cdots \subset \sigma_q$ , then  $\hat{\sigma}_0 \cdots \hat{\sigma}_q$  is the simplex spanned by the barycenters. If  $\alpha$  is a simplex of  $L$ , then  $D(\alpha, L)$ , the dual to  $\alpha$  in  $L$ , and its subcomplex  $\dot{D}(\alpha, L)$  are defined by  $D(\alpha, L) = \{\hat{\sigma}_0 \cdots \hat{\sigma}_q | \alpha \subset \sigma_0 \subset \cdots \subset \sigma_q\}$ ,  $\dot{D}(\alpha, L) = \{\hat{\sigma}_0 \cdots \hat{\sigma}_q | \alpha \not\subseteq \sigma_0 \subset \cdots \subset \sigma_q\}$ .  $D(\alpha, f)$ , the dual to  $\alpha$  with respect to  $f$ , is a subcomplex of  $K'$  defined by  $D(\alpha, f) = \{\hat{\tau}_0 \cdots \hat{\tau}_q | \alpha \subset f(\tau_0), \tau_0 \subset \cdots \subset \tau_q\}$ ; similarly for  $\dot{D}(\alpha, f)$ . Each dual  $D(\alpha, L)$  is a finite subcomplex of  $L'$ , and since  $f$  is a compact surjection each  $D(\alpha, f)$  is also finite and non-empty. Clearly  $D(\alpha, L)$  is the join  $\hat{\alpha}\dot{D}(\alpha, L)$ . It is known [4] that  $D(\alpha, f) = f^{-1}D(\alpha, L)$ ,  $\dot{D}(\alpha, f) = f^{-1}\dot{D}(\alpha, L)$ , and  $D(\alpha, f)$  collapses to  $f^{-1}(\hat{\alpha})$ .

Set  $\{K_\alpha\} = \{D(\alpha, f)\}$  and  $\{L_\alpha\} = \{D(\alpha, L)\}$ , where  $\alpha$  runs through all the simplexes of  $L$ . Then  $\{K_\alpha\}$  and  $\{L_\alpha\}$  are isomorphic locally finite covers of  $K$  and  $L$ , and  $\{L_\alpha\}$  refines  $\mathcal{U}$ . Each dual  $D(\alpha, L)$  is contractible, and since  $f$  is contractible each dual  $D(\alpha, f)$  is contractible. It follows from West's theorem (see § 2) that each dual is a  $Q$ -factor. If  $D(\alpha, L) \cap D(\beta, L) \neq \phi$ , then  $\alpha$  and  $\beta$  span a simplex  $\gamma$  and  $D(\alpha, L) \cap D(\beta, L) = D(\gamma, L)$ . In this case  $D(\alpha, f) \cap D(\beta, f) = f^{-1}D(\alpha, L) \cap f^{-1}D(\beta, L) = f^{-1}(D(\alpha, L) \cap D(\beta, L)) = f^{-1}D(\gamma, L) = D(\gamma, f)$ . If  $D(\beta, L) \not\subseteq D(\alpha, L)$ , then  $\alpha \not\subseteq \beta$ ,  $D(\beta, L) \subset \dot{D}(\alpha, L)$ , and  $D(\beta, f) \subset \dot{D}(\alpha, f)$ . Since  $D(\alpha, L) = \hat{\alpha}\dot{D}(\alpha, L)$  there exists for each  $\varepsilon > 0$  a map  $r: D(\alpha, L) \rightarrow D(\alpha, L) \setminus \dot{D}(\alpha, L)$  with  $d(r, \text{id}) < \varepsilon$ , and such that  $x$  and  $r(x)$  have the same carrier in  $L$ . Since  $f: K \rightarrow L$  is simplicial, the map  $r$  can be lifted to a map  $\bar{r}: D(\alpha, f) \rightarrow D(\alpha, f) \setminus \dot{D}(\alpha, f)$  such that  $f \circ \bar{r} = r \circ f$  on  $D(\alpha, f)$  and  $d(\bar{r}, \text{id}) < \varepsilon$ . It follows from (2.1) that  $\dot{D}(\alpha, L)$  and  $\dot{D}(\alpha, f)$ , and therefore  $D(\beta, L)$  and  $D(\beta, f)$ , are  $Z$ -sets in  $D(\alpha, L)$  and  $D(\alpha, f)$ , respectively. This completes the proof that  $\{K_\alpha\}$  and  $\{L_\alpha\}$  are  $Q$ -factor decompositions.

3.2. THEOREM. *A simplicial map  $f: K \rightarrow L$  stabilizes to a near-homeomorphism if and only if  $f$  is a compact contractible surjection.*

PROOF. Suppose  $f$  is a compact contractible surjection. Let  $\mathcal{W}$  be an open cover of  $L \times Q$ . There exists an open cover  $\mathcal{U}$  of  $L$  and a function  $m: \mathcal{U} \rightarrow Z^+$  such that for  $(x_1, q_1)$  and  $(x_2, q_2)$  in  $L \times Q$  with  $\{x_1, x_2\} \subset U \in \mathcal{U}$  and  $\tau^{m(U)}(q_1) = \tau^{m(U)}(q_2)$ ,  $\{(x_1, q_1), (x_2, q_2)\} \subset W \in \mathcal{W}$ . By (3.1)

there exist isomorphic  $Q$ -factor decompositions  $\{K_\alpha\}$  of  $K$  and  $\{L_\alpha\}$  of  $L$  such that  $\{L_\alpha\}$  refines  $\mathcal{U}$  and  $K_\alpha = f^{-1}(L_\alpha)$ . Define  $p : A \rightarrow Z^+$  by  $p(\alpha) = \min \{m(U) \mid L_\alpha \subset U \in \mathcal{U}\}$ . By (2.4) there exists a homeomorphism  $H : K \times Q \rightarrow L \times Q$  such that  $H(K_\alpha \times Q) = L_\alpha \times Q$  and  $\tau^{p(\alpha)} K_\alpha \times Q = \tau^{p(\alpha)} H / K_\alpha \times Q$  for each  $\alpha$ . Clearly  $H$  and  $f \times \text{id}$  are  $\mathcal{W}$ -close.

Conversely, suppose that  $f \times \text{id}$  is a near-homeomorphism. Since the image of  $f \times \text{id}$  must be dense in  $L \times Q$ ,  $f$  is surjective. Consider a point  $x$  in  $L$  and the inverse  $f^{-1}(x) \subset K$ . Since there exists a homeomorphism of  $K \times Q$  onto  $L \times Q$  taking  $f^{-1}(x) \times Q$  into a compact neighborhood of  $\{x\} \times Q$ ,  $f^{-1}(x)$  is compact. (The same argument shows that the inverse image of every compact set is compact.) Since  $f$  is simplicial  $f^{-1}(x)$  is polyhedral and therefore a retract of some neighborhood  $U$  in  $K$ . Using compactness of the inverse image of a compact neighborhood of  $x$ , we obtain a neighborhood  $V$  of  $x$  such that  $f^{-1}(V) \subset U$ . Then there exists a contractible neighborhood  $W$  of  $x$  and a homeomorphism  $H : K \times Q \rightarrow L \times Q$  such that  $H(f^{-1}(x) \times Q) \subset W \times Q \subset H(U \times Q)$ . Thus  $f^{-1}(x) \times Q$  is contractible in the neighborhood  $U \times Q$  which retracts onto it, and therefore  $f^{-1}(x)$  is contractible.

A non-piecewise linear map  $f : K \rightarrow L$  which stabilizes to a near-homeomorphism may not be contractible (although it follows from the proof above that point-inverses must have the shape of a point). For example, it is easily seen that there exists a map  $f : I^2 \rightarrow I$  such that  $f^{-1}(t)$  is an arc if  $t \neq \frac{1}{2}$ ,  $f^{-1}(\frac{1}{2})$  is a topologist's sine curve containing  $I \times \{0, 1\}$ , and  $f$  is the uniform limit of piecewise-linear maps satisfying the conditions of (3.2). Hence  $f$  itself stabilizes to a near-homeomorphism.

#### 4. Inverse limit applications

Brown's theorem (see § 1) and Theorem (3.2) imply that if  $(K_i, f_i)$  is an inverse sequence of finite complexes with simplicial contractible surjections as bonding maps, then  $\text{Lim } (K_i, f_i) \times Q$  is homeomorphic to  $K_i \times Q$ . Since a dendron is an inverse limit of finite trees with elementary collapses as bonding maps, this technique provides a quick proof of the fact, announced in [1] and demonstrated in [8], that every dendron is a  $Q$ -factor. Let  $J^\infty = \prod_{i=1}^\infty [-1, 1]_i$ , and let  $J^\infty/R$  be the quotient space obtained by identifying  $(x_i)$  with  $(-x_i)$ . Schori and Barit have recently used the same technique to show that  $J^\infty/R$  is a  $Q$ -factor.

The following extension of Brown's theorem to complete metric spaces permits the application of (3.2) in the non-compact case.

4.1. THEOREM. *If  $(X_i, f_i)$  is an inverse sequence of copies of a complete metric space  $X$  with near-homeomorphisms as bonding maps, then  $\text{Lim } (X_i, f_i)$  is homeomorphic to  $X$ .*

PROOF. We inductively choose homeomorphisms  $h_i : X_{i+1} \rightarrow X_i$ ,  $i \geq 1$ , such that  $\text{Lim}(X_i, f_i)$  is homeomorphic to  $\text{Lim}(X_i, h_i)$ . For  $i < j$  let  $f_{ij} = f_i \circ \cdots \circ f_{j-1}$  and  $h_{ij} = h_i \circ \cdots \circ h_{j-1}$  be compositions of the bonding maps, and let  $f_{i\infty} : \text{Lim}(X_i, f_i) \rightarrow X_i$  and  $h_{i\infty} : \text{Lim}(X_i, h_i) \rightarrow X_i$  be the projections. Suppose that  $h_1, \dots, h_{j-1}$  have been chosen. Then there exists an open cover  $\mathcal{U}_j$  of  $X_j$  such that  $\text{mesh } f_{ij}(\mathcal{U}_j) < 2^{-j}$  and  $\text{mesh } h_{ij}(\mathcal{U}_j) < 2^{-j}$  for  $1 \leq i < j$ . Choose a homeomorphism  $h_j : X_{j+1} \rightarrow X_j$  such that  $f_j$  and  $h_j$  are  $\mathcal{U}_j$ -close.

A straight-forward verification shows there exists a map  $F : \text{Lim}(X_i, f_i) \rightarrow \text{Lim}(X_i, h_i)$  such that  $h_{i\infty}F(x) = \lim_{n \rightarrow \infty} h_{in}f_{n\infty}(x)$  for each  $i$ . Likewise there exists a map  $H : \text{Lim}(X_i, h_i) \rightarrow \text{Lim}(X_i, f_i)$  such that  $f_{i\infty}H(x) = \lim_{n \rightarrow \infty} f_{in}h_{n\infty}(x)$ . We show that  $H \circ F$  and  $F \circ H$  are the identity maps. Let  $1 \leq i < n$  and  $x \in \text{Lim}(X_i, f_i)$  be given. Then  $d(f_{i\infty}HF(x), f_{in}h_{n\infty}F(x)) < 2^{-n+1}$ , and for each  $m > n$ ,  $d(f_{i\infty}(x), f_{in}h_{nm}f_{m\infty}(x)) < 2^{-n+1}$ . Since  $h_{n\infty}F(x) = \lim_{m \rightarrow \infty} h_{nm}f_{m\infty}(x)$ , there exists  $m > n$  such that  $d(f_{in}h_{n\infty}F(x), f_{in}h_{nm}f_{m\infty}(x)) < 2^{-n}$ . Thus  $d(f_{i\infty}HF(x), f_{i\infty}(x)) < 3 \cdot 2^{-n+1}$ , and since  $n$  was arbitrary  $H \circ F = \text{id}$ . Similarly  $F \circ H = \text{id}$ .

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