

# COMPOSITIO MATHEMATICA

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**Correction : “On a class of starlike functions”**

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## Correction

### On a class of starlike functions

by

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Compositio Mathematica 19 (1968), 78-82

### Introduction

In a recent paper (Comp. Math., 19 (1968), 78-82) with the above title we investigated the class  $\bar{S}$  of functions

$$f(z) = z + a_2 z^2 + \dots$$

that are analytic and starlike in  $|z| < 1$  and satisfy the condition  $|\{z f'(z)/f(z)\} - 1| \leq 1$  in  $|z| < 1$ . Theorem 4 of that paper, giving the radius of convexity of the class  $\bar{S}$ , contains an error and as such the result obtained is not sharp. Below we give the correct proof of that theorem.

**THEOREM.** *Each function  $f(z) \in \bar{S}$  maps*

$$|z| < \frac{(3 - \sqrt{5})}{2}$$

*onto a convex domain.*

**PROOF.** If  $f(z) \in \bar{S}$ , we can write

$$(1) \quad z \frac{f'(z)}{f(z)} = 1 + \psi(z),$$

when  $\psi(z)$  is analytic in  $|z| < 1$  and satisfies  $|\psi(z)| \leq 1$  and  $\psi(0) = 0$ . Logarithmic differentiation of (1) yields:

$$z \frac{f''(z)}{f'(z)} = \frac{z\psi'(z)}{1 + \psi(z)} + \psi(z).$$

Making use of the fact that  $|\psi(z)| \leq |z|$  and

$$|\psi'(z)| \leq \frac{1 - |\psi(z)|^2}{(1 - |z|^2)},$$

we find that

$$\begin{aligned} \left| z \frac{f''(z)}{f'(z)} \right| &\leq \frac{|z|(1-|\psi(z)|^2)}{(1-|\psi(z)|)(1-|z|^2)} + |\psi(z)| \\ &= \frac{|z|(1+|\psi(z)|)}{(1-|z|^2)} + |\psi(z)| \\ &\leq \frac{2|z|-|z|^2}{(1-|z|)}. \end{aligned}$$

Therefore,  $f(z)$  is convex if

$$\frac{2|z|-|z|^2}{1-|z|} < 1$$

or

$$|z| < \frac{(3-\sqrt{5})}{2}.$$

To show that  $(3-\sqrt{5})/2$  is the exact radius of convexity, we consider the function

$$f_0(z) = z \exp(z).$$

A little calculation shows that for this function

$$1 + \frac{zf_0''(z)}{f_0'(z)} = \frac{z^2+3z+1}{1+z}$$

vanishes when  $z = (\sqrt{5}-3)/2$ .