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CORRECTION FOR THE PAPER "S^3-BUNDLES AND EXOTIC ACTIONS" BULL. SOC. MATH. FRANCE 112 (1984), 69–92

by T.E. Barros

Abstract. — In [R] explicit representatives for S^3-principal bundles over S^7 are constructed, based on these constructions explicit free S^3-actions on the total spaces are described, with quotients exotic 7-spheres. To describe these actions a classification formula for the bundles is used. This formula is not correct. In Theorem 1 below, we correct the classification formula and in Theorem 2 we exhibit the correct indices of the exotic 7-spheres that occur as quotients of the free S^3-actions described above.

Résumé (*Correction à l'article "S^3-bundles and exotic actions", Bull. Soc. Math. France 112 (1984), 69–92*)

On construit dans [R] des représentants explicites des fibrés principaux de fibre S^3 au-dessus de S^7 ; on en déduit des actions libres et explicites de S^3, dont les quotients sont des sphères exotiques de dimension 7. La description de ces actions est basée sur une formule de classification des fibrés. Or cette formule est incorrecte. Le théorème 1 corrige cette formule ainsi que la formule de classification ; le théorème 2 exhibe les indices corrects des sphères exotiques de dimension 7 qui apparaissent comme quotient des actions libres de S^3.

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1. The correction Formulas

The classification formula in page 76 of [R] is wrong. In fact, applying induction on the Hilton Theorem [R, p. 75] and the formula

$$[\iota,\iota] = 2h \pm \Sigma\omega$$

(see [Hu, p. 330]), where ι is the class of the identity map of S^4 and $[\,,\,]$ is the Whitehead product, we have

$$\begin{aligned} f_n \circ h = (n\iota)\circ h &= n(\iota\circ h) + \frac{1}{2}n(n-1)[\iota,\iota] \\ &= n^2 h \pm \frac{1}{2}n(n-1)\Sigma\omega \equiv \Big(n^2, \frac{1}{2}n(n-1)\Big) \end{aligned}$$

thus the correct classification of $\tilde{P}_n$ is

$$\tilde{P}_n \cong E_{\varphi(n-1) \bmod 12}$$

where $\varphi(n-1) = \binom{n}{2} = \frac{1}{2}n(n-1)$.

It can be easily shown that $\varphi(n-1) \equiv \varphi((n-1)+24t) \bmod 12$, $t \in \mathbb{Z}$. So we have

n	1	2	3	4	5	6	7	8	9	10	11	12
$\varphi(n-1)$	0	1	3	6	10	3	9	4	0	9	7	6

n	13	14	15	16	17	18	19	20	21	22	23	24
$\varphi(n-1)$	6	7	9	0	4	9	3	10	6	3	1	0

With this we conclude

THEOREM 1. — *One has:*

$$\begin{aligned} E_0 &\cong \tilde{P}_{24t} \cong \tilde{P}_{1+24t} \cong \tilde{P}_{9+24t} \cong \tilde{P}_{16+24t}, \\ E_1 &\cong \tilde{P}_{2+24t} \cong \tilde{P}_{23+24t}, \\ E_3 &\cong \tilde{P}_{3+24t} \cong \tilde{P}_{6+24t} \cong \tilde{P}_{19+24t} \cong \tilde{P}_{22+24t}, \\ E_4 &\cong \tilde{P}_{8+24t} \cong \tilde{P}_{17+24t}, \\ E_6 &\cong \tilde{P}_{4+24t} \cong \tilde{P}_{12+24t} \cong \tilde{P}_{13+24t} \cong \tilde{P}_{21+24t}, \\ E_7 &\cong \tilde{P}_{11+24t} \cong \tilde{P}_{14+24t}, \\ E_9 &\cong \tilde{P}_{7+24t} \cong \tilde{P}_{10+24t} \cong \tilde{P}_{15+24t} \cong \tilde{P}_{18+24t}, \\ E_{10} &\cong \tilde{P}_{5+24t} \cong \tilde{P}_{20+24t}, \end{aligned}$$

$t \in \mathbb{N}$ *and in* $\tilde{P}_{24t}$ *and* $\tilde{P}_{1+24t}$, $t \neq 0$.

Consequently $\tilde{P}_{13}$ is not trivial, instead $\tilde{P}_9$ is and we note at this point that the bundles E_2, E_5, E_8 and E_{11} can not be realized as $\tilde{P}_n$ for any choice of n. Their total spaces however are diffeomorphic respectively to E_{10}, E_7, E_4 and E_1 that are $\tilde{P}_n$'s.

Using Theorem 1 we can correct the corollaries of page 84 of [R] obtaining:

Theorem 2. — *There are free S^3-actions on:*

(a) E_1, E_9 *with quotient* $\Sigma^7_{[r]}$, $r = 2, 26, 34, 42$*;*

(b) E_3, E_7 *with quotient* $\Sigma^7_{[k]}$, $k = 6, 14, 30, 54$*;*

(c) E_0, E_4 *with quotient* $\Sigma^7_{[s]}$, $s = 0, 16, 40, 48$*;*

(d) E_6, E_{10} *with quotient* $\Sigma^7_{[m]}$, $m = 12, 20, 28, 44$.

Remark. — Theorem 2 is used in [GZ] to show that all principal S^3-bundles over S^7 admit Riemannian metrics with non-negative sectional curvature.

The same calculations of [R] imply that an exotic free action of S^3 on $S^7 \times S^3$ with quotient $\Sigma^7_{[16]}$ is

$$q \star \left(\begin{pmatrix} a \\ b \end{pmatrix}, \nu\right) \equiv \Phi^{-1}\left(q \star \Phi\left(\begin{pmatrix} a \\ b \end{pmatrix}, \nu\right)\right) = \left(\begin{pmatrix} \bar{q}aq \\ \bar{q}bq \end{pmatrix}, q \diamond \nu\right)$$

where,

$$q \diamond \nu = \begin{cases} \bar{q}\,\nu & \text{if } \ \frac{5}{12}\pi \le \cos^{-1}\big(|a|\big) \le \frac{1}{2}\pi, \\ \overline{(F\circ\alpha)\begin{pmatrix} \bar{q}aq \\ \bar{q}bq \end{pmatrix}}\,\bar{q}\,(F\circ\alpha)\dfrac{a}{b}\nu & \text{if } \ \frac{1}{12}\pi \le \cos^{-1}\big(|a|\big) \le \frac{5}{12}\pi, \\ \bar{q}\,\nu & \text{if } \ 0 \le \cos^{-1}\big(|a|\big) \le \frac{1}{12}\pi, \end{cases}$$

and

$$F : S^3 \times S^3 \times \left[0, \frac{1}{2}\pi\right] \longrightarrow S^3$$

is a C^∞ homotopy between the map $(A, B) \mapsto B^8(A\bar{B})^8\bar{A}^8$ and the constant map $(A, B) \mapsto 1$ from $S^3 \times S^3$ to S^3 (see [BR]), α is the map defined on page 86 of [R] and $\Phi : S^7 \times S^3 \to \tilde{P}_9$ is a diffeomorphism.

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