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Corrections to: “A conjecture in the arithmetic theory of differential equations”


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Corrections to

A CONJECTURE IN THE ARITHMETIC THEORY
OF DIFFERENTIAL EQUATIONS (*)

BY

NICHOLAS M. KATZ

I am indebted to O. GABBER for pointing out the following errors.

Section 10.6. — Delete the sentence “the Lie algebra \( \text{Lie} (G_{\text{gal}}) \) is the smallest algebraic Lie sub-algebra of \( M(n) \) which contains the endomorphism

\[
\begin{pmatrix}
2\pi i \lambda_1 & 0 \\
0 & 2\pi i \lambda_n
\end{pmatrix}
\]

It is both false (think of the case of all \( \lambda_i \in \mathbb{Z} \)), and irrelevant to the correct calculation which follows.

Theorem 11.2 is incomplete in both hypotheses and proof as it is stated in the text. Here is a correct version.

**Theorem 11.2.** — Let \((M, \nabla)\) be a rank two equation on an arbitrary \(X\), whose determinant becomes trivial on a finite etale covering of \(X\). Then we have \( \mathcal{G} = \text{Lie} (G_{\text{gal}}) \) if any of the following conditions holds.

(A) The Lie algebra \( \mathcal{G} \) contains non-nilpotent endomorphisms.
(B) \((M, \nabla)\) has non-nilpotent \( \psi_p \) for infinitely many \( p \).
(C) \((M, \nabla)\) does not have regular singular points.
(D) \((M, \nabla)\) has regular singular points but it does not have rational exponents at infinity i.e., it does not have quasi-unipotent local monodromy at infinity.

(*) Addendum à l’article de N. M. KATZ paru dans le fascicule II, tome 110, 1982, p. 203-239.
(E) \( \mathcal{G} \neq 0 \), and Grothendieck's conjecture holds for all rank one equations on all open subsets of \( X \).

Proof. — Recall first that both (D), (C) imply (B), and (B) implies (A). If (A) holds, then the proof in the text is complete. If (A) does not hold, but (E) holds, then \( \mathcal{G} \) is the unipotent radical \( \mathcal{U} \) of a Borel. (This is the case overlooked in the text.) Then there exists a unique line \( L \subset M \otimes \mathbb{C}(X) \) which is \( \mathcal{G} \)-stable, and this line is killed by \( \mathcal{G} \). As in case 2 in the text, this \( L \) must be horizontal. Then both \( L \) and \( M \) modulo \( L \) have nilpotent, hence zero, \( \psi_p \) for almost all \( p \). Applying Grothendieck's conjecture to \( L \) and to \( M \) mod \( L \), we find that \( \text{Lie}(G_{\text{gal}}) \) lies in \( \mathcal{U} \).

Q.E.D.

Example 11.3. — Replace "of infinite order" (true but irrelevant) by "not quasi-unipotent".