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Corrections to : "A conjecture in the arithmetic theory of differential equations"

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110, 1982, p. 347-348.

Corrections to

A CONJECTURE IN THE ARITHMETIC THEORY OF DIFFERENTIAL EQUATIONS (*)

BY

NICHOLAS M. KATZ

I am indebted to O. GABBER for pointing out the following errors.

Section 10.6. – Delete the sentence "the Lie algebra Lie (G_{gal}) is the smallest algebraic Lie sub-algebra of $M(n)$ which contains the endomorphism

$$\begin{pmatrix} 2\pi i\lambda_1 & & 0 \\ & \ddots & \\ 0 & & 2\pi i\lambda_n \end{pmatrix}"$$

It is both false (think of the case of all $\lambda_i \in \mathbb{Z}$), and irrelevant to the correct calculation which follows.

Theorem 11.2 is incomplete in both hypotheses and proof as it is stated in the text. Here is a correct version.

THEOREM 11.2. – *Let* (M, ∇) *be a rank two equation on an arbitrary* X, *whose determinant becomes trivial on a finite etale covering of* X. *Then we have* $\mathscr{G} = \text{Lie}(G_{\text{gal}})$ *if* any *of the following conditions holds.*

(A) *The Lie algebra* $\mathscr{G}$ *contains non-nilpotent endomorphisms.*

(B) (M, ∇) *has non-nilpotent* ψ_p *for infinitely many* p.

(C) (M, ∇) *does not have regular singular points.*

(D) (M, ∇) *has regular singular points but it does not have rational exponents at infinity i. e., it does not have quasi-unipotent local monodromy at infinity.*

(*) Addendum à l'article de N. M. KATZ paru dans le fascicule II, tome 110, 1982, p. 203-239.

(E) $\mathscr{G} \neq 0$, *and Grothendieck's conjecture holds for all rank one equations on all open subsets of* X.

Proof. — Recall first that both (D), (C) imply (B), and (B) implies (A). If (A) holds, then the proof in the text is complete. If (A) does *not* hold, but (E) holds, then $\mathscr{G}$ is the unipotent radical $\mathscr{U}$ of a Borel. (This is the case overlooked in the text.) Then there exists a unique line $L \subset M \otimes \mathbb{C}(X)$ which is $\mathscr{G}$-stable, and this line is killed by $\mathscr{G}$. As in case 2 in the text, this L must be horizontal. Then both L and M modulo L have nilpotent, hence zero, ψ_p for almost all p. Applying Grothendieck's conjecture to L and to M mod L, we find that $\mathrm{Lie}(G_{\mathrm{gal}})$ lies in $\mathscr{U}$.

Q.E.D.

Example 11.3. — Replace "of infinite order" (true but irrelevant) by "not quasi-unipotent".

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