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ON A THEOREM OF CHARLES AND ERDÉLYI ;

BY

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The original purpose of the following was to give a short proof of a theorem of CHARLES [1]. CHARLES then indicated that the proof resembled the proof of a theorem of ERDÉLYI [2], p. 81, and that if modified slightly, would cover both theorems. The same proof however proves also a theorem of J. IRWIN and E. WALKER [3]. In the following the three theorems are combined together.

Let G be a primary group, and if $x \in G$ let $h(x)$ denote the ordinary height of x in G . Also let a represent either an integer or the first infinite ordinal; and if a is the first infinite ordinal, let $p^a G$ represent any subgroup

of $\bigcap_{n=1}^{\infty} p^n G$. Then :

THEOREM (CHARLES, ERDÉLYI, IRWIN and WALKER). — Let M be a subgroup of G maximal with respect to disjointness from $p^a G$. Then M is pure in G .

PROOF. — Deny the theorem. Then there is a least positive integer $n < a$ for which there is an equation $p^n x = y$, $y \in M$ having a solution x in G but not in M . Then there exists an integer m , $0 \leq m < n$, and $z \in M$ such that $0 \neq p^m x + z \in p^a G$. Then $h(z) = h(p^m x) \geq m$, since $h(p^m x) \geq m$ and $h(p^m x + z) \geq a$. Since $m < n$, there is an element $z_1 \in M$ with $p^m z_1 = z$. However $p^{n-m}(p^m x + z) \in M \cap (p^a G)$, and hence is zéro. Thus $p^{n-m}(-z) = p^n x = y$. Thus,

$$p^n(-z_1) = p^{n-m}(p^m(-z_1)) = p^{n-m}(-z) = y, \quad \text{with } -z_1 \in M.$$

The author would like to thank J. IRWIN and E. WALKER for letting him read their manuscripts [3] and [4] which are to appear shortly in the *Pacific Journal of Mathematics*.

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