SAMIR KHABBAZ

On a theorem of Charles and Erdélyi

Bulletin de la S. M. F., tome 89 (1961), p. 103-104
<http://www.numdam.org/item?id=BSMF_1961__89__103_0>

Bull. Soc. math. France,
89, 1961, p. 103 à 104.

ON A THEOREM OF CHARLES AND ERDÉLYI ;

BY

SAMIR KHABBAZ.

(Lehigh University).

The original purpose of the following was to give a short proof of a theorem of CHARLES [1]. CHARLES then indicated that the proof resembled the proof of a theorem of ERDÉLYI [2], p. 81, and that if modified slightly, would cover both theorems. The same proof however proves also a theorem of J. IRWIN and E. WALKER [3]. In the following the three theorems are combined together.

Let G be a primary group, and if $x \in G$ let $h(x)$ denote the ordinary height of x in G. Also let a represent either an integer or the first infinite ordinal; and if a is the first infinite ordinal, let $p^a G$ represent any subgroup of $\bigcap_{n=1}^{\infty} p^n G$. Then :

THEOREM (CHARLES, ERDÉLYI, IRWIN and WALKER). — Let M be a subgroup of G maximal with respect to disjointness from $p^a G$. Then M is pure in G.

PROOF. — Deny the theorem. Then there is a least positive integer $n < a$ for which there is an equation $p^n x = y$, $y \in M$ having a solution x in G but not in M. Then there exists an integer m, $0 \leqq m < n$, and $z \in M$ such that $0 \neq p^m x + z \in p^a G$. Then $h(z) = h(p^m x) \geqq m$, since $h(p^m x) \geqq m$ and $h(p^m x + z) \geqq a$. Since $m < n$, there is an element $z_1 \in M$ with $p^m z_1 = z$. However $p^{n-m}(p^m x + z) \in M \cap (p^a G)$, and hence is zéro. Thus $p^{n-m}(-z) = p^n x = y$. Thus,

$$p^n(-z_1) = p^{n-m}(p^m(-z_1)) = p^{n-m}(-z) = y, \qquad \text{with} \quad -z_1 \in M.$$

The author would like to thank J. IRWIN and E. WALKER for letting him read their manuscripts [3] and [4] which are to appear shortly in the *Pacific Journal of Mathematics.*

BIBLIOGRAPHY.

[1] CHARLES (Bernard). — Étude sur les sous-groupes d'un groupe abélien, *Bull. Soc. math. France*, t. 88, 1960, p. 217-227.

[2] FUCHS (L.). — *Abelian groups*. — Budapest, Hungarian Academy of Sciences, 1958.

[3] IRWIN (J) and WALKER (E). — On N-high subgroups of abelian groups, *Pacific J. of Math.* (to appear shortly).

[4] IRWIN (J.). — High subgroups of abelian torsion groups, *Pacific J. of Math.* (to appear shortly).

(Manuscrit reçu le 25 novembre 1960.)

Samir KHABBAZ,
Mathematics Department,
Lehigh University,
Bethlehem, Penn. (États-Unis).