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FROM PROBABILITY TO GEOMETRY (II)

VOLUME IN HONOR OF THE 60th BIRTHDAY OF JEAN-MICHEL BISMUT

Xianzhe DAI, Rémi LÉANDRE, Xiaonan MA and Weiping ZHANG, editors

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FROM PROBABILITY TO GEOMETRY (II)
VOLUME IN HONOR OF THE 60th BIRTHDAY OF JEAN-MICHEL BISMUT

Xianzhe Dai, Rémi Léandre, Xiaonan Ma and Weiping Zhang, editors

Abstract. — These two volumes contain original research articles submitted by colleagues and friends to celebrate the 60th birthday of Jean-Michel Bismut.

These articles cover a wide range of subjects in probability theory, in global analysis and in arithmetic geometry, to which Jean-Michel Bismut has made fundamental contributions.

Résumé **(De Probabilité à Géométrie, volume en l'honneur du 60e anniversaire de Jean-Michel Bismut)**

Ces deux volumes regroupent des articles originaux soumis par des collègues et amis à l'occasion des 60 ans de Jean-Michel Bismut.

Ces articles portent sur la théorie des probabilités, sur l'analyse sur les variétés et sur la géométrie arithmétique, domaines où Jean-Michel Bismut a fait des contributions fondamentales.

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TABLE OF CONTENTS

Jochen Brüning — ***The signature operator on manifolds with a conical singular stratum*** 1

Introduction 1
1. Dirac operators on fibrations 7
2. Representation of the signature operator near the singularity 13
3. Spectral decomposition of the cone coefficient 18
4. A self-adjoint extension 21
5. The index calculation 31
   5.1. Perturbations of regular projections 33
   5.2. A vanishing theorem 38
   5.3. Generalized Thom spaces 41
References 43

Ulrich Bunke & Thomas Schick — ***Smooth K-Theory*** 45

1. Introduction 46
   1.1. The main results 46
   1.2. A short introduction to smooth cohomology theories 48
   1.3. Related constructions 53
2. Definition of smooth K-theory via cycles and relations 56
   2.1. Cycles 56
   2.2. Relations 59
   2.3. Smooth K-theory 63
   2.4. Natural transformations and exact sequences 65
   2.5. Comparison with the Hopkins-Singer theory and the flat theory 69
3. Push-forward 72
   3.1. K-orientation 72
   3.2. Definition of the Push-forward 75
   3.3. Functoriality 84
4. The cup product 88
   4.1. Definition of the product 88
   4.2. Projection formula 91

4.3. Suspension 93
5. Constructions of natural smooth K-theory classes 94
5.1. Calculations 94
5.2. The smooth K-theory class of a mapping torus 95
5.3. The smooth K-theory class of a geometric family with kernel bundle 96
5.4. A canonical $\hat{K}^1$-class on S^1 98
5.5. The product of S^1-valued maps and line-bundles 99
5.6. A bi-invariant $\hat{K}^1$- class on $SU(2)$ 100
5.7. Invariant classes on homogeneous spaces 102
5.8. Bordism 104
5.9. $\mathbb{Z}/k\mathbb{Z}$-invariants 105
5.10. $Spin^c$-bordism invariants 106
5.11. The e-invariant 108
6. The Chern character and a smooth
Grothendieck-Riemann-Roch theorem 110
6.1. Smooth rational cohomology 110
6.2. Construction of the Chern character 112
6.3. The Chern character is a rational isomorphism and multiplicative .. 118
6.4. Riemann Roch theorem 121
7. Conclusion 132
References 133

Henri Gillet & Fatih M. Ünlü — ***An explicit proof of the generalized Gauss-Bonnet formula*** 137
Introduction 137
1. Superobjects 140
2. Sheaves on Real Analytic Manifolds 142
3. Superconnections and the Chern Character 143
4. The Grothendieck Fundamental Class 144
5. Koszul Factorizations 145
6. Generalized Supertraces 148
7. Twisted Complexes 150
8. Koszul-Dolbeault twisted resolutions 152
9. Koszul Factorizations II 155
10. Comparison with the Grothendieck Class 157
References 159

Sebastian Goette — ***Torsion invariants for families*** 161
Introduction 161
1. An Index Theorem for Flat Vector Bundles 164
1.1. Characteristic classes for flat vector bundles 165
1.2. The cohomological index theorem 167
1.3. A refined index theorem 168

2. Construction of the Bismut-Lott torsion 169
2.1. A finite-dimensional model 169
2.2. The Bismut-Lott torsion form 172
2.3. Elementary Properties 174
3. Properties of Bismut-Lott torsion 175
3.1. A transfer formula 175
3.2. Lott's Secondary K-theory of flat bundles 176
3.3. Rigidity of Kamber-Tondeur classes 178
3.4. Equivariant analytic torsions 179
3.5. The Ma-Zhang subsignature operator 181
4. Igusa-Klein torsion 183
4.1. Generalised Morse functions and filtered complexes 184
4.2. Filtered chain complexes and the Whitehead space 185
4.3. Properties of the Igusa-Klein torsion 186
5. Bismut-Lott = Igusa-Klein 188
5.1. The Witten deformation 188
5.2. Analytic Igusa-Klein torsion 189
5.3. Axioms for higher torsions 191
6. Dwyer-Weiss-Williams torsion 192
6.1. The topological index theorem 192
6.2. Topological higher Reidemeister torsion 194
7. Exotic smooth bundles 196
7.1. Hatcher's example 196
7.2. The space of stable exotic smooth structures 197
7.3. Heitsch-Lazarov torsion for foliations 199
8. The hypoelliptic Laplacian and Bismut-Lebeau torsion 200
8.1. The hypoelliptic Laplacian on the cotangent bundle 200
8.2. Bismut-Lebeau torsion 202
References 203

F. Reese Harvey & H. Blaine Lawson, Jr — *Boundaries of positive holomorphic chains and the relative Hodge question* 207

1. Introduction 207
2. The Characterization Theorem 211
2.1. Solving $dT = \Gamma$ for T positive 212
2.2. Replacing the positive solution by a holomorphic chain 212
3. Relative Hodge Classes and Representability 214
3.1. Relative Hodge Question (Positive Version) 215
4. A Real Hodge Filtration on $H_{2p}(X, M; \mathbf{R})$ 218
5. Residual Currents 218
Case 1 219
Case 2 220

References 220

Kefeng Liu & Hao Xu — ***Mirzakharni's recursion formula is equivalent to the Witten-Kontsevich theorem*** 223

1. Introduction 223
2. Proof of Mirzakhani's recursion formula 225
3. Higher Weil-Petersson volumes 229

References 234

Vincent Maillot & Damian Rössler — ***Formes automorphes et théorèmes de Riemann-Roch arithmétiques*** 237

1. Introduction 237
2. Les formes modulaires de Yoshikawa de premier type 238
3. Les formes modulaires d'Igusa « produit des thêta constantes paires » .. 244
4. Les formes modulaires de Yoshikawa de deuxième type 246
5. Appendice : une formule du point fixe singulière conjecturale en théorie d'Arakelov 251

Références 252

Varghese Mathai, Richard B. Melrose & Isadore M. Singer — ***The index of projective families of elliptic operators : the decomposable case*** 255

Introduction 256
1. Trivialization by the circle bundle 260
2. Geometry of the decomposed class 263
3. Smooth Azumaya bundle 264
4. Stable Azumaya isomorphism 268
5. Analytic index 270
6. The topological index 273
7. Twisted Chern character 275
8. Semiclassical quantization 280
9. The index theorem 285
10. The Chern character of the index 286

Appendix A. Differential characters 289
Appendix B. Čech class of the Azumaya bundle 290
Appendix C. The universal case 291
References 295

Paul-Émile Paradan & Michèle Vergne — ***Index of transversally elliptic operators*** 297

1. Introduction 297
2. The analytic index 302
   2.1. Generalized functions 302

2.2. Symbols and pseudo-differential operators 302
2.3. Functoriality properties of the analytic index 304
2.3.1. Excision 305
2.3.2. Exterior product 305
2.3.3. Free action 306
2.4. Basic examples 306
2.4.1. Bott symbols 306
2.4.2. Atiyah symbol 307
2.5. Unicity of the index 308
3. The cohomological index 308
3.1. Restrictions of generalized functions 309
3.2. Integration of bouquet of equivariant forms 311
3.3. The Chern character with support 313
3.4. The Chern character of a transversally elliptic symbol 316
3.5. Definition of the cohomological index 318
4. The cohomological index coincides with the analytic one 325
4.1. Free action 325
4.2. Multiplicative property 330
4.3. Normalization conditions 331
4.3.1. Atiyah symbol 331
4.3.2. Bott symbols 333
5. Examples 333
5.1. Pushed symbols 333
5.2. Contact manifolds 336
References 337

Sean Timothy Paul & Gang Tian — *CM Stability And The Generalized Futaki Invariant II* 339
1. Introduction 339
1.1. Statement of results 339
2. Background and Motivation 342
3. Algebraic potentials 344
4. Singular Hermitian metrics 346
4.1. Bott Chern secondary classes 348
4.2. Properness Implies that $F_1(\lambda) < 0$ 351
References 354

Ken-ichi Yoshikawa — *Calabi–Yau threefolds of Borcea–Voisin, analytic torsion, and Borcherds products* 355
1. Introduction 355
2. Lattices and orthogonal modular varieties 359
2.1. 2-elementary lattices 359
2.2. Lorentzian lattices 360

2.3. Lattices of signature $(2, n)$ 360
3. Calabi–Yau threefolds of Borcea–Voisin 361
3.1. $K3$ surfaces with involution and their moduli space 361
3.2. Elliptic curves and elliptic fibrations 362
3.3. Borcea–Voisin threefolds and their moduli space 363
3.4. Degenerations of Borcea–Voisin threefolds 365
3.5. Borcea–Voisin threefolds of exceptional type 368
4. Odd unimodular lattices and Borcherds products 369
4.1. Automorphic forms 369
4.2. Borcherds product associated with 2-elementary lattices 370
4.3. A Borcherds product associated with Λ_m 371
4.4. Borcherds products associated with the odd unimodular lattices 374
4.5. The Borcherds Φ-function and Φ_{10} 375
5. The BCOV invariant of Borcea–Voisin threefolds 376
5.1. The BCOV invariant of Calabi–Yau threefolds 376
5.2. The singularity of the BCOV invariant 377
5.3. The BCOV invariant of Borcea–Voisin threefolds of type Λ_m 378
5.4. Proof of Theorem 5.7 382
5.5. Factorization of the BCOV invariant for Borcea–Voisin threefolds 383
6. Automorphic forms on the Kähler moduli of a Del Pezzo surface 385
6.1. Del Pezzo surfaces 385
6.2. An automorphic form on the Kähler moduli of V 387
6.3. The functional equations of Φ_V 389
6.4. Borcherds Φ-function as an analogue of Φ_V for Enriques surfaces 390
References 391

This volume is dedicated to Jean-Michel Bismut.

The editors : Xianzhe Dai, Rémi Léandre, Xiaonan Ma and Weiping Zhang.