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FROM PROBABILITY TO GEOMETRY (I)

VOLUME IN HONOR OF THE 60$^{\text{th}}$ BIRTHDAY OF JEAN-MICHEL BISMUT

Xianzhe DAI, Rémi LÉANDRE, Xiaonan MA and Weiping ZHANG, editors

SOCIÉTÉ MATHÉMATIQUE DE FRANCE

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FROM PROBABILITY TO GEOMETRY (I)

VOLUME IN HONOR OF THE 60^{th} BIRTHDAY OF JEAN-MICHEL BISMUT

Xianzhe Dai, Rémi Léandre, Xiaonan Ma and Weiping Zhang, editors

Abstract. — These two volumes contain original research articles submitted by colleagues and friends to celebrate the 60^{th} birthday of Jean-Michel Bismut.

These articles cover a wide range of subjects in probability theory, in global analysis and in arithmetic geometry, to which Jean-Michel Bismut has made fundamental contributions.

***Résumé* (De Probabilité à Géométrie, volume en l'honneur du 60^e anniversaire de Jean-Michel Bismut)**

Ces deux volumes regroupent des articles originaux soumis par des collègues et amis à l'occasion des 60 ans de Jean-Michel Bismut.

Ces articles portent sur la théorie des probabilités, sur l'analyse sur les variétés et sur la géométrie arithmétique, domaines où Jean-Michel Bismut a fait des contributions fondamentales.

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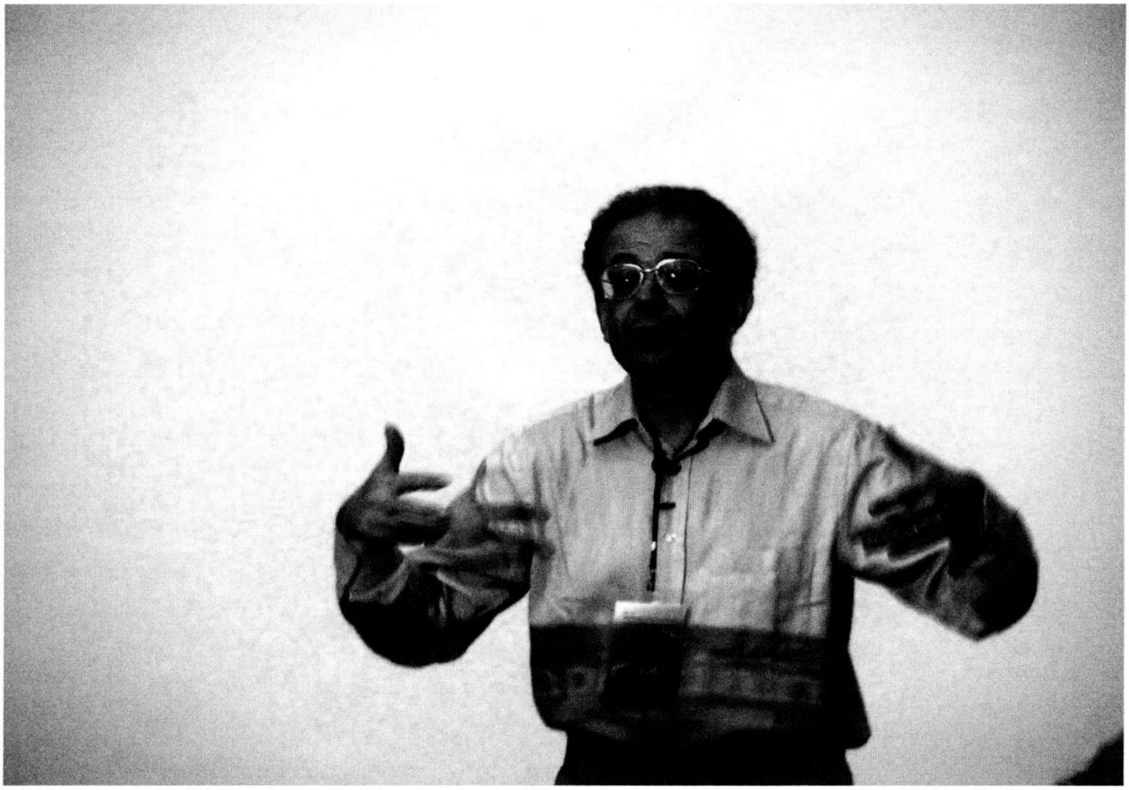

This volume is dedicated to Jean-Michel Bismut.

The editors : Xianzhe Dai, Rémi Léandre, Xiaonan Ma and Weiping Zhang.

PREFACE BY PAUL MALLIAVIN

Jean-Michel Bismut has had a carrier with an incredibly wide scope. At the beginning of the seventies, after a strong scientific curriculum at École Polytechnique, he received from the French state a life appointment as an engineer in charge of supervising the strategic French national stockpile of crude oil.

To fulfill the forecasting duties of his position, he started econometrical studies which led him to Statistics and subsequently to Probability. He discovered in the mid seventies an object which is now called the *backward stochastic differential equations;* this concept was so innovative that referees of probability Journals to which he submitted the corresponding paper rejected it "as obviously wrong". Finally he succeeded by publishing several papers on the subject. Fifteen years after this publication, Pardoux, starting from Bismut's paper, built a full theory which stands now as a major tool of Mathematical Finance, on which superhedging theory in a risky market is based.

In 1980 Jean-Michel Bismut published a six hundred pages Lecture Notes on *Random Mechanics* which marks the turn of his carrier to Pure Mathematics. In 1984 Bismut published a two hundred pages book "Large Deviations and the Malliavin Calculus" in which he merges Differential Calculus on the Wiener space with extremal problems associated to computing probability of rare events. Fifty pages of this book are devoted to the Stochastic Calculus of Variations of Brownian motion on a Riemannian manifold; he enriched the classical orthonormal frame bundle approach to this problem by introducing *infinitesimal rotations of the Wiener space.* This innovative idea remained unnoticed for around five years until Daniel Stroock and Bruce Driver showed its full potential. In many papers dealing with infinite dimensional stochastic analysis, it appears as the *Bismut formula.*The same year, Bismut proved the Atiyah-Singer index formula by an intrinsic computation of the small time expansion of the heat kernel through stochastic analysis.

In the beginning of the eighties, I had the pleasure to see Jean-Michel Bismut every week at my seminar at École Normale. Then Bismut moved towards *Global Analysis* and our common mathematical discussions became, unfortunately for me, less frequent. The extraordinary drive who brought Bismut from Petroleum Industry to Probability, has, once more, taken him towards new mathematical horizons.

Paul Malliavin,
June 2008

PREFACE BY SIR MICHAEL ATIYAH

Many years ago I first heard the name of Jean-Michel Bismut from my colleagues in the probability world. He was an up and coming young man in the field. My first personal encounter with him was at the colloquium held in Paris in 1983 in honour of Laurent Schwartz. In my lecture I described Witten's approach to the index theorem via a fixed-point argument in the loop-space. This was not rigorous mathematics but it was a beautiful idea. Bismut later told me that this was what converted him from a probabilist into a differential geometer.

Moreover much of Bismut's subsequent work consisted of providing rigorous proofs for the heuristic physical ideas coming from Witten and other theoretical physicists. It turned out to be an extremely fruitful field and a logical follow-on to Bismut's earlier work in probability.

As index theory developed, focus shifted from global topological formulae to more precise local differential-geometric ones. By integration these reproduced the topological formulae but they contained more information. This was the area in which Bismut specialized and his work embodied many subtle geometric and analytic ideas.

Perhaps the most striking application of the more precise local formulae was to Arakelov theory, also called arithmetic geometry. This involves algebraic geometry defined over number fields in which the infinite prime has to be treated by complex analysis. As yet the theory is in its infancy but its potential as a link from number theory via geometry and analysis to theoretical physics holds out enormous promise. If this aim is eventually achieved, Bismut's many contributions will be seen as essential links in the chain.

Although Bismut has moved beyond classical probability theory, his background in that field certainly fits well with the probabilistic ideas embodied in quantum theory and the Feynman integral. He has shown the broader outlook, ignoring disciplinary boundaries, which is characteristic of great mathematics.

Michael Atiyah
February, 2008

A LETTER FROM A FRIEND [1]

Dear Jean-Michel,

I'm very sorry that I can't be present to help celebrate your birthday. But it helps a lot that we have been able to spend the last few months together. We have had many good times.

You are one of the world's deepest and most original mathematicians, with a unique perspective and unique abilities. In your chosen field, you are simply orders of magnitude beyond everyone else. In my view, you are a true and worthy successor to Atiyah and Singer.

The work that we did together was one of the great mathematical experiences of my life. In some ways it was also one of the most humbling—particularly at those times when I would be sitting with a blank look on my face and you would be writing away, while at the same time, singing gaily to yourself, just loud enough so that I could hear it. Really, that was too much!

But the truth is that the close friendship that we have shared—especially all the heated discussions—is by far more important to me than the math that we did. You are a wonderful friend.

So enjoy your party and remember that you are still a young man—well maybe not so young—anyhow, at least you can say that you are younger than I am.

Jeff Cheeger

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[1] Dai, Ma and Zhang organized a conference "Geometry and Analysis on Manifolds" at the Chern Institute of Mathematics from April 8 until April 14, 2007, in honour of Jean-Michel Bismut. We reproduce here a letter of Jeff Cheeger, which was read by Jean-Pierre Bourguignon during the conference.

CURRICULUM VITÆ OF JEAN-MICHEL BISMUT

Jean-Michel Bismut, born on 26^{th} February 1948 in Lisbon (Portugal). French. Married, 3 children.

Education

1967-1970	Graduate from École polytechnique
1973	Docteur d'État in Mathematics, Université Paris VI, with a thesis entitled "Analyse convexe et probabilités".

Career

1970-1976	"Ingénieur du Corps des Mines"
1975-1987	Maître de Conférences at École polytechnique
1976-1980	Associate professor at Department of Mathematics, Université Paris-Sud (Orsay)
1981-	Professor at Department of Mathematics, Université Paris-Sud (Orsay)
1984	Member of I.A.S. (Princeton)
1986	Invited lecture in the Geometry section, International Congress of Mathematicians (ICM) in Berkeley
1987-1988	Visitor at I.H.É.S
1994	Member of I.A.S. (Princeton)
1989-2008	Editor of *Inventiones Mathematicae*
1996-2008	Managing Editor of *Inventiones Mathematicae* (with Gerd Faltings)
1998	Plenary speaker, International Congress of Mathematicians (ICM) in Berlin
1990-1998	Member of the scientific committee of the Isaac Newton Institute for Mathematical Sciences at Cambridge (UK)
1992-2002	Senior member of Institut Universitaire de France
1998-2002	Member of the Executive Committee, International Mathematical Union (IMU)

2000-2006	Chairman of Fachbeirat of the Max-Planck Institut für Mathematik of Bonn
2002-2006	Vice-President of the International Mathematical Union (IMU)

Honors and Prizes

Montyon Prize of Académie des Sciences (1984)
Ampère Prize of Académie des Sciences (1990)
Corresponding member of Académie des Sciences (1990)
Member of Académie des Sciences (1991)
Member of Academia Europaea (1998)
Member of Deutsche Akademie Leopoldina (2004)

The Ph.D. students of Jean-Michel Bismut

June 1984	Patrick Cattiaux	Université Paris-Sud
	Dissertation title :	Diffusions avec une condition frontière: hypoellipticité du semi-groupe associé conditionnement et filtrage
October 1984	Rémi Léandre	Université de Franche-Comté
	Dissertation title:	Extension du théorème de Hörmander à divers processus de sauts
May 1985	Carl Graham	Université Paris VI (Co-advised with Michel Métivier)
	Dissertation title :	Systèmes de particules en interaction dans un domaine à paroi collante et problèmes de martingales avec réflexion
April 1993	Weiping Zhang	Université Paris-Sud
	Dissertation title :	Invariant êta et torsion analytique
May 1993	Kai Köhler	Université Paris-Sud
	Dissertation title :	Torsion analytique complexe
July 1993	Alain Berthomieu	Université Paris-Sud
	Dissertation title :	Métrique de Quillen et suite spectrale de Leray

April 1998	Xiaonan Ma	Université Paris-Sud
	Dissertation title :	Formes de torsion analytique et familles de submersions
October 2003	Dimitri Zvonkine	Université Paris-Sud
	Dissertation title :	Enumeration of Ramified Coverings of Riemann Surfaces