J. Thévenaz
P. J. Webb

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Astérisque, tome 181-182 (1990), p. 263-272
<http://www.numdam.org/item?id=AST_1990__181-182__263_0>

A Mackey Functor Version of a Conjecture of Alperin

J. THÉVENAZ AND P.J. WEBB

The conjecture of Alperin we consider has to do with modular representations of a finite group G over an algebraically closed field k of characteristic p.

CONJECTURE 1 (Alperin [2]). *The number of weights for G equals the number of simple kG-modules.*

Alperin makes his definition of a *weight* for G in [2]. There now exist various equivalent forms of this conjecture, and we will work with the one which appeared first after Alperin's original version. We let $\mathrm{np}(G)$ denote the number of non-projective simple kG-modules, and Δ the simplicial complex of chains of non-identity p-subgroups of G (see [10]).

CONJECTURE 2 (Knörr–Robinson [5]). *For all finite groups G,*

$$\mathrm{np}(G) = \sum_{\sigma \in \Delta/G} (-1)^{\dim \sigma}\, \mathrm{np}(G_\sigma).$$

The notation $\mathrm{np}(G)$ differs from what appears in print elsewhere. In the notation of [5] and [10] one has $\mathrm{np}(G) = \ell(G) - f_0(G)$ where $\ell(G)$ is the number of simple kG-modules and $f_0(G)$ is the number of blocks of defect zero. In comparing different printed versions it may help to notice that for the stabilizer groups G_σ we have $\mathrm{np}(G_\sigma) = \ell(G_\sigma)$ because the G_σ always have a non-trivial normal p-subgroup, and so $f_0(G_\sigma) = 0$.

We will show that Conjectures 1 and 2 are equivalent to the next conjecture.

CONJECTURE 3. *For every finite group G, for every prime p, there exist Mackey functors M_1, M_2 so that $M_1(H)$ and $M_2(H)$ are vector spaces over a field R whose characteristic is 0 or prime to $|G|$, satisfying*

(i) For every subgroup H, the restrictions $M_1 \downarrow^G_H$ and $M_2 \downarrow^G_H$ are projective relative to p-local subgroups of H.

(ii) For every subgroup H, $\dim M_1(H) - \dim M_2(H) = \mathrm{np}(H)$.

THEOREM 4. *Conjectures 2 and 3 are equivalent.*

The interesting thing about this conjecture is that somehow it conveys the information of Alperin's conjecture in the structure of the Mackey functors. In other forms of the conjecture there is a sum over some reasonably complicated indexing set, be it conjugacy classes of p-subgroups as in Alperin's original version, or conjugacy classes of chains of p-subgroups as in Robinson's version. In Conjecture 3 there is no such sum. The hope is that because arithmetic is somehow replaced by structure there may come about an understanding of what is going on. With the exception of the groups of Lie type in defining characteristic p, it seems that every verification of Alperin's conjecture for a particular class of groups has been an observation of a numerical equality. The conjecture may eventually be completely proved in such a fashion, but there will still be the need for an explanation, and here the Mackey functors may come in.

In this note we first prove Theorem 4, which relies on the theorem in [11] combined with a description of the irreducible Mackey functors [9]. Someone familiar with [11] will quickly see the connection between Conjecture 3 and Conjecture 2, and in fact much of the motivation behind [11] came from trying to prove Alperin's conjecture in this way. From this point of view we are really interested in the implication that Conjecture 3 implies Conjecture 2. For the proof of the converse statement we have to construct Mackey functors M_1 and M_2 satisfying the conditions of Conjecture 3, but the construction we have is rather artificial. Our interest in this implication is the consequence that so long as one believes that Alperin's conjecture is true then it is not a waste of time to study Conjecture 3. It seems probable that if one can prove Alperin's conjecture using Mackey functors then it will be done by some natural construction and artificial constructions will not do.

Conjecture 3 arose out of a less complicated set of conditions, which unfortunately turned out to have a counterexample. We present this set of conditions as Question 13, and give the counterexample after that.

We conclude in the last section by presenting the most general situation in which we have constructed Mackey functors satisfying Conjecture 3. This is the case of groups which have a cyclic Sylow p-subgroup, and we use the existing theory of modular representations of such groups to construct the Mackey functors. It is exactly this theory which can be used directly to establish Alperin's Conjecture and so we should make it clear that we have no new approach here. The interest in our construction of these Mackey functors is that it may give some indication of what one should expect in general.

1. Preliminaries on Mackey functors

The reader should refer to [3], [4] and [11] for the definition of a Mackey functor and the notion of relative projectivity. We will regard a Mackey functor as being defined on the set of all subgroups of G (rather than on G-sets, which is the approach taken in [3] and [4]). We write induction, restriction and conjugation mappings as I_H^G, R_H^G and c_g.

We will use the operations on Mackey functors of *restriction, induction* and *inflation.* When N is a Mackey functor for G and $H \leq G$ we obtain a Mackey functor $N \downarrow_H^G$ on H by restricting attention to subgroups of H. Thus $N \downarrow_H^G (K) = N(K)$. Induction first appears in [6], where it is attributed to Yoshida. Suppose M is a Mackey functor defined for a subgroup H of G. Using temporarily the G-set notation, we define $M \uparrow_H^G$ to be the Mackey functor for G which is given by $M \uparrow_H^G (\Omega) = M(\Omega \downarrow_H^G)$, where Ω is a G-set and $\Omega \downarrow_H^G$ is its restriction as an H-set. In subgroup notation this becomes

$$M \uparrow_H^G (K) = \bigoplus_{x \in H\backslash G/K} M(H \cap {}^xK), \qquad K \leq G.$$

We now define the notion of inflation. Suppose that G has a normal subgroup N with $G/N = Q$ and M is a Mackey functor defined on the subgroups of Q. We construct a Mackey functor $\mathrm{Inf}_Q^G M$ defined on subgroups K of G by

$$\mathrm{Inf}_Q^G M(K) = \begin{cases} 0 \text{ if } K \not\geq N \\ M(K/N) \text{ if } K \geq N. \end{cases}$$

Here K/N is a subgroup of Q. Restriction, induction and conjugation mappings are necessarily zero except between subgroups containing N, when they are defined to be the mappings $I_{H/N}^{K/N}, R_{H/N}^{K/N}, c_{gN}$ with $H, K \geq N, g \in G$. The fact that one extends M by zero on subgroups not containing N is at first a surprise, but it is the canonical way to make such an extension.

PROPOSITION 5. *Let H be a subgroup of G, M a Mackey functor for $N_G(H)/H$. Put $L = (\mathrm{Inf}_{N_G(H)/H}^{N_G(H)} M) \uparrow_{N_G(H)}^G$. Then*
(i) $L(K) = 0$ unless K contains a conjugate of H,
(ii) $L(H) = M(1)$.

Proof. From the definition,

$$L(K) = \bigoplus_{g \in N_G(H)\backslash G/K} \mathrm{Inf}_{N_G(H)/H}^{N_G(H)} M(N_G(H) \cap {}^gK).$$

The terms are zero unless ${}^gK \geq H$, whence condition (i). Furthermore, when $K = H$ we only get a non-zero contribution when ${}^gH \geq H$, but this means $g \in N_G(H)$ so there is only one such term and it occurs with $g = 1$.

COROLLARY 6. *Given any subgroup $H \leq G$ there exists a Mackey functor L on G such that*
(i) $L(H)$ is a one-dimensional vector space over a field R,
(ii) $L(K) = 0$ unless $H \leq {}^gK$ for some $g \in G$,
(iii) If $O_p(H) \neq 1$ then for all subgroups K of G, $L \downarrow_K^G$ is projective relative to p-local subgroups of K.

Proof. To prove (i) and (ii), apply Proposition 5 with M the Mackey functor on $N_G(H)/H$ which has $M(K/H) = R$ for all subgroups $K \geq H$. Restriction and conjugation mappings are the identity and induction mappings are the multiplication by the index.

For (iii) we use the formula

$$L \downarrow_K^G = (\operatorname{Inf} M) \uparrow_{N_G(H)}^G \downarrow_K^G = \bigoplus_{g \in K \backslash G / N_G(H)} c_g((\operatorname{Inf} M) \downarrow_{N_G(H) \cap {}^{g^{-1}}K}^{N_G(H)}) \uparrow_{{}^gN_G(H) \cap K}^K .$$

Each summand is zero unless ${}^{g^{-1}}K \supseteq H$, i.e. $K \supseteq {}^gH$, in which case it is induced from ${}^gN_G(H) \cap K$, which has a normal p-subgroup $O_p({}^gH)$. Thus each non-zero term is also induced from the larger subgroup $N_K(O_p({}^gH))$, which is a p-local subgroup of K. Hence the result.

We need to quote a description of the irreducible Mackey functors which we do not prove here. We state the result in the case of Mackey functors over R where R is a field of characteristic 0 or prime to $|G|$. This means Mackey functors M for which $M(H)$ is always a vector space over R. This is a special case of a more general result without this restriction on R which is proved in [9]. As is usual in an abelian category, a Mackey functor is said to be *irreducible* if it has no non-trivial proper subfunctors.

THEOREM 7. *Let R be a field whose characteristic is either 0 or prime to $|G|$. Up to isomorphism, the irreducible Mackey functors over R biject with pairs (H, V) where H is a subgroup of G determined up to conjugacy and V is an irreducible $RN_G(H)/H$-module. The irreducible Mackey functor $S_{H,V}$ corresponding to such a pair is constructed as follows. Let M be the Mackey functor on $N_G(H)/H$ given by $M(K) = V^K$ for $K \leq N_G(H)/H$. Then $S_{H,V} = (\operatorname{Inf}_{N_G(H)/H}^{N_G(H)} M) \uparrow_{N_G(H)}^G$.*

We will also need to quote the following result, which is obtained by combining Theorem 7 with 12.2 and 12.3 of [7] (see also [9]).

THEOREM 8. *Let M be a Mackey functor over R, where R is a field whose characteristic is 0 or prime to $|G|$. Then M is a direct sum of irreducible Mackey functors.*

2. Proof of Theorem 4

We first show that Conjecture 3 implies Conjecture 2. We use an elaboration of the argument which proves the theorem in [11]. To shorten the exposition we assume the reader to be familiar with the theorem of [11] and its notation, and do not explain this notation here. We suppose that Mackey functors M_1 and M_2 exist as specified in Conjecture 3. By Theorem 8, each of M_1 and M_2 is a direct sum of irreducible Mackey functors $S_{H,V}$ with $H \leq G$ and V an irreducible $RN_G(H)/H$-module.

LEMMA 9. *Every subgroup H such that $S_{H,V}$ is a summand of M_1 or M_2 has a non-trivial normal p-subgroup.*

Proof. Suppose $S_{H,V}$ is a summand of M_i. By hypothesis, $M_i \downarrow_H^G$ is projective relative to p-local subgroups of H, so $S_{H,V} \downarrow_H^G$ is projective relative to p-local subgroups of H. But $S_{H,V} \downarrow_H^G$ is non-zero only on H, so H is a p-local subgroup of H, that is, $O_p(H) \neq 1$.

Let Δ be the simplicial complex of chains of non-identity p-subgroups of G. We show that for each summand $S_{H,V}$ of M_i, $i = 1, 2$, there is a split exact sequence

$$\begin{aligned} 0 \to S_{H,V}(G) \to \bigoplus_{\sigma\in\Gamma_0(\Delta)/G} S_{H,V}(G_\sigma) \to \bigoplus_{\sigma\in\Gamma_1(\Delta)/G} S_{H,V}(G_\sigma) \to \\ \cdots \to \bigoplus_{\sigma\in\Gamma_d(\Delta)/G} S_{H,V}(G_\sigma) \to 0 \end{aligned} \tag{10}$$

From this it will follow by adding up over the irreducible summands that there are split exact sequences

$$\begin{aligned} 0 \to M_i(G) \to \bigoplus_{\sigma\in\Gamma_0(\Delta)/G} M_i(G_\sigma) \to \bigoplus_{\sigma\in\Gamma_1(\Delta)/G} M_i(G_\sigma) \to \\ \cdots \to \bigoplus_{\sigma\in\Gamma_d(\Delta)/G} M_i(G_\sigma) \to 0 \end{aligned}$$

for $i = 1, 2$. The alternating sum of the dimensions vanishes, from which we extract

$$\dim M_1(G) - \dim M_2(G) = \sum_{\sigma\in\Delta/G} (-1)^{\dim\sigma}(\dim M_1(G_\sigma) - \dim M_2(G_\sigma)).$$

By condition (ii) of Conjecture 3 this becomes the statement of Conjecture 2.

We demonstrate the existence of the split exact sequence (10). In the notation of [11] this sequence is $S_{H,V}(\tilde{\Gamma}(\Delta))$. Applying Theorem 7 we have

$$S_{H,V} = (\mathrm{Inf}_{N_G(H)}^{N_G(H)/H} L) \uparrow_{N_G(H)}^G$$

for a certain Mackey functor L on $N_G(H)/H$, so by the definition of Mackey functor induction the sequence is

$$\mathrm{Inf}_{N_G(H)}^{N_G(H)/H} L(\tilde{\Gamma}(\Delta) \downarrow^G_{N_G(H)}).$$

We may now apply the theorem of [11] in the situation of the Mackey functor $\mathrm{Inf}_{N_G(H)}^{N_G(H)/H} L$ for $N_G(H)$ applied to the H-simplicial complex $\Delta \downarrow^G_{N_G(H)}$. In the statement of the theorem there are sets of subgroups $\mathcal{X}$ and $\mathcal{Y}$ which we take to be

$$\mathcal{X} = \{J \leq N_G(H) \mid H \leq J\}$$
$$\mathcal{Y} = \{J \leq N_G(H) \mid H \not\leq J\}.$$

The conditions of the theorem of [11] are satisfied since all subgroups in $\mathcal{X}$ have a normal p-subgroup and so have contractible fixed points on Δ, and $\mathrm{Inf}_{N_G(H)}^{N_G(H)/H} L$ vanishes on subgroups in $\mathcal{Y}$ by the definition of inflation. The theorem now states that (10) is split exact.

We now prove that Conjecture 2 implies Conjecture 3 and to do this we construct M_1 and M_2 inductively working from the bottom to the top in the poset of subgroups of G. The argument works for an arbitrary field R, not just one of characteristic 0 or prime to $|G|$. Consider sets $\mathcal{Z}$ of subgroups of G satisfying the following properties

(i) $\mathcal{Z}$ is closed under conjugation and taking subgroups,
(ii) there exist Mackey functors M_1 and M_2 for G such that on restriction to every subgroup H, $M_1 \downarrow^G_H, M_2 \downarrow^G_H$ are projective relative to p-local subgroups of H, and for all $H \in \mathcal{Z}$, $\dim M_1(H) - \dim M_2(H) = \mathrm{np}(H)$.

We show by an inductive argument that the set of all subgroups of G is such a set $\mathcal{Z}$.

First it is clear that the set $\mathcal{Z}$ of all p'-subgroups satisfies (i) and (ii) on taking M_1 and M_2 to be the zero functors. Next suppose we have some such set $\mathcal{Z}$ (containing all p'-subgroups) with Mackey functors M_1 and M_2, and let K be a subgroup of G minimal subject to not being in $\mathcal{Z}$. We show that the union of $\mathcal{Z}$ and the conjugacy class of K also satisfies (i) and (ii). Since this union immediately satisfies (i) by definition, we only have to verify (ii). Note that K is not a p'-subgroup. There are two cases to consider, depending on whether or not $O_p(K) = 1$.

First suppose $O_p(K) = 1$. We apply the theorem of [11] to the situation of K acting on the simplicial complex Δ of p-subgroups of K, and with the restrictions of M_1 and M_2 to K as Mackey functors. These are projective relative to p-local subgroups of K by condition (ii) for $\mathcal{Z}$ and vanish on p'-subgroups, so by an argument we have seen previously we have

$$\dim M_i(K) = \sum_{\sigma \in \Delta/K} (-1)^{\dim \sigma} \dim M_i(K_\sigma) \qquad i = 1, 2. \tag{11}$$

Each subgroup K_σ has a normal p-subgroup and so it is a proper subgroup of K and lies in $\mathcal{Z}$. Thus we know that

$$\dim M_1(K_\sigma) - \dim M_2(K_\sigma) = \mathrm{np}(K_\sigma) \tag{12}$$

for all σ. Assuming Conjecture 2 we deduce from (11) and (12) that

$$\dim M_1(K) - \dim M_2(K) = \mathrm{np}(K).$$

This demonstrates condition (ii) for the union of $\mathcal{Z}$ and the conjugacy class of K.

The second case is when $O_p(K) \neq 1$. Let L be the irreducible Mackey functor constructed in Corollary 6 satisfying $\dim L(K) = 1, L(H) = 0$ unless a conjugate of H contains K, and such that every restriction $L \downarrow^G_J$ is projective relative to p-local subgroups of J. In particular $L(H) = 0$ for $H \in \mathcal{Z}$. Consider the value of $n = \dim M_1(K) - \dim M_2(K) - \mathrm{np}(H)$. If $n \leq 0$, replace M_1 by

$$M_1 \oplus \underbrace{L \oplus \cdots \oplus L}_{-n \text{ copies}},$$

and if $n \geq 0$ replace M_2 by

$$M_2 \oplus \underbrace{L \oplus \cdots \oplus L}_{n \text{ copies}}.$$

With these modifications condition (ii) is satisfied for the union of $\mathcal{Z}$ and the conjugacy class of K.

It follows from this that the set of all subgroups of G satisfies conditions (i) and (ii) above. This verifies the conditions of Conjecture 3, which finishes the proof of Theorem 4. Note that in the proof above, we only had to modify the Mackey functors M_1 and M_2 in case $O_p(K) \neq 1$. Thus the construction of the Mackey functors actually stops as soon as $\mathcal{Z}$ contains all p-local subgroups.

3. A counterexample

Given that one is thinking along the lines of Conjecture 3, it is natural to consider not so much Conjecture 3 itself, which involves the difference of two Mackey functors, as the corresponding statement with only one Mackey functor. This gives a set of conditions which is both simpler and conceptually more appealing. We pose this statement as a question.

QUESTION 13. *Does there exist for every finite group G, for every prime p, a Mackey functor M so that $M(H)$ is always a vector space over a field R, satisfying the conditions*
(i) for every subgroup H, $M\downarrow_H^G$ is projective relative to p-local subgroups of H,
(ii) for every subgroup H, $\dim M(H) = \mathrm{np}(H)$?

Evidently a positive answer to this question would imply Conjecture 3 since we only need take $M_1 = M$ and $M_2 = 0$ in that conjecture for it to be satisfied. Thus such a positive answer would establish Conjecture 2 provided R has characteristic 0 or prime to $|G|$. We did at one time think there was a chance that Question 13 might indeed have a positive answer, but even if one forgets about condition (i) one can still find a counterexample with a very uncomplicated group, as the following result shows.

PROPOSITION 14. *With $p = 3$ there exists a group G with a non-trivial normal 3-subgroup for which there is no Mackey functor M satisfying* $\dim M(H) = \mathrm{np}(H)$ *for every subgroup H of G.*

Proof. Let G be the Frobenius group $(C_3 \times C_3)\cdot C_2$ where C_2 acts on $C_3 \times C_3$ as inversion. G has 4 subgroups C_3 and each is contained in 3 subgroups Σ_3 obtained by adjoining elements of the non-trivial coset of $C_3 \times C_3$. Call these 12 subgroups $A_{ij}, 1 \le i \le 4, 1 \le j \le 3$, where for fixed i, A_{i1}, A_{i2}, A_{i3} are supposed to be the three subgroups isomorphic to Σ_3 containing a fixed subgroup C_3. These three subgroups form a single conjugacy class. They satisfy

$$A_{ij} \cap A_{kl} = \begin{cases} C_3 & \text{if } i = k, j \neq l \\ C_2 & \text{if } i \neq k. \end{cases}$$

We have $N_G(A_{ij}) = A_{ij}$.

Suppose that M is a Mackey functor on G for which $M(H)$ is a vector space over a field R, and such that $\dim M(H) = \mathrm{np}(H)$ for every subgroup H. Thus $\dim M(G) = \dim M(A_{ij}) = 2$, $\dim M(C_3) = 1$. We work with subgroups A_{i1}. The Mackey decomposition formula takes the form

$$R^G_{A_{i1}} I^G_{A_{i1}} = 1 + I^{A_{i1}}_{C_3} c_x R^{A_{i1}}_{C_3}$$
$$R^G_{A_{j1}} I^G_{A_{i1}} = 0 \text{ if } i \neq j \text{ since } {}^g A_{i1} \cap A_{j1} = C_2.$$

Here x is any 3-element not in A_{i1}. In the first equation the mapping $I^{A_{i1}}_{C_3} c_x R^{A_{i1}}_{C_3}$ has rank ≤ 1 since it factors through the space $M(C_3)$ of dimension 1. Hence $\operatorname{rank} R^G_{A_{i1}} I^G_{A_{i1}} \geq 1$, and so $\operatorname{rank} R^G_{A_{i1}} \geq 1$ and $\operatorname{rank} I^G_{A_{i1}} \geq 1$. This holds for all i. The second equation says $\operatorname{Im} I^G_{A_{i1}} \subseteq \ker R^G_{A_{j1}}$ for all $i \neq j$ and combining this with the rank condition gives $\operatorname{Im} I^G_{A_{i1}} = \ker R^G_{A_{j1}}$ for all $i \neq j$. We deduce $\operatorname{Im} I^G_{A_{i1}} = \operatorname{Im} I^G_{A_{j1}} = \ker R^G_{A_{j1}}$ for all j since this suffix can take more than 2 values. But this means $R^G_{A_{i1}} I^G_{A_{i1}} = 0$, contradicting the first equation above. We deduce that no Mackey functor M can exist.

4. Groups with cyclic Sylow p-subgroups

In this situation we construct a Mackey functor M on G which does in fact satisfy the conditions of Question 13. By taking $M_1 = M, M_2 = 0$ we also satisfy the conditions of Conjecture 3.

Suppose that G has a cyclic Sylow p-subgroup, and let C denote any subgroup of order p. Let Br be the 'ring of Brauer characters' Mackey functor on $N_G(C)/C$, and in the notation used previously let $M = (\operatorname{Inf}^{N_G(C)}_{N_G(C)/C} \operatorname{Br}) \uparrow^G_{N_G(C)}$. Then by the argument of Corollary 6(iii) M is a Mackey functor such that restriction to every subgroup is projective relative to p-local subgroups. We will show that it satisfies condition (i) of Question 13, namely, for every subgroup H, $\dim M(H) = \operatorname{np}(H)$. The argument will follow after the next lemma, which is little more than Proposition 5.

LEMMA 15. *(i) $M(H) = 0$ unless H contains C up to conjugacy.*
(ii) If $C \leq H$ then $M(H) = M(N_H(C)) = \operatorname{Br}(N_H(C))$.

Proof. We have

$$M(H) = \bigoplus_{x \in N_G(C)\backslash G/H} \operatorname{Inf}^{N_G(C)}_{N_G(C)/C} \operatorname{Br}(N_G(C) \cap {}^x H).$$

(i) All the terms are zero unless $N_G(C) \cap {}^x H \geq C$ for some x, i.e. ${}^x H \geq C$.

(ii) Suppose $C \leq H$. We get a non-zero term whenever $C \leq N_G(C) \cap {}^x H$. In this case $C \leq {}^x H$, ${}^{x^{-1}}C \leq H$. Now $C = {}^{hx^{-1}}C$ for some $h \in H$ since Sylow p-subgroups are cyclic, so $hx^{-1} = n \in N_G(C)$ and $x = n^{-1}h$ lies in the double coset $N_G(C) \cdot 1 \cdot H$. Thus only the summand represented by $x = 1$ is non-zero and it takes the value $\mathrm{Br}(N_G(C) \cap H) = \mathrm{Br}(N_H(C))$. We get the same answer if we do this calculation with $N_H(C)$ instead of H, so $M(H) = M(N_H(C))$.

We now show that $\dim M(H) = \mathrm{np}(H)$ for all subgroups H. Each Sylow p-subgroup of G contains a unique subgroup of order p which is a conjugate of C, and so H has order divisible by p if and only if H contains a conjugate of C. Thus when H is a p'-group we have $M(H) = 0$ by (i) , and also $\mathrm{np}(H) = 0$ because all modules are projective. This is the correct answer. Suppose now that H has order divisible by p. Without loss of generality H contains C. It follows from the theory of blocks with cyclic defect that $\mathrm{np}(H) = \mathrm{np}(N_H(C))$ using the fact that $N_H(C)$ contains up to conjugacy the normalizer of every non-identity p-subgroup [1, p.150]. The latter number is $\dim \mathrm{Br}(N_H(C))$ since $N_H(C)$ has a normal p-subgroup. By (ii) this is also $\dim M(H)$. This completes the proof that M satisfies the conditons of Question 13.

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Jacques THÉVENAZ

Institut de Mathématiques
Université de Lausanne
CH-1015 Lausanne-Dorigny
Switzerland

Peter WEBB

Mathematical Sciences Research Institute
1000 Centennial Drive
Berkeley CA 94720
USA