

Astérisque

AST

**Actes du colloque de théorie de Hodge - Luminy,
1-6 Juin 1987 - Pages préliminaires**

Astérisque, tome 179-180 (1989), p. 1-11

http://www.numdam.org/item?id=AST_1989__179-180__1_0

© Société mathématique de France, 1989, tous droits réservés.

L'accès aux archives de la collection « Astérisque » (<http://smf4.emath.fr/Publications/Asterisque/>) implique l'accord avec les conditions générales d'utilisation (<http://www.numdam.org/conditions>). Toute utilisation commerciale ou impression systématique est constitutive d'une infraction pénale. Toute copie ou impression de ce fichier doit contenir la présente mention de copyright.

NUMDAM

Article numérisé dans le cadre du programme
Numérisation de documents anciens mathématiques

<http://www.numdam.org/>

179-180

ASTÉRISQUE

1989

**ACTES DU COLLOQUE
DE
THÉORIE DE HODGE**

Luminy, 1-6 Juin 1987

Comité d'organisation :
D. BARLET, H. ESNAULT, F. ELZEIN,
†J.L. VERDIER, E. VIEHWEG.

SOCIÉTÉ MATHÉMATIQUE DE FRANCE

Publié avec le concours du CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

A.M.S. subjects Classification :

32J25, 14C30

**(par article) 32C40-32B30-32J25 ; 32J25-14C30-14F35 ; 32J25-14C30-32C35-32G20 ;
32L20-14F40-14F05-32L10 ; 14F40-14F45-32L10 ; 32J25-14C30-14F40 ; 14B05-32C40-
14J17-32J25 ; 14C30-14J10-32G20 ; 32J25-14C30-32C35.**

*Jean-Louis VERDIER était coorganisateur du colloque de
Théorie de Hodge et coéditeur de ce volume qui en rassemble
quelques contributions.*

Sa disparition accidentelle nous affecte profondément.

Nous dédions ces Proceedings à sa mémoire.

INTRODUCTION

Dans les années précédant la tenue du Colloque, diverses directions en "théorie de Hodge" avaient connu un développement spectaculaire : Cattani-Kaplan-Schmid et Kashiwara-Kawai avaient démontré par des méthodes L^2 que la cohomologie d'intersection à valeur dans une variation de structure de Hodge polarisée sur une variété lisse kählérienne possède une structure de Hodge. A la demande des organisateurs, Cattani-Kaplan ont rédigé pour ces Proceedings une introduction au sujet. Looijenga et Saper-Stern avaient étendu ce résultat au cas des variétés localement symétriques pour la cohomologie d'intersection à coefficients constants. Une présentation en est donnée dans la contribution de Zucker.

Ces travaux ne peuvent être compris que dans le cadre général de la théorie de la cohomologie d'intersection et des faisceaux pervers, théorie dont un des fondateurs est O. Gabber. Sa présence à la conférence fut déterminante : ses remarques et critiques très constructives ont animé et clarifié beaucoup d'exposés.

Un des développements récents en théorie de Hodge, dont on mesure encore mal toutes les conséquences, est la théorie des modules de Hodge mixtes de M. Saito. Celle-ci devrait correspondre "philosophiquement" à celle des faisceaux pervers mixtes de Beilinson-Bernstein - Deligne-Gabber.

Dans un autre ordre d'idées, Deligne-Illusie avaient décomposé le complexe de De Rham d'une variété lisse en caractéristique $p > 0$ sous l'hypothèse de relèvement modulo p^2 de la variété, comblant ainsi un "trou" de la Géométrie Algébrique (exposé non rédigé). Diverses contributions aux "bords" de la théorie de Hodge furent présentées à la conférence : extensions de variations de structure de Hodge, cycles algébriques, théorème de comparaison analytique/algébrique, ainsi que des applications de la théorie de Hodge aux théorèmes d'annulation et aux singularités.

Different directions in "Hodge Theory" were developed in a spectacular way during the years before the conference : Cattani-Kaplan-Schmid and Kashiwara-Kawai had shown via L^2 -methods that the intersection cohomology with values in a polarized variation of Hodge structure on a kählerian manifold carries a Hodge structure. At the request of the organizers, Cattani-Kaplan contributed an introduction to the subject for these Proceedings. Looijenga and Saper-Stern had extended these results to the case of locally symmetric spaces for the intersection cohomology with constant coefficients. A presentation is given in Zucker's contribution.

These works can only be understood in the general framework of intersection cohomology and perverse sheaves, a theory one of whose founder was O. Gabber. His participation to the conference was determinant, his very constructive remarks and criticisms enlivened and clarified many talks.

One of the recent development in Hodge theory all of whose consequences are not yet understood is M. Saito's theory of mixed Hodge modules. This should correspond "philosophically" to the theory of mixed perverse sheaves of Beilinson-Bernstein-Deligne-Gabber.

In a different direction, Deligne-Illusie had decomposed the De Rham complex of a manifold in characteristic $p > 0$ under the assumption that the variety lifts modulo p^2 , filling in a "gap" in Algebraic Geometry (contribution not written down). Several subjects at the border line of Hodge theory were presented at the conference : extensions of variations of Hodge structure, algebraic cycles, comparison theorems analytic versus algebraic, as well as applications of Hodge theory to vanishing theorems and to singularities.

TABLE DES MATIÈRES

<p>ASYMPTOTIC FILTRATION and POLES of $\int_X f ^{2\lambda} \square$ by Daniel BARLET</p>	13
<p>EXTENSIONS OF VARIATIONS OF MIXED HODGE STRUCTURE by James A. CARLSON and Richard M. HAIN</p>	39
<p>DEGENERATING VARIATIONS OF HODGE STRUCTURE by Eduardo CATTANI and Aroldo KAPLAN</p>	67
<p>VANISHING AND NON VANISHING THEOREMS by Hélène ESNAULT and Eckart VIEHWEG</p>	97
<p>COHOMOLOGY WITH COEFFICIENTS IN $\mathbb{Z}(p)$ AND DE RHAM COHOMOLOGY : EXAMPLES by Helmut A. HAMM</p>	113
<p>"INTRODUCTION TO MIXED MODULES" by Morihiko SAITO</p>	145
<p>THE SPECTRUM OF HYPERSURFACE SINGULARITIES by Joseph H.M. STEENBRINK</p>	163
<p>TYPE I DEGENERATION OF KUNEV SURFACES by Sampei USUI</p>	185
<p>L²-COHOMOLOGY AND INTERSECTION HOMOLOGY OF LOCALLY SYMMETRIC VARIETIES, III by Steve ZUCKER</p>	245

LISTE DES PARTICIPANTS

BALLICO E.
Dip. di Matematica
Univ. di Trento
38050 POVO
Italie

BARLET D.
UER de Mathématiques
Univ. de Nancy I
B.P. 239
54506 VANDEUVRE LES NANCY

BEAUVILLE A.
Mathématiques, Bât. 425
Univ. de Paris-Sud
91405 ORSAY CEDEX

BUCHWEITZ R.O.
Institut f. Mathematik
Univ. Hannover
Welfengarten 1
D 3000 HANNOVER 1
R.F.A.

CARLSON J.
The University of Utah
Dpt of Maths
Salt Lake City
Utah 84112
USA

CATTANI E.
Dept of Mathematics
University of Massachusetts
AMHERST, Ma. 01003
USA

DU BOIS P.
Dépt. de Mathématiques
Faculté des Sciences
2, Chemin de la Houssinière
44072 NANTES CEDEX 03

ELZEIN F.
Dépt. de Mathématiques
Univ. Paris 7
2, Place Jussieu
75251 PARIS

ESNAULT H.
Max-Planck Institut f. Mathematik
Gottfried-Claren-Str. 26
D-53 BONN 3
R.F.A.

GABBER O.
I.H.E.S.
35, Rte de Chartres
91440 BURES SUR YVETTE

GRANGER J.M.
Mathématiques
Univ. d'Angers
2, Bd Lavoisier
49 ANGERS

GUILLEN J.M.
ETSEIB
Univ. Politecnica de Cataluna
Diagonal 647
08028 BARCELONA
Espagne

HAMM H.A.
Math Institut
W.W.U. Münster
Einsteinstr. 62
D 4400 MUNSTER
R.F.A.

HÖFER T.
Math. Institut
Universität Bonn
Beringstr. 4
D 5300 BONN 1
R.F.A.

HOMANN W.
Math. Institut
W.W.U. Münster
Einsteinstr. 64
D 4400 MUNSTER
R.F.A.

ILLUSIE L.
Mathématiques, Bât. 425
Université de Paris Sud
Centre d'Orsay
91405 ORSAY CEDEX

IVINSKIS K.
FB 6 Mathematik
Universität GHS Essen
Universitätstr. 3
4300 ESSEN 1
R.F.A.

KAPLAN A.
Dept of Mathematics
University of Massachusetts
AMHERST, Ma. 01003
U.S.A.

THÉORIE DE HODGE

KASHIWARA M.
Research Institute for Mathematical Studies
Kyoto University,
KYOTO - Japon

KNÖRRER H.
Mathematisches Institut
Universität Bonn
Wegelerstrasse 10
D-5300 BONN
R.F.A.

LAURENT Y.
Mathématiques, Bât. 425
Université de Paris Sud
91405 ORSAY CEDEX

LEJEUNE-JALABERT M.
Université de Grenoble
B.P. 74
38402 ST MARTIN D'HERES CEDEX

LOESER F.
Centre de Mathématiques
Ecole Polytechnique
91128 PALAISEAU CEDEX

MAISONOBE P.
Dept de Mathématiques
Université de Nice
Parc Valrose
06034 NICE CEDEX

MEBKHOUT Z.
C.N.R.S.
PARIS

MERINDOL J.Y.
Dépt de Mathématiques
Univ. d'Angers
Bd Lavoisier
49045 ANGERS CEDEX

NAKAYAMA N.
Max Planck Institut
f. Mathematik
Gottfried Claren Str. 26
D 53 BONN 3
R.F.A.

ROIG A.
Univ. Politecnica de Catalunya
ETSEIB
Diagonal 647
08028 BARCELONA
Espagne

VERDIER J.L.
Ecole Normale Supérieure
45, Rue d'Ulm
75005 PARIS

VIEHWEG E.
FB 6 Mathematik
Essen Universität
Universitätstr. 2
D 43 ESSEN
R.F.A.

SABBACH C.
Centre de Mathématiques
Ecole Polytechnique
91128 PALAISEAU CEDEX

SAITO M.
Reserach Inst. for Math. Sciences
Kyoto University
Sakyo-ku
KYOTO 606
Japon

STEENBRINK J.
Math. Institut
Université de Reiden
Niels Eohrweg 1
2333 CA LEIDEN
Pays Bas

SZPIRGLAS A.
Dept de Math
Centre Scientifique et polytechnique
Université Paris Nord
Avenue J.B. Clément
93430 VILLETANEUSE - France

TIMMERSCHIEDT K.
FB 6 Mathematik
Universität Essen
Universitätstr. 3
4300 ESSEN 14
R.F.A.

USUI S.
Dpt. of Mathematics
Fac. of Sciences, Kochi University
KOCHI 606
Japon

VAN STRATEN D.
Rÿusburgerweg 31 D
2334 LEIDEN
Pays Bas

VAQUIE M.
Centre de Mathématiques
Ecole Polytechnique
91128 PALAISEAU CEDEX

VOGT A.
Math. Institut
Universität Regensburg
NWF 1, Universitätstr. 31
D 8400 REGENSBURG
R.F.A.

ZUCKER S.
Dept. of Math./JHU
John Hopkins Univ.
BALTIMORE, Md. 21218
U.S.A

LISTE DES EXPOSÉS

Lundi 1^{er} juin :

- 9 h 30 A. Kaplan : Degeneration of Hodge structures.
11 h 15 S. Zucker : Hodge theory with degenerating coefficients.
16 h 00 L. Illusie : Relèvements mod. p^2 et dégénérescence de la suite spectrale de Hodge.
17 h 45 H. Esnault : Complexe de De Rham logarithmique et théorème d'annulation.

Mardi 2 juin :

- 9 h 30 E. Cattani : L^2 and intersection cohomology.
11 h 15 M. Kashiwara : Hodge modules with normal crossing singularities.
16 h 00 M. Saito : Mixed Hodge modules.
17 h 45 K. Timmerscheidt : Mixed Hodge theory for unitary systems.

Mercredi 3 juin :

- 9 h 30 J. Steenbrink : Courbes polaires et cycles évanescents.
11 h 15 J. Carlson : Extension of variations of Hodge structures.
16 h 00 R. Buchweitz : Algebraic theory of algebraic cycles.
17 h 45 A. Beauville : Théorème d'annulation générique de Green-Lazarsfeld.

Jeudi 4 juin :

- 9 h 30 F. Elzein : Théorie de Hodge à coefficients dans une variation de structure de Hodge mixte.
11 h 15 Z. Mebkhout : Théorème de comparaison entre cohomologies de De Rham.

Vendredi 5 juin :

- 9 h 30 H. Hamm : Cohomologie à valeurs dans $Z_{(p)}$ et cohomologie de De Rham.
11 h 15 Usui : Degeneration of Kunev varieties.
14 h C. Sabbah : Proximité évanescence.
15 h 30 D. Barlet : Symétrie de Hodge pour les racines du polynôme de Bernstein-Sato.

ASYMPTOTIC FILTRATION and POLES of $\int_X |f|^{2\lambda} \square$

par Daniel BARLET

Abstract : We consider an "Asymptotic filtration" F^\bullet , following A.N. Varchenko's idea, on the cohomology of the Milnor fiber of a germ $f : (\mathbb{C}^{n+1}, 0) \rightarrow (\mathbb{C}, 0)$ without assumption on the singularity of $\{f = 0\}$.

Then we give a relation between the nilpotent logarithm N of the unipotent part of the monodromy, the filtrations F^\bullet and \bar{F}^\bullet and the poles of the meromorphic continuation of the distribution $\int |f|^{2\lambda} \square$.

This contains estimates of the integral shift of the roots of the Bernstein-Sato polynomial of f in term of the action of N on the filtration F^\bullet .

EXTENSIONS OF VARIATIONS OF MIXED
HODGE STRUCTURE

by James A. CARLSON and Richard M. HAIN

Abstract : We compute the Yoneda extension groups of unipotent variations of mixed Hodge structure. When the variations are \mathbb{Q} and $\mathbb{Q}(p)$, we show that the group of extensions is the continuous Deligne-Beilinson cohomology of the fundamental group of the base space.

DEGENERATING VARIATIONS OF HODGE STRUCTURE

by Eduardo CATTANI and Aroldo KAPLAN

Abstract : This is a survey on some techniques and results on degenerations of polarized, pure Hodge structures. We review Schmid's orbit theorems and their generalization to variations of several variables emphasizing the relationships : nilpotent orbit \leftrightarrow polarized mixed Hodge structure and SL_2 -orbit \leftrightarrow polarized mixed Hodge structure split over \mathbb{R} . These are then used to describe the asymptotic behavior of a period mapping and to give estimates on the degeneration of the Hodge norm. We conclude with a brief sketch of the proof of the isomorphism between the L_2 and intersection cohomologies of a compact Kähler manifold with values on a local system underlying a polarizable VHS.

VANISHING AND NON VANISHING THEOREMS

by Hélène ESNAULT and Eckart VIEHWEG

Abstract : We present several applications of our vanishing theorem for cohomology groups of sheaves of twisted logarithmic differentials. We improve A. Sommese's vanishing theorem for k -ample sheaves. Moreover we generalize A.N. Varchenko's and our own description by global logarithmic differential forms of certain non trivial cohomology classes with values in local constant systems.

COHOMOLOGY WITH COEFFICIENTS IN $\mathbb{Z}(p)$
AND DE RHAM COHOMOLOGY : EXAMPLES

by Helmut A. HAMM

Abstract : If p is a prime number and X a smooth complete intersection over $\mathbb{Z}(p)$ one can use a calculation of P. Deligne to show that the groups $H^r(X^{an}; \mathbb{Z}(p))$ and $H^r(X, \Omega_{X/\mathbb{Z}(p)}^r)$ have the same rank and torsion. This result is extended to similar classes of examples ; here the De Rham complex is modified in view of the fact that the spaces in question may not be smooth nor proper.

"INTRODUCTION TO MIXED MODULES"

by Morihiko SAITO

Abstract : We introduce the notion of Mixed Hodge Modules which corresponds philosophically to that of mixed perverse sheaf of Beilinson-Bernstein-Deligne-Gabber, and explain some consequences of the stability of its bounded derived categories $D^b\text{MHM}(X)$ by the standard functors f_* , $f_!$, f^* , $f^!$, ψ_g , $\phi_{g,1}$, D , \boxtimes , \otimes , Hom , and the relation with Hodge cycle, Griffiths normal function, Deligne cohomology, admissible variation of mixed Hodge structure in the sense of Steenbrink-Zucker and Kashiwara, etc.

THE SPECTRUM OF HYPERSURFACE SINGULARITIES

by Joseph H.M. STEENBRINK

Abstract : The notion of the spectrum is generalized from isolated to arbitrary hypersurface singularities. A formula for the behaviour of the spectrum in Yomdin series of isolated singularities is conjectured and proven in some special cases : curve singularities and certain types of surface singularities and homogeneous ones. Examples are given of curve singularities with different imbedded topological types and equal spectra, and of surface singularities with equal spectra but different absolute topological type. A formula is given for the spectrum of a homogeneous singularity with one-dimensional singular locus.

TYPE I DEGENERATION OF KUNEV SURFACES

by Sampei USUI

Abstract : We determine here completely the main components of the degenerations of Kunev surfaces with finite local monodromy on the second cohomology. There are two series of degenerations. One is $\mathcal{S}_0 \cap \mathcal{F}_0 := \{\text{Kunev surfaces}\} \rightarrow \mathcal{S}_1 := \{\text{numerical K3 surfaces with one double fiber}\} \rightarrow \mathcal{S}_2 := \{\text{K3 surfaces}\}$, and the other is $\mathcal{S}_0 \cap \mathcal{F}_0 \rightarrow \mathcal{F}_1 \cap \mathcal{S}_0 := \{\text{elliptique surfaces with } P_g = q = 1\} \rightarrow \mathcal{F}_2 := \{\text{splitting abelian surfaces}\}$. In particular, this explains uniformly the appearance of positive dimensional fibers of the period maps of the surfaces $\mathcal{S}_0 \cap \mathcal{F}_0$, \mathcal{S}_1 and $\mathcal{F}_1 \cap \mathcal{S}_0$ by degeneration.

L²-COHOMOLOGY AND INTERSECTION HOMOLOGY OF LOCALLY SYMMETRIC VARIETIES, III

by Steve ZUCKER

Abstract : In 1987, two proofs were announced for the so-called Zucker Conjecture, giving a topological interpretation of the L^2 -cohomology of a locally symmetric variety (Shimura variety). One, by E. Looijenga, is algebro-geometric in character ; the other, by L. Saper and M. Stern, uses no algebraic geometry at all! In this article, we give an exposition of both proofs, and discuss their relation to previous developments. As the title suggests, this is the third article in a series, each of which was different in tone : the first was a brief account of the author's results that led to the conjecture ; the second was written as a "last testament" on the subject... .