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Cats’ tales

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Consider the following: there are cats in some of the places of a two-sided infinite sequence. Then start, step by step, this process: at each step each cat jumps, independently of the others, with probability $\frac{1}{2}$ to each of the two neighbouring places. If two cats land on the same place, they disappear (imagine each second cat to be an anti-cat).

Define $A \equiv \{ 0 \text{ is visited an infinite number of times} \}$.

**Question:** $p(A) = ?$

(there are some versions of this problem, that can be handled in a very similar way).

The answer depends, of course, on the initial distribution of the cats.

We can immediately get $p(A) = 1$ and $p(A) = 0$ in the cases of odd and even number of cats, respectively.

Denote by $i(n)$ the initial number of cats in the block $1, \ldots, n$, and suppose the negatives are initially empty. Then in the infinite cats case in which $\frac{i(n)}{n} \to 0$ simple examples can be found for which $p(A) = 1$, as well as others for which $p(A) = 0$.

The general case $\lim_{n \to \infty} \frac{i(n)}{n} > 0$ is unsolved yet, but there is a large class for which the answer can be proved to be $p(A) = 1$. This class contains, as a typical sub-class, those sequences in which there is some $n$ such that there are infinitely many $n_k$'s such that
the block \( n, \ldots, n+2r_k \) is, in the beginning, symmetric with respect to reflection about \( n+n_k \), and contains an odd number of cats (the sequence in which all the naturals are initially occupied \( \frac{i(n)}{n} \equiv 1 \) is, of course, contained in this subclass).

The proof to the last claim is rather long, but its basic idea is the same as that in the following proof of \( p(A) \) being 1 when there is one cat only.

Suppose the cat is in the \( n \)'th place. By symmetry, there is probability \( \frac{1}{2} \) that \( 2n \) is visited before \( 0 \). If that happens, then there is probability \( \frac{1}{2} \) that \( 4n \) is visited before \( 0 \), and so on. But \( \left( \frac{1}{2} \right)^\infty = 0 \), so \( 0 \) will a.s. be visited, so it will a.s. be visited an infinite number of times.

In the case of finite number of cats, a similar method can be applied to the \( n \)-dimensional analogous problem (\( p(A) \) found, as is known, to vanish for \( n > 2 \)), but I don't know how to treat the general \( n \)-dimensional infinite cats problem (excluding some special cases).

REFERENCES


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