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On the set of complex points of a 2-sphere

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Abstract. L G r $\mathbb{C}^2$ C^∞- r ∂G. L S 2- r ∂G. D $\mathcal{E}$ f S. P n r f $\mathcal{E}$ f S.

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1. Introduction

L M 2- C^1- f $\mathbb{C}^2$. A p M *complex point*, f T_pM M p . D $\mathcal{E}$ f M. I f M r , $\mathcal{E}$ f . I f M r r f r f M r f $\mathcal{E}$ (11). T n r f M r $\mathcal{E}$, r ff r f (, f r , 2, 3, 9, 10, 12, 15 . I r n r f $\mathcal{E}$, M 2- r , S f , r ∂G f C^∞- r G $\mathbb{C}^2$ (r f r , 5 6). I r G, $\mathcal{E}$ r r S. O r r r r f $\mathcal{E}$ f S. , f f C^{2-} f r r r f f L $^{1,\alpha}$ f r $\alpha < 1$. O r r f r f .

Theorem 1.1. *Let G be a strictly pseudoconvex domain in $\mathbb{C}^2$ with C^∞-smooth boundary ∂G. Let S be a 2-dimensional sphere embedded into ∂G. Then, depending on the smoothness of S, the following holds*:

1) *If S is of class C^2, then the set $\mathcal{E}$ of complex points of S is contained in a C^1-smooth nonclosed curve $\gamma \subset S$.*

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2) *There exists a 2-dimensional sphere $S \subset \partial G$ of class C^{2-} such that the set $\mathcal{E}$ contains a Jordan curve of positive 2-dimensional measure.*

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2. Proof of the first part of the theorem

P r r B 2 (9). N f p $\mathcal{E}$, f r f r p r r r S

$$D = \Big\{(z, f(z)) \in \mathbb{C}^2 : z \in \triangle\Big\}$$

$\triangle$ r r f C^2- f M r r, f r f G, f r f r f f f r

$$f(z) = \frac{1}{2}|z|^2 - \beta \quad z^2 + o(|z|^2), \qquad \beta \geq 0$$

r r , r *elliptic point* f $0 \leq \beta < \frac{1}{2}$, *hyperbolic point* f $\beta > \frac{1}{2}$ *parabolic point* f $\beta = \frac{1}{2}$. E r- r $\mathcal{E}$. I f r r r $z = x + iy$ r r f $f(z) = y^2 + o(|z|^2)$. H $\partial_{\bar{z}} f(z) = iy + o(|z|)$, f r -

$i\vec{u}(p) \in T_p^{\mathbb{C}}(\partial G)$ … $\vec{n}(p) \in T_p(\partial G)$ … $T_p^{\mathbb{C}}(\partial G)$ … $(\vec{u}(p), i\vec{u}(p), \vec{n}(p))$ … ∂G … p. L … $O(p)$ … $T_p(\partial G)$ … $\vec{u}(p)$ … $i\vec{u}(p)$ … $\vec{n}(p)$. … 8, … 1.4) … S' … Γ' … 2- … $S'' \subset \partial G$, C^∞- … Γ' … $\Gamma' \subset S''$ … $\vec{n}(p) \in T_p(S'')$ … $p \in \Gamma'$. I … S'' … Γ'. … S'' … Γ' … C^∞- … 2- … $\tilde{S} \subset \partial G$ … Gr … 7, … 342). N … B … K … 3 … Kn … 10 … 3- $\mathcal{B}$ … $\{D_\alpha\}$, … $\partial \mathcal{B} = \tilde{S}$. B … C … 4 … D_α … C^∞- … ∂D_α … $\tilde{S}$ (*i.e.* … $\tilde{S}$) … ∂D_α … D_α … C^∞- … $\tilde{S}$ … $\{T_p^{\mathbb{C}}(\partial G)\}$ … ∂G. C … D_{α_0} … $\{D_\alpha\}$ … ∂D_{α_0} – … Γ' … – … p … $\partial D_{\alpha_0} \cap \Gamma'$. M … $D_{\alpha_0} \subset \mathcal{B}$ … $\partial D_{\alpha_0} \cap \Gamma' \neq \emptyset$ … $\mathcal{B} \backslash D_{\alpha_0}$, … D_α, … $\partial D_\alpha \cap \Gamma' = \emptyset$. N … ∂D_{α_0} … Γ' … p, … Γ' … ∂D_{α_0} … p. O … p … $\tilde{S}$, … ∂D_{α_0} … $T_p^{\mathbb{C}}(\partial G)$. …

Remark 2.1. I … G … 3- … $\Gamma \subset S \subset \partial G$ … 1, … 1 .

3. Proof of the second part of the theorem

… F … $E \subset \mathbb{R}^2_{x,y}$ … 2- … G … E … $G \in C^{2-}(E)$ … $G'_x(x, y) = y$ … $G'_y(x, y) = 0$ … G … E. N … H. … (… 14), … $H \in C^{2-}(E)$ … $H'_x(x, y) = 0$ … $H'_y(x, y) = 0$. … G … H … $F \in C^{2-}(E)$ … $F'_x(x, y) = y$ … $F'_y(x, y) = 0$ … E. … E, … J … $\tilde{E} \subset \mathbb{R}^2_{x,y}$ … 2- … F, … $\tilde{F}$ … $\tilde{E}$ … $C^{2-}(\tilde{E})$ …

$\tilde{F}'_x(x,y) = y$ $\tilde{F}'_y(x,y) = 0$. N ... P ... $\tilde{F}$, ... 2- ... $S^2 \subset \mathbb{R}^3_{x,y,z}$ ... C^{2-} ... J ... 2- ... S^2 ... $\mathbb{R}^3_{x,y,z}$. F ... D ... bG ... G.

3.1. Construction of the arc E

F ... $\alpha \in (0,1)$ ...

$$\mathbb{E}^\alpha = \left(\left[0, \frac{1-\alpha}{2}\right] \cup \left[\frac{1+\alpha}{2}, 1\right]\right) \times \left(\left[0, \frac{1-\alpha}{2}\right] \cup \left[\frac{1+\alpha}{2}, 1\right]\right)$$
$$\cup\,(\{0\} \times [0,1]) \cup \left([0,1] \times \left\{\frac{1+\alpha}{2}\right\}\right) \cup (\{1\} \times [0,1])\,.$$

P ... $A = (0,0)$, $B = (1,0)$, $Q_0 = [0, \frac{1-\alpha}{2}] \times [0, \frac{1-\alpha}{2}]$, $Q_1 = [0, \frac{1-\alpha}{2}] \times [\frac{1+\alpha}{2}, 1]$, $Q_2 = [\frac{1+\alpha}{2}, 1] \times [\frac{1+\alpha}{2}, 1]$, $Q_3 = [\frac{1+\alpha}{2}, 1] \times [0, \frac{1-\alpha}{2}]$. F ... $A_0 = A = (0,0)$, $B_0 = (0, \frac{1-\alpha}{2})$, $A_1 = (0, \frac{1+\alpha}{2})$, $B_1 = (\frac{1-\alpha}{2}, \frac{1+\alpha}{2})$, $A_2 = (\frac{1+\alpha}{2}, \frac{1+\alpha}{2})$, $B_2 = (1, \frac{1+\alpha}{2})$, $A_3 = (1, \frac{1-\alpha}{2})$, $B_3 = B = (1,0)$ (... $\mathbb{E}^\alpha$ ... F ... 3.1).

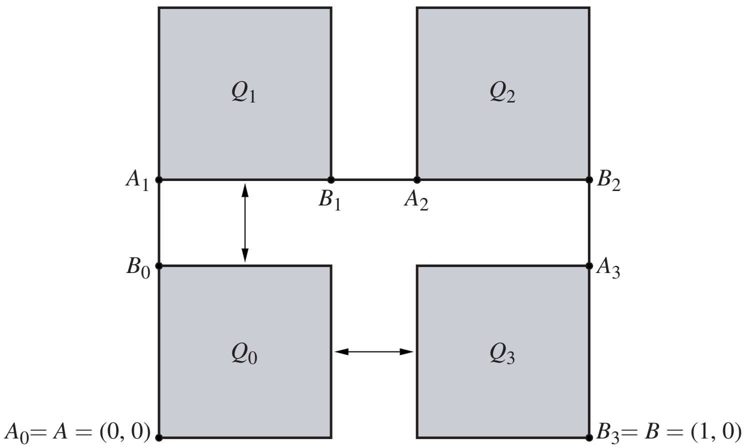

Figure 3.1. T ... $\mathbb{E}^\alpha$.

T ... E ... $\alpha_n = \frac{1}{(n+1)^2}$, $n = 1, 2, \ldots$ P ... E ... E_n ... $E_1 = \mathbb{E}^{\alpha_1}$. T ... E_2 ... $\tilde{\mathbb{E}}^{\alpha_2}$ ... $\mathbb{E}^{\alpha_2}$ ... $\frac{1-\alpha_1}{2}$.

T … $i = 0, 1, 2, 3$ … $\mathbb{E}_i$ … $\tilde{\mathbb{E}}^{\alpha_2}$ … (…) … A … B … A_i … B_i, … $i = 0, 3$. T … E_2 … E_1 … $i = 0, 1, 2, 3$ … Q_i … $\mathbb{E}_i$. F r … $j = 0, 1, 2, 3$ … Q_{ij}, A_{ij} … B_{ij} … Q_j … A_j, B_j … $\mathbb{E}_i$, … (… E_2 … F r 3.2).

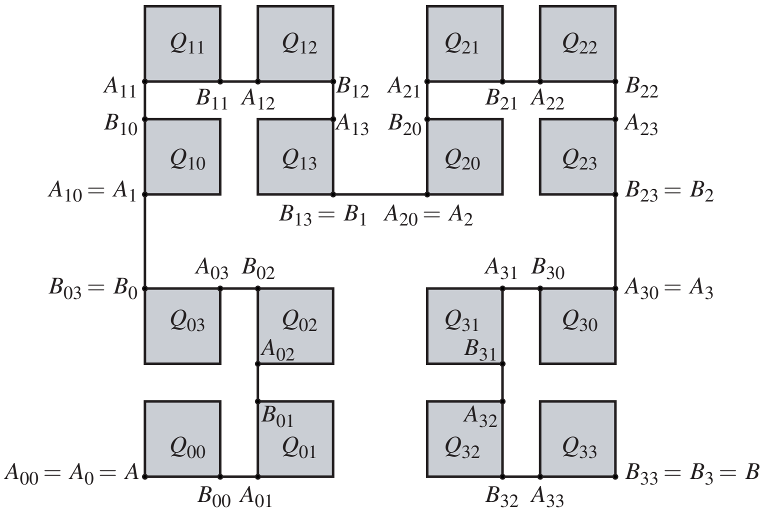

Figure 3.2. T … E_2.

T … E_n … E_{n+1}. C … $\tilde{\mathbb{E}}^{\alpha_{n+1}}$ … $\mathbb{E}^{\alpha_{n+1}}$ … $\prod_{i=1}^{n}\left(\frac{1-\alpha_i}{2}\right)$. T … $(i_1, i_2, \ldots, i_n)$, $i_j = 0, 1, 2, 3$, $j = 1, 2, \ldots, n$, … $\mathbb{E}_{i_1 \ldots i_n}$ … $\tilde{\mathbb{E}}^{\alpha_{n+1}}$ … (…) … A … B … $A_{i_1 \ldots i_n}$ … $B_{i_1 \ldots i_n}$, … T … E_{n+1} … E_n … $Q_{i_1 \ldots i_n}$ … $\mathbb{E}_{i_1 \ldots i_n}$. F r … $i_j = 0, 1, 2, 3$, $j = 1, 2, \ldots, n+1$, … $Q_{i_1 \ldots i_{n+1}}$, $A_{i_1 \ldots i_{n+1}}$ … $B_{i_1 \ldots i_{n+1}}$ … $Q_{i_{n+1}}$ … $A_{i_{n+1}}$, $B_{i_{n+1}}$ … $\mathbb{E}_{i_1 \ldots i_n}$, … N … $(i_1, \ldots, i_n)$ … $A_{i_1 \ldots i_n 0} = A_{i_1 \ldots i_n}$ … $B_{i_1 \ldots i_n 3} = Bt.$ 2 2 .2 . . Tm Tj F Tf . 2 . . 2

J_n ... r ... s. F r $m = n - 1$ ... r ... fr ... f ... f ... G_n ... Gr ... r ... $G_n(q) - G_n(p) - \int_{[p,q]} y\,dx$ r ... r ... (...) ... f ... r ... f ... r $J_n^{p,q}$ ... $[p, q]$ (... F r 3.4).

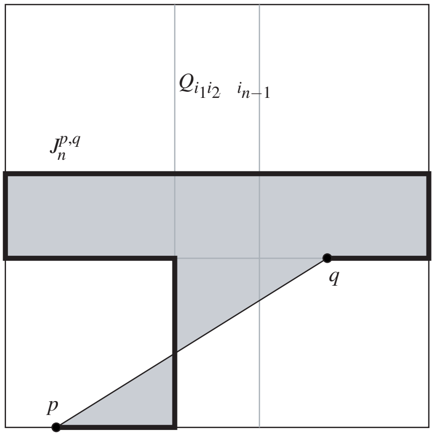

Figure 3.4. T ... r $J_n^{p,q}$ ... r $Q_{i_1i_2\ldots i_{n-1}}$.

... r ... r $Q_{i_1i_2\ldots i_{n-1}}$, ... (3.1) ... n ... T ... n ... f ... r m ... f ... rr ... r ... , r f r, r r ... T ... f ... r r r m ... r ... n ... r ... r .

A ... (3.1) ... n f r ... r $Q_{i_1i_2\ldots i_m}$ ... $p, q \in J_n \cap Q_{i_1i_2\ldots i_m}$ ... $m \geq m_0 + 1$ (n ...). C ... r ... r $Q_{i_1i_2\ldots i_{m_0}}$ ... $p, q \in J_n \cap Q_{i_1i_2\ldots i_{m_0}}$. O ... r ... r ... $[p, q]$ ... r ... r ... f ... r $Q_{i_1i_2\ldots i_{m_0}i_{m_0+1}}$, $i_{m_0+1} = 0, 1, 2, 3$. P ... f ... r ... r ... f ... (... r ... n ... r, ... r ... r ... r), ... r $Q_{i_1i_2\ldots i_{m_0}0}$, $Q_{i_1i_2\ldots i_{m_0}1}$ ... $Q_{i_1i_2\ldots i_{m_0}2}$ (... f ... r r ... r ... r ... r). C ... r ... r ... f ... $[p, q]$ fr ... p ... q ... fr ... $i = 0, 1, 2$ ... p_i ... p_i' ... r ... , r ... , (... r ... r ... f$[p, q]$) ... f ... $[p, q] \cap J_n^{p,q} \cap Q_{i_1i_2\ldots i_{m_0}i}$. N ... , ... $p_0 = p$ ... $p_2' = q$. F r ... $i = 0, 1, 2$ ... r ...

$$\left| G_n(p_i') - G_n(p_i) - \int_{[p_i, p_i']} y\,dx \right| < \text{Ar}\quad (Q_{i_1i_2\ldots i_{m_0}i}) < \frac{1}{4}\,\text{Ar}\quad (Q_{i_1i_2\ldots i_{m_0}}). \quad (3.2)$$

... $[p_0', p_1]$ ... r ... r J_n ... , ... - ... fr ... Gr ... r ... $G_n(p_1) - G_n(p_0') - \int_{[p_0', p_1]} y\,dx$ r ... r ...

r f Σ1 r $J_n^{p'_0,\,p_1}$ $[p'_0, p_1]$ (
F r 3.5).

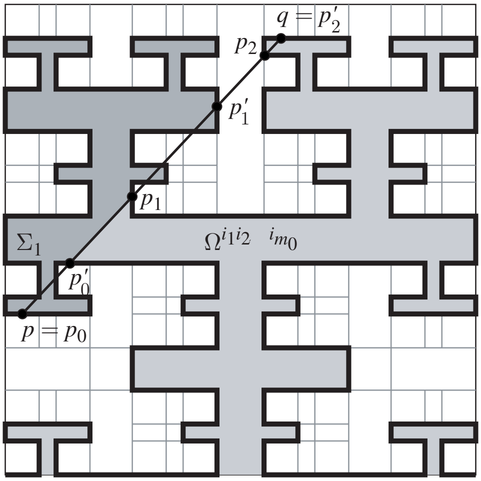

Figure 3.5. T Σ_1 $\Omega_n^{i_1 i_2 \dots i_{m_0}}$ r $\Omega_{i_1 i_2 \dots i_{m_0}}$.

Σ_1 $\Omega_n^{i_1 i_2 \dots i_{m_0}} = \Omega_n \cap Q_{i_1 i_2 \dots i_{m_0}}$,
$\Omega_n \cap Q_{i_1 i_2 \dots i_{m_0}} \subset Q_{i_1 i_2 \dots i_{m_0}} \setminus E_n$, f

$$0 < G_n(p_1) - G_n(p'_0) - \int_{[p'_0,\, p_1]} y\,dx = \mathrm{Ar}\quad(\Sigma_1) < \mathrm{Ar}\quad(\Omega_n \cap Q_{i_1 i_2 \dots i_{m_0}})$$

$$< \mathrm{Ar}\quad(Q_{i(}$$

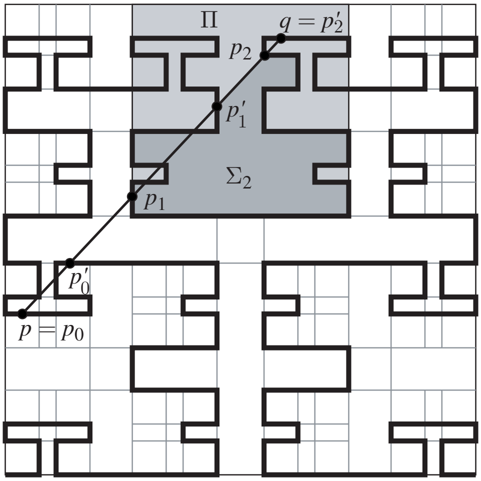

Figure 3.6. T Σ_2 Π r $\Omega_{i_1 i_2 \dots i_{m_0}}$.

Σ_2 f Π, $\mathrm{Ar}(\Pi) < \frac{1}{4}\mathrm{Ar}(Q_{i_1 i_2 \dots i_{m_0}})$,

$$0 > G_n(p_2) - G_n(p_1') - \int_{[p_1', p_2]} y dx = -\mathrm{Ar}(\Sigma_2) > -\mathrm{Ar}(\Pi)$$
$$> -\frac{1}{4}\mathrm{Ar}(Q_{i_1 i_2 \dots i_{m_0}}). \tag{3.4}$$

T ff r f r (3.3) (3.4) f r $m_0 \geq 6$ f

$$\left|\left(G_n(p_1) - G_n(p_0') - \int_{[p_0', p_1]} y dx\right) + \left(G_n(p_2) - G_n(p_1') - \int_{[p_1', p_2]} y dx\right)\right|$$
$$< \frac{1}{4}\mathrm{Ar}(Q_{i_1 i_2 \dots i_{m_0}}). \tag{3.5}$$

F , (3.2) (3.5)

$$\left|G_n(q) - G_n(p) - \int_{[p,q]} y dx\right| < \sum_{i=0}^{2}\left|(G_n(p_i') - G_n(p_i) - \int_{[p_i, p_i']} y dx\right|$$
$$+ \left|\left(G_n(p_1) - G_n(p_0') - \int_{[p_0', p_1]} y dx\right) + (G_n(p_2) - G_n(p_1') - \int_{[p_1', p_2]} y dx\right|$$
$$< \frac{3}{4}\mathrm{Ar}(Q_{i_1 i_2 \dots i_{m_0}}) + \frac{1}{4}\mathrm{Ar}(Q_{i_1 i_2 \dots i_{m_0}}) = \mathrm{Ar}(Q_{i_1 i_2 \dots i_{m_0}}).$$

T r f f . □

N ... G ... E. L ... p ... E. C ... $p_n \in J_n$, $n = 1, 2, \ldots$, ... $p_n \to p$, ... $n \to \infty$ ... $G(p) = \lim_{n\to\infty} G_n(p_n)$.

T ... G ... C ... $\mathbb{Q} = \bigcap_{n=1}^{\infty} \bigcup_{(i_1,i_2,\ldots,i_n)} Q_{i_1 i_2 \ldots i_n}$ ... $E \setminus \mathbb{Q}$ ... G_n, ... $G_n(p_n)$ ... $p_n \to p$ ... p, $p_n \in \mathbb{Q}$, $n = 1, 2, \ldots$ L ... p_{n_1} ... p_{n_2}, $n_1 > n_2$, ... $p_n \in J_n \bigcap \mathbb{Q}$, $n = 1, 2, \ldots$, ... $Q_{i_1 i_2 \ldots i_m}$ ... O ... $A_{i_1 i_2 \ldots i_m}$ ... $Q_{i_1 i_2 \ldots i_m}$ ... J_{n_1} ... J_{n_2}. I ... Gr ... $G_{n_1}(A_{i_1 i_2 \ldots i_m}) - G_{n_2}(A_{i_1 i_2 \ldots i_m})$ ... $J_{n_1}^{A, A_{i_1 i_2 \ldots i_m}}$ ... $J_{n_2}^{A, A_{i_1 i_2 \ldots i_m}}$. ... $\Omega_{n_1} \setminus \bar{\Omega}_{n_2}$, ...

$$| G_{n_1}(A_{i_1 i_2 \ldots i_m}) - G_{n_2}(A_{i_1 i_2 \ldots i_m}) | < \mathrm{Ar}\ (\Omega_{n_1} \setminus \bar{\Omega}_{n_2}). \tag{3.6}$$

F ... $j = 1, 2$ ... (...):

$$\left| G_{n_j}(p_{n_j}) - G_{n_j}(A_{i_1 i_2 \ldots i_m}) - \int_{[A_{i_1 i_2 \ldots i_m}, p_{n_j}]} y dx \right| < \mathrm{Ar}\ (Q_{i_1 i_2 \ldots i_m}).$$

I ... l_m ... $Q_{i_1 i_2 \ldots i_m}$ (... m), ...

$$\begin{aligned} \sum_{j=1}^{2} | G_{n_j}(p_{n_j}) - G_{n_j}(A_{i_1 i_2 \ldots i_m})| &< \sum_{j=1}^{2} \left| \int_{[A_{i_1 i_2 \ldots i_m}, p_{n_j}]} y dx \right| + 2\,\mathrm{Ar}\ (Q_{i_1 i_2 \ldots i_m}) \\ &< 2 l_m + 2\,\mathrm{Ar}\ (Q_{i_1 i_2 \ldots i_m}) < 3 l_m = 3 \prod_{i=1}^{m} \left(\frac{1 - \alpha_i}{2} \right) = \frac{3}{2^{m+1}} \cdot \frac{m+2}{m+1}. \end{aligned} \tag{3.7}$$

I ... (3.6) ... (3.7) ...

$$\begin{aligned} | G_{n_1}(p_{n_1}) - G_{n_2}(p_{n_2}) | &\le \sum_{j=1}^{2} | G_{n_j}(p_{n_j}) - G_{n_j}(A_{i_1 i_2 \ldots i_m}) | \\ &+ | G_{n_1}(A_{i_1 i_2 \ldots i_m}) - G_{n_2}(A_{i_1 i_2 \ldots i_m}) | < \frac{3}{2^{m+1}} \cdot \frac{m+2}{m+1} + \mathrm{Ar}\ (\Omega_{n_1} \setminus \bar{\Omega}_{n_2}). \end{aligned}$$

... p, p_{n_1}, $p_{n_2} \in \mathbb{Q}$ ... m ... $p_{n_1}, p_{n_2} \to p$, ... Ar $(\Omega_{n_1} \setminus \bar{\Omega}_{n_2}) \to 0$ ... $n_1, n_2 \to \infty$, ... $G_n(p_n)$ ... $n \to \infty$ ... G ...

N ., r . $G \in C^{2-}(E)$ $G'_x(x, y) = y$ $G'_y(x, y) = 0$ r. r. f G $(x, y) \in E$. T r r r f f $C^{2-}(E)$ (H. P . F r. r f , f r 13).

Definition 3.2. L E f $\mathbb{R}^2_{x,y}$ f f E. P f $C^{2-}(E)$ f r f f'_x f'_y E r r f r $\varepsilon > 0$ r M

$$\begin{aligned} &|f(x + \Delta x, y + \Delta y) - f(x, y) - f'_x(x, y)\Delta x - f'_y(x, y)\Delta y| \\ &\leq M(|\Delta x| + |\Delta y|)^{2-\varepsilon} \end{aligned} \tag{3.8}$$

f r $(x, y), (x + \Delta x, y + \Delta y) \in E$.

T r $G \in C^{2-}(E)$ r $p, p + \Delta p \in E$. f G f $[B_{i_1 \dots i_n}, A_{i_1 \dots (i_n+1)}]$, $i_n = 0, 1, 2,$ (3.8) $G'_x(x, y) = y$ $G'_y(x, y) = 0$ r, r , $p, p + \Delta p \in E$, r p $p + \Delta p$ r C r $\mathbb{Q}$. T f $p, p + \Delta p \in \mathbb{Q}$. C r r m $p, p + \Delta p$ r $Q_{i_1 \dots i_m}$ f r $(i_1, \dots, i_m)$, r r $Q_{i_1, \dots i_m i_{m+1}}$, $i_{m+1} = 0, 1, 2, 3$. p $p + \Delta p$ ff r r $Q_{i_1 \dots i_m i_{m+1}}$ $Q_{i_1 \dots i_m i'_{m+1}}$, f $Q_{i_1 \dots i_m i_{m+1}}$ $Q_{i_1 \dots i_m i'_{m+1}}$,

$$\begin{aligned} |\Delta p| \geq \alpha_{m+1} \left(\frac{1 - \alpha_1}{2}\right) \dots \left(\frac{1 - \alpha_m}{2}\right) &= \frac{1}{(m + 2)^2} \cdot \frac{1}{2^m} \prod_{k=1}^{m} \left(1 - \frac{1}{(k + 1)^2}\right) \\ &= \frac{1}{2^{m+1}} \cdot \frac{1}{(m + 1)(m + 2)}. \end{aligned} \tag{3.9}$$

N f f (3.8) f r r f G

$$\begin{aligned} \mathcal{L}_G(p, p + \Delta p) &\stackrel{\text{f}}{=} G(p + \Delta p) - G(p) - G'_x(p)\Delta x - G'_y(p)\Delta y \\ &= G(p + \Delta p) - G(p) - y\Delta x, \end{aligned}$$

r $p = (x, y)$ $\Delta p = (\Delta x, \Delta y)$. I

$$\int_{[p, p+\Delta p]} y dx = y\Delta x + \frac{1}{2}\Delta x \Delta y,$$

$$|\mathcal{L}_G(p, p+\Delta p)| \leq \left| G(p + \Delta p) - G(p) - \int_{[p, p+\Delta p]} y dx \right| + \frac{1}{2} |\Delta x||\Delta y|. \tag{3.10}$$

I[illegible]

$$\left|G(p+\Delta p)-G(p)-\int_{[p,p+\Delta p]} y\,dx\right| \le \frac{1}{2^{2m+2}}\left(\frac{m+2}{m+1}\right)^2. \tag{3.11}$$

[illegible] $p, p+\Delta p \in Q_{i_1\ldots i_m}$, [illegible] $|\Delta x|$ [illegible] $|\Delta y|$ [illegible] $Q_{i_1\ldots i_m}$, [illegible] $\frac{1}{2^{m+1}}\left(\frac{m+2}{m+1}\right)$. [illegible] (3.10) [illegible] (3.11) [illegible]

$$|\mathcal{L}_G(p,p+\Delta p)| \le \frac{3}{2}\cdot\frac{1}{2^{2m+2}}\left(\frac{m+2}{m+1}\right)^2. \tag{3.12}$$

F[illegible] (3.9) [illegible] (3.12) [illegible] G [illegible] (3.8) [illegible] $\varepsilon > 0$ [illegible] M [illegible]

$$\frac{3}{2}\cdot\frac{1}{2^{2m+2}}\left(\frac{m+2}{m+1}\right)^2 \le M\left(\frac{1}{2^{m+1}}\cdot\frac{1}{(m+1)(m+2)}\right)^{2-\varepsilon} \quad [illegible]\ m\to\infty$$

[illegible]

$$\frac{1}{(2^{\varepsilon})^{m+1}} \le M\,\frac{2}{3}\left(\frac{1}{(m+1)(m+2)}\right)^{2-\varepsilon}\left(\frac{m+1}{m+2}\right)^2 \quad [illegible]\ m\to\infty.$$

T[illegible] $m\to\infty$. T[illegible] G [illegible] $C^{2-}(E)$.

3.3. Definition and properties of the functions H and F

F[illegible] H [illegible] C[illegible] $\mathbb{Q}$. E[illegible] p [illegible] $Q_{i_1} \supset Q_{i_1i_2} \supset Q_{i_1i_2i_3} \supset \ldots$ [illegible] $Q_{i_1\ldots i_n}$. T[illegible] H [illegible] p [illegible] $H(p)=\sum_{n=1}^{\infty}\frac{i_n}{4^n}$. I[illegible] $i_n = 0,1,2$ [illegible] $H(A_{i_1\ldots i_{n-1}(i_n+1)}) = \sum_{k=1}^{n}\frac{i_k}{4^k}+\frac{1}{4^n}$ [illegible] $H(B_{i_1\ldots i_{n-1}i_n}) = \sum_{k=1}^{n}\frac{i_k}{4^k}+\sum_{k=n+1}^{\infty}\frac{3}{4^k} = \sum_{k=1}^{n}\frac{i_k}{4^k}+\frac{1}{4^n}$, [illegible] H [illegible] $[B_{i_1\ldots i_{n-1}i_n}, A_{i_1\ldots i_{n-1}(i_n+1)}]$, $i_n = 0,1,2$, [illegible] $\sum_{k=1}^{n}\frac{i_k}{4^k}+\frac{1}{4^n}$ [illegible] T[illegible] H [illegible] E.

N[illegible] $H\in C^{2-}(E)$ [illegible] $H'_x(x,y)=0$ [illegible] $H'_y(x,y)=0$ [illegible] H [illegible] E. P[illegible] G, [illegible] $p, p+\Delta p \in E$. [illegible] H [illegible] $E\setminus\mathbb{Q}$, [illegible] H [illegible] (3.8) [illegible] $p, p+\Delta p \in \mathbb{Q}$. L[illegible], m [illegible] $p, p+\Delta p \in Q_{i_1\ldots i_m}$, [illegible]

$p, p + \Delta p \notin Q_{i_1 \dots i_m i_{m+1}}$ f[illegible]r [illegible] $i_{m+1} = 0, 1, 2, 3$. T[illegible] f H [illegible] $|H(p + \Delta p) - H(p)| \le \frac{1}{4^m}$. H[illegible], [illegible] (3.9) [illegible] f[illegible]r [illegible] $\varepsilon > 0$ [illegible] M [illegible]

$$\frac{1}{4^m} \le M \left(\frac{1}{2^{m+1}} \cdot \frac{1}{(m+1)(m+2)} \right)^{2-\varepsilon} \quad m \to \infty,$$

[illegible] f[illegible] G.

T[illegible] F [illegible] E [illegible] f H [illegible] $H(A) = 0$ [illegible] $H(B) = 1$. T[illegible] f G [illegible] $G(A) = 0$, [illegible] C [illegible] $F = G + CH$ [illegible] $F(A) = 0$ [illegible] $F(B) = 0$. F[illegible], [illegible] $G \in C^{2-}(E)$ [illegible] $G'_x(x, y) = y$ [illegible] $G'_y(x, y) = 0$, [illegible] $H \in C^{2-}(E)$ [illegible] $H'_x(x, y) = 0$ [illegible] $H'_y(x, y) = 0$, [illegible] $F \in C^{2-}(E)$ [illegible] $F'_x(x, y) = y$ [illegible] $F'_y(x, y) = 0$ [illegible] $(x, y) \in E$.

3.4. Construction of the sphere $S \subset \partial G$

L[illegible] $\mathbb{A}$ [illegible] f $\mathbb{R}^2_{x,y}$ [illegible] $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. C[illegible] $E^1 = E + \vec{e}_y$, $E^2 = -\mathbb{A}E + \vec{e}_x + \vec{e}_y$, $E^3 = -E + \vec{e}_x$ [illegible] $E^4 = \mathbb{A}E$, [illegible] $\vec{e}_x$ [illegible] $\vec{e}_y$ [illegible] x [illegible] y, [illegible] $\tilde{E}$ [illegible] $\tilde{E} = \bigcup_{i=1}^4 E^i$ ([illegible] $\tilde{E}$ [illegible] F[illegible] 3.7).

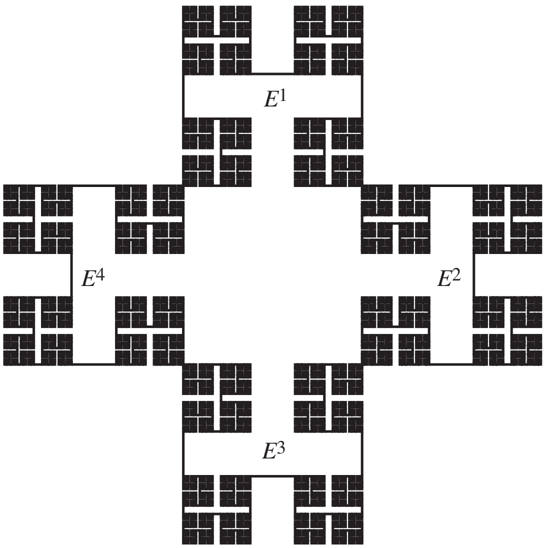

Figure 3.7. T[illegible] $\tilde{E}$.

I $\tilde{E}$ J r r f 2- r $\mathbb{R}^2_{x,y}$. A f $E^i, i = 1, 2, 3, 4,$ r f r f F $E,$ f F^i rr E^i r r :

1) $F^i \in C^{2-}(E^i)$,

2) $\frac{\partial F^i}{\partial x}(x, y) = y$ $\frac{\partial F^i}{\partial y}(x, y) = 0$ f r $(x, y) \in E^i$,

3) F^i r f r E^i.

H , f $\tilde{F}$ $\tilde{E}$ $\tilde{F}(p) = F^i(p)$ f r $p \in E^i, i = 1, 2, 3, 4,$ f r f $\tilde{F} \in C^{2-}(\tilde{E})$ $\frac{\partial \tilde{F}}{\partial x}(x, y) = y$ $\frac{\partial \tilde{F}}{\partial y}(x, y) = 0$ $(x, y) \in \tilde{E}$. T , r fP (, f r , 13, T r 4 177), r f $\approx{F} \in C^{2}(\mathbb{R}^2_{x,y})$ $\approx{F} \in C^\infty(\mathbb{R}^2_{x,y} \setminus \tilde{E})$ $\approx{F}(p) = \tilde{F}(p)$ f r $p \in \tilde{E}$. If r r f $\approx{F}$ $\mathbb{D} \subset \mathbb{R}^2_{x,y}$ $\tilde{E} \subset \mathbb{D}$ r f r f r r 2- r S^2 $\mathbb{R}^3_{x,y,z}$, r $\Gamma(\tilde{F})$ f $\tilde{F}$ J r r S^2 f 2- r f r - S^2 rr f r r $\{dz - ydx = 0\}$.

N G r $\mathbb{C}^2$ C^∞- r q f ∂G. T , r f D r , r r U f q ∂G C^∞- ff r Φ f U r V f r $\mathbb{R}^3_{x,y,z}$ r f $\{T^{\mathbb{C}}_p(\partial G)\}$ r f r Φ r r $\mathbb{R}^3_{x,y,z}$. P f r $S^2 \subset V$ (f , r r r f r $x \to cx, y \to cy, z \to c^2 z$ f $\mathbb{R}^3_{x,y,z}$ r r r r f S^2 r r f r $c > 0$ f f S^2). T $S = \Phi^{-1}(S^2)$ 2- r ∂G f C^{2-} $\Gamma = \Phi^{-1}(\Gamma(\tilde{F})) \subset S$ J r r f 2- r $p \in \Gamma$ $T_p S$ S T r r f r □

References

1 D. BENNE IN, *Entrelacements et équations de Pfaff*, A r **107-108** (1983), 87 161.

2 E. BI HOP, *Differentiable manifolds in complex Euclidean space*, D M . J. **32** (1965), 1 21.

3 E. BEDFO D P. KLINGENBE G, *On the envelope of holomorphy of a 2-sphere in $\mathbb{C}^2$*, J. A r. M **4** (1991), 623 643.

4 E. M. CHI KA, *Regularity of the boundaries of analytic sets*, M . **117** (1982), 291 336; E r . M . **45** (1983), 291 335.

5 A. ELIA HBE G, *Filling by holomorphic discs and its applications, Geometry of low-dimensional manifolds*, 2 (D r 1989), L M L r N r., . 151, C r. . Pr , C r. , 1990, 45 67.
6 O. G. E O HKIN, *On a topological property of the boundary of an analytic subset of a strictly pseudoconvex domain in* $\mathbb{C}^2$, M **49** (1991), 149 151; E r M . N **49** (1991), 546 547.
7 M. G OMO , *Pseudoholomorphic curves in symplectic manifolds*, I M **82** (1985), 307 347.
8 M. . HI CH, D ff r T – Gr T M , . 33, r r- r , N r , 1976.
9 B. JO ICKE, *Local polynomial hulls of discs near isolated parabolic points*, I . M . J. **46** (1997), 789 826.
10 N. G. K ILIN, *Two-dimensional spheres on the boundaries of pseudoconvex domains in* $\mathbb{C}^2$, I . A . N r. M . **55** (1991), 1194 1237; E r M . I . **39** (1992), 1151 1187.
11 H. F. LAI, *Characteristic classes of real manifolds immersed in complex manifolds*, Tr . A r. M . **172** (1972), 1 33.
12 . NEMI O KI, *Complex analysis and differential topology on complex surfaces*, M . N **54** (1999), 47 74; E r M r **54** (1999), 729 752.
13 E. TEIN, r I r D ff r Pr r f F –, Pr . Pr , Pr , 1970.
14 H. HITNE , *A function not constant on a connected set of critical points*, D M . J. **1** (1935), 514 517.
15 J. IEGE INCK, *Local polynomially convex hulls at degenerated* CR *singularities of surfaces in* $\mathbb{C}^2$, I . M . J. **44** (1995), 897 915.

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