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Isomorphisms of H^*-Triple Systems [1]

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Let A be a module over the commutative unit ring ϕ. We say that A is a ϕ-triple system if it is endowed with a trilinear map $\langle\cdots\rangle$ of $A \times A \times A$ to A. If $\phi = \mathbb{R}$ or $\mathbb{C}$ the map $^* : A \to A$ that to each x assigns x^* is said to be a multiplicative involution if it satisfies $\langle xyz\rangle^* = \langle x^*y^*z^*\rangle$ for any x, y, $z \in A$ and it is involutive linear if $\phi = \mathbb{R}$ or involutive antilinear if $\phi = \mathbb{C}$. The ϕ-triple system A is said an H^*-triple system if its underlying ϕ-module ($\phi = \mathbb{R}$ or $\mathbb{C}$) is a Hilbert ϕ-space of inner product $(\cdot|\cdot)$ endowed with a multiplicative involution $x \mapsto x^*$ satisfying

$$(\langle xyz\rangle|t) = (x|\langle tz^*y^*\rangle) = (y|z^*tx^*\rangle) = (z|\langle y^*x^*t\rangle)$$

for any x, y, $z \in A$. A ϕ-linear map F between the triple systems V and V' is said to be a morphism if $F\langle xyz\rangle = \langle F(x)F(y)F(z)\rangle$ for any x, y, $z \in V$. The concepts of isomorphism, automorphism between triple systems and *-morphism, *-isomorphism and *-automorphism between H^*-triple systems are defined in an obvious way. A *-isomorphism F between the H^*-triple systems V and V' is said to be an isogeny if there exists a real positive number λ, called the constant of the isogeny, such that $(F(x)|F(y)) = \lambda(x|y)$ for any x, $y \in V$. If F is a continuous morphism between the H^*-triple systems V and V', we denote by $F^\square : V' \to V$ the adjoint operator of F and by $F^* : V \to V'$ the morphism given by $F^* : x \mapsto [F(x^*)]^*$. A subspace I of a triple system V is said to be an ideal of V if it satisfies $\langle IVV\rangle + \langle VIV\rangle + \langle VVI\rangle \subseteq I$. An H^*-triple system V is topologically simple if the triple product is non-zero and contains no proper closed ideals. In an H^*-triple system V we define the annihilator Ann(V) of V to be the set $\{x \in V : \langle xVV\rangle = 0\}$. We observe that if V is an H^*-triple system, then Ann(V) $= \{x \in V : \langle VxV\rangle = 0\} = \{x \in V : \langle VVx\rangle = 0\}$, and the involution * is isometric if Ann(V) $= 0$ ([3], [4] and [5]). Ann(V) is a closed self-adjoint ideal of V. The centroid $Z(V)$ of an H^*-triple system V is the set of

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linear maps $F : V \to V$ such that $F\langle xyz\rangle = \langle F(x)yz\rangle = \langle xF(y)z\rangle = \langle xyF(z)\rangle$ for any x, y, $z \in V$. In [6] we proved that if V is a topologically simple H^*-triple system then $(Z(V), \Box)$ is $(\mathbb{C}\,\mathrm{Id}, ^-)$ in the complex case and either $(\mathbb{C}\,\mathrm{Id}, ^-)$ or $(\mathbb{R}\,\mathrm{Id}, \mathrm{Id})$ in the real case.

PROPOSITION 1. *Let V and V' be two H^*-triple systems with continuous involution and $F : V \to V'$ a continuous morphism with dense range. Then $F^* \circ F^\Box$ lies in $Z(V')$.*

PROOF. For any x, y, $z \in V$, $t \in V'$, we have

$$(F\langle xyz\rangle | t) = (\langle xyz\rangle | F^\Box(t)) = (x | \langle F^\Box(t) z^* y^*\rangle)$$

and on the other hand

$$\begin{aligned}(F\langle xyz\rangle | t) &= (\langle F(x)F(y)F(z)\rangle | t) = (F(x) | \langle tF(z)^* F(y)^*\rangle) \\ &= (x | F^\Box\langle tF(z)^* F(y)^*\rangle).\end{aligned}$$

Hence $F^\Box\langle tF(z)^* F(y)^*\rangle = \langle F^\Box(t) z^* y^*\rangle$. Substituting y for y^* and z for z^*, we can write $F^\Box\langle tF^*(z)F^*(y)\rangle = \langle F^\Box(t) zy\rangle$, and by applying F^* to both members, we obtain

$$F^* F^\Box\langle tF^*(z)F^*(y)\rangle = \langle F^* F^\Box(t) F^*(z) F^*(y)\rangle.$$

It follows, from continuity of F and the fact that F^* is a morphism, that F^* has dense range. Taking into account the above equality we have

$$F^* F^\Box\langle tuv\rangle = \langle F^* F^\Box(t) uv\rangle,$$

for any u, $v \in V'$. Analogously we can prove that

$$F^* F^\Box\langle tuv\rangle = \langle tF^* F^\Box(u) v\rangle = \langle tuF^* F^\Box(v)\rangle,$$

for any u, $v \in V'$ and therefore $F^* F^\Box \in Z(V')$.

PROPOSITION 2. *(a) Let V be an H^*-triple system with zero annihilator of norm $\|\cdot\|$. If V is endowed with another norm $\|\cdot\|_1$ such that $(V, \|\cdot\|_1)$ is a complete normed triple system, then $\|\cdot\|$ and $\|\cdot\|_1$ are equivalent.*

(b) Let V and V' be two H^*-triple systems with zero annihilator and $F : V \to V'$ an (algebraic) isomorphism. Then F is continuous.

PROOF. (a) Let V_1 be the triple system V with the norm $\|\cdot\|_1$. For any x, $y \in V$, we have $L(x, y) \in BL(V) \cap BL(V_1)$. So

$$L(x, y)^\Box L(x, y) \in BL(V) \cap BL(V_1).$$

From the Banach inverse map theorem, we obtain

$$r(BL(V), L(x,y)^{\square}L(x,y)) = r(BL(V_1), L(x,y)^{\square}L(x,y)).$$

So

$$r(BL(V_1), L(x,y)^{\square}L(x,y)) \leq \|L(x,y)^{\square}L(x,y)\|_1,$$

and therefore

$$\|L(x,y)\|^2 \leq \|L(x,y)^{\square}\|_1 \|L(x,y)\|_1 \leq \|x^*\|_1 \|y^*\|_1 \|x\|_1 \|y\|_1. \tag{3}$$

As in [7, (1-2-36)], it can be shown that * is continuous for the topology induced by the norm $\|\cdot\|_1$, hence there exists a positive real k such that

$$\|x^*\|_1 \leq k\|x\|_1, \tag{4}$$

for any $x \in V_1$. Let $\{z_n\}$ be a sequence of elements of V with $\lim_{n\to\infty} z_n = 0$ in V_1 and $\lim_{n\to\infty} z_n = z$ in V. It follows from (3) and (4) that

$$\|L(x,y)\| \leq k\|x\|_1\|y\|_1,$$

and therefore

$$\|L(z_n,y)\| \leq k\|z_n\|_1\|y\|_1.$$

The limit of the sequence $\{L(z_n,y)\}$ with the norm $\|\cdot\|$ is therefore zero. Since relative to the norm $\|\cdot\|$, we have $\lim_{n\to\infty} L(z_n,y) = L(z,y)$, it follows that $L(z,y) = 0$ and $z = 0$. The closed map theorem implies that $x \mapsto x$ is a continuous map of V_1 to V. The Banach inverse map theorem finishes the proof.

(b) We define a new norm $\|\cdot\|_1$ on V by means $\|x\|_1 = \|F(x)\|$. Then $(V, \|\cdot\|_1)$ is a complete normed triple system. Part (a) proves that $\|\cdot\|$ and $\|\cdot\|_1$ are equivalent and this implies the continuity of F.

COROLLARY 5. *Let V and V' be topologically simple H^*-triple systems and $G : V \to V'$ a *-isomorphism. Then G is an isogeny.*

PROOF. By Proposition 3, G is continuous. It follows from Proposition 1 and [6, Teorema 14] that $GG^{\square} = \lambda\,\mathrm{Id}$ ($\lambda \in \mathbb{R}$). For any x, $y \in V'$, we have

$$(G^{\square}(x)|G^{\square}(y)) = (x|GG^{\square}(y)) = (x|\lambda y) = \lambda(x|y).$$

Hence λ is a positive real number and G is an isogeny of constant λ.

DEFINITION 6. Let V be a triple system over $\mathbb{K}$ and $D : V \to V$ a linear map. We say that D is a derivation if it satisfies

$$D\langle xyz\rangle = \langle D(x)yz\rangle + \langle xD(y)z\rangle + \langle xyD(z)\rangle,$$

for any $x,\ y,\ z \in V$.

LEMMA 7. *Let V be a complex H^*-triple system with non-zero triple product, F a continuous automorphism of V and D a continuous derivation of V. Then*

(a) *there exist $\lambda,\ \mu,\ \nu \in \mathrm{Sp}(F)$, such that $\lambda\mu\nu \in \mathrm{Sp}(F)$;*

(b) *there exist $\lambda,\ \mu,\ \nu \in \mathrm{Sp}(D)$, such that $\lambda+\mu+\nu \in \mathrm{Sp}(D)$;*

(c) $\mathrm{Sp}(F)$ *cannot be contained in a halfline of origin* 0 *different from* $\mathbb{R}^+$ *and* $\mathbb{R}^-$*;*

(d) $\mathrm{Sp}(D)$ *cannot be contained in a line other than for the lines containing the origin.*

PROOF. Let F be the Banach space of the continuous trilinear maps of $V \times V \times V$ into V. For any $T \in BL(V)$ we define the maps $\overset{a}{T},\ \overset{b}{T},\ \overset{c}{T},\ \overset{d}{T} \in BL(F)$ by

$$\overset{a}{T}(f)(x,y,z) = T(f(x,y,z))$$

$$\overset{b}{T}(f)(x,y,z) = f(T(x),y,z)$$

$$\overset{c}{T}(f)(x,y,z) = f(x,T(y),z)$$

$$\overset{d}{T}(f)(x,y,z) = f(x,y,T(z)).$$

If $f_0(x,y,z) = \langle xyz\rangle$ then T is an automorphism iff $\overset{a}{T}(f_0) = \overset{b}{T}\overset{c}{T}\overset{d}{T}(f_0)$ and T is a derivation iff $\overset{a}{T}(f_0) = \big(\overset{b}{T}+\overset{c}{T}+\overset{d}{T}\big)(f_0)$. The map $T \mapsto \overset{a}{T}$ is a continuous morphism which preserves the unity and the maps $T \mapsto \overset{x}{T}$, $x \in \{b,c,d\}$, are continuous skewmorphisms which preserve the unity. Hence

$$\mathrm{Sp}\big(\overset{x}{T}\big) \subset \mathrm{Sp}(T), \qquad x \in \{a,b,c,d\}. \tag{8}$$

If F is an automorphism then $\overset{a}{F}(f_0) = \overset{b}{F}\overset{c}{F}\overset{d}{F}(f_0)$, that is, $\big(\overset{a}{F} - \overset{b}{F}\overset{c}{F}\overset{d}{F}\big)\ (f_0) = 0$, and therefore $0 \in \mathrm{Sp}\big(\overset{a}{F} - \overset{b}{F}\overset{c}{F}\overset{d}{F}\big)$. It follows from the fact that $\big\{\overset{x}{F}\big\}_{x\in\{a,b,c,d\}}$ is a commutative set, that $0 \in \mathrm{Sp}\big(\overset{a}{F}\big) - \mathrm{Sp}\big(\overset{b}{F}\big)\mathrm{Sp}\big(\overset{c}{F}\big)\mathrm{Sp}\big(\overset{d}{F}\big)$ (see [11, p. 280]). So there exist $\rho \in \mathrm{Sp}\big(\overset{a}{F}\big)$, $\lambda \in \mathrm{Sp}\big(\overset{b}{F}\big)$, $\mu \in \mathrm{Sp}\big(\overset{c}{F}\big)$, $\nu \in \mathrm{Sp}\big(\overset{d}{F}\big)$, such that $0 = \rho - \lambda\mu\nu$. Part (a) now follows from (8). In a similar way part (b) can be obtained. Parts (c) and (d) are consequence of (a) and (b).

Let V be a Hilbert space and $F \in BL(V)$. We recall that F is a positive operator if F is self-adjoint and $(F(x)|x) \geq 0$ for any $x \in V$. In the complex case the self-adjointness follows from the last condition.

LEMMA 9. *Let V and V' be two topologically simple complex H^*-triple systems and $F : V \to V'$ an isomorphism. Then either $(F^*)^{-1} \circ F$ or $-(F^*)^{-1} \circ F$ is a positive operator.*

PROOF. By [6, Teorema 14] and Proposition 1 we have $F^* \circ F^\square = \lambda \operatorname{Id}$ ($\lambda \in \mathbb{C}$). Firstly we prove that $\lambda \in \mathbb{R} - \{0\}$. The fact that $\lambda \neq 0$ is obtained from the invertibility of F. So

$$(F^*)^{-1} \circ F = \frac{1}{\lambda} F^\square \circ F.$$

Since $F^\square \circ F$ is a positive operator

$$\operatorname{Sp}\left(\frac{1}{\lambda} F^\square \circ F\right) \subset \frac{1}{\lambda} \mathbb{R}^+,$$

and, by Lemma 7, $\lambda \in \mathbb{R} - \{0\}$. Then we have either $(F^*)^{-1} \circ F$ or $-(F^*)^{-1} \circ F$ is a positive operator. This finishes the proof.

From the following lemmata we shall prove that the unique positive root of $(F^*)^{-1} \circ F$ or $-(F^*)^{-1} \circ F$ is also a morphism. Next we generalize a well know result (see [10, Lemme 8 p. 313]).

LEMMA 10. *Let E be a complex Banach space and $\hat{F}$ the Banach space of the continuous multilinear maps of E^n into E. Let $D \in BL(E)$ and $F = e^D$. We define $D' \in BL(F)$ and $F' \in BL(\hat{F})$ by*

$$\begin{aligned}(D'f)(\varsigma_1, \varsigma_2, \dots, \varsigma_n) = D(f(\varsigma_1, \varsigma_2, \dots, \varsigma_n)) &- f(D\varsigma_1, \varsigma_2, \dots, \varsigma_n)) \\ &- f(\varsigma_1, D\varsigma_2, \dots, \varsigma_n) - \cdots - f(\varsigma_1, \varsigma_2, \dots, D\varsigma_n) \\ (F'f)(\varsigma_1, \varsigma_2, \dots, \varsigma_n) = F(f(F^{-1}\varsigma_1, F^{-1}\varsigma_2, &\dots, F^{-1}\varsigma_n))\end{aligned}$$

with $\varsigma_1, \varsigma_2, \dots, \varsigma_n \in E$, $f \in \hat{F}$. Then

(a) $D'f = 0$ *implies* $F'f = f$.

(b) *If* $F'f = f$, *and* $\operatorname{Sp} D \subset \left\{z \in \mathbb{C} : |\operatorname{Im}(z)| < \dfrac{2\pi}{(n+1)}\right\}$, *then* $D'f = 0$.

PROOF. We define maps $x_1, x_1, \dots, x_n \in BL(\hat{F})$ by

$$(x_0 f)(\varsigma_1, \dots, \varsigma_n) = D(f(\varsigma_1, \dots, \varsigma_n)),$$
$$(x_i f)(\varsigma_1, \dots, \varsigma_n) = -f(\varsigma_1, \dots, D\varsigma_i, \dots, \varsigma_n), \qquad i \in \{1, \dots, n),$$

taking into account that $D' = \sum_{i=0}^{n} x_i$ and that the x_i pairwise commute, we can conclude the proof as in [10, Lemme 8 p. 314].

LEMMA 11. *Let V be a complex complete normed triple system and F a continuous automorphism of V such that*

$$\mathrm{Sp}(F) \subset \left\{ z \in \mathbb{C} : |\arg(Z)| < \frac{\pi}{2} \right\}.$$

There there exists a unique continuous derivation $D : V \to V$ such that $e^D = F$ and

$$\mathrm{Sp}(D) \subset \left\{ z \in \mathbb{C} : |\mathrm{Im}(z)| < \frac{\pi}{2} \right\}.$$

PROOF. Let f be the triple product of V and log the principal determination of the logarithm. Let $D = \log(F)$. Because F is an automorphism, it follows that $F'f = f$, with F' as in Lemma 10. The condition on F implies that

$$\mathrm{Sp}(D) = \log(\mathrm{Sp}(F)) \subset \left\{ z \in \mathbb{C} : |\mathrm{Im}(z)| < \frac{\pi}{2} \right\},$$

and by Lemma 10 (b) we have that $D'f = 0$, that is D is a derivation.

PROPOSITION 12. *Let V be a topologically simple H^*-triple system and F a positive automorphism of V. Then there exists a unique positive automorphism G of V such that $G^2 = F$. Moreover if $(F^*)^{-1} = F$, then $(G^*)^{-1} = G$.*

PROOF. Let V be a complex topologically simple H^*-triple system. Since $\mathrm{Sp}(F) \subseteq \mathbb{R}^+$, by Lemma 11 there exists a unique continuous derivation $D : V \to V$ such that $e^D = F$. Obviously $\frac{1}{2} D$ is a derivation of V, and by Lemma 10 (a) and the spectral mapping theorem, $G = e^{(1/2)D}$ is a positive automorphism of V such that $G^2 = F$. If V is a real H^*-triple system with $(Z(V), \square) = (\mathbb{R}\,\mathrm{Id}, \mathrm{Id})$, the unique positive automorphism $\hat{G}$ of $\mathcal{C}(V)$, with $\hat{G}^2 = \mathcal{C}(F)$ can be obtained by arguing over the automorphism $\mathcal{C}(F)$ of the complexified $\mathcal{C}(V)$ of V given by $\mathcal{C}(F) : (a+bi) \mapsto F(a)+F(b)i$. A direct calculation proves that $\tau\hat{G}\tau$ is another positive root of $\mathcal{C}(F)$, where τ is the (real) involutive $\mathbb{C}$-antilinear automorphism of V given by $\tau : (a+bi) \mapsto (a-bi)$. So $\hat{G}(V) \subseteq V$ and $G = \hat{G}|_V$ is the unique positive automorphism of V such that $G^2 = F$. Finally, if V is a real H^*-triple system with $(Z(V), \square) = (\mathbb{C}\,\mathrm{Id}, ^-)$, that is, V is the realization of a complex H^*-triple system, then as in [1, Lemma 1.4.3] it can be proved that F is either $\mathbb{C}$-linear or $\mathbb{C}$-antilinear. But the (real) positivity of F implies that F must be a $\mathbb{C}$-linear automorphism of V. Hence this case follows from the complex one.

Suppose now that $(F^*)^{-1} = F$. By Proposition 1, we obtain $(G^*)^{-1} = \mu G$, for some $\mu \in Z(V)$ ($Z(V) = \mathbb{R}$ or $\mathbb{C}$), since G and G^* are positive operators $\mu > 0$. On the other hand, we have

$$\mu^2 F = (\mu G) \circ (\mu G) = (G^*)^{-1} \circ (G^*)^{-1} = (F^*)^{-1} = F$$

so $\mu = 1$ and the proposition is proved.

MAIN THEOREM 13. *Let V and V' be two topologically simple H^*-triple systems and $F : V \to V'$ an isomorphism. Then either $F : V \to V'$,* *or* $-F : V^b \to V'$ *splits in a unique way*

$$\sigma F = F_2 \circ F_1, \qquad \sigma \in \{1, -1\},$$

where F_1 *is a positive automorphism,* F_2 *is a* **-isomorphism and* V^b *is the twin of* V*, that is, the* H^**-triple system with the same Hilbert space and triple product as* V *and involution* $x \mapsto -x^*$.

In particular if V *and* V' *are isomorphic, then either* V *or* V^b *is* **-isomorphic to* V'.

PROOF. Let $H = (F^*)^{-1} \circ F$. By Lemma 9 and the proposition above, we obtain that either H or $-H$ has a unique positive root F_1. First we suppose that H is a positive operator. A direct calculation prove that $(H^*)^{-1} = H$, and by Proposition 12 we have $(F_1^*)^{-1} = F_1$. Then $F_2 = F \circ F_1^{-1}$ is a *-isomorphism. Indeed from

$$F_1^{-1} \circ (F^*)^{-1} \circ F_2 = F_1^{-1} \circ (F^*)^{-1} \circ F \circ F_1^{-1} = F_1^{-1} \circ F_1^2 \circ F_1^{-1} = \mathrm{Id},$$

we obtain

$$F_1^{-1} \circ (F^*)^{-1} = F_2^{-1},$$

or equivalently

$$F_2^* = F \circ F_1^*.$$

It follows from $(F_1^*)^{-1} = F_1$ that $F_2^* = F_2$, and F_2 is a *-isomorphism. If H is a negative operator we argue over $-\mathrm{Id} \circ H$ in a similar way, taking into account that $-\mathrm{Id} : V \to V^b$ shows that $(-\mathrm{Id})^* = \mathrm{Id}$. Finally we prove the uniqueness of the factorization. Arguing as in the proof of Lemma 9, we have that $G^\square = \lambda(G)(G^*)^{-1}$ for every automorphism G, where $\lambda(G)$ is a non-zero real number. Moreover $\lambda(G) > 0$ if G is positive. Suppose that $F = F_2 \circ F_1$ with F_1 a positive automorphism and F_2 a *-isomorphism. Then

$$\begin{aligned}(F^*)^{-1} \circ F &= (F_2^* \circ F_1^*)^{-1} \circ F_2 \circ F_1 = (F_1^*)^{-1} \circ (F_2^*)^{-1} \circ F_2 \circ F_1 \\ &= (F_1^*)^{-1} \circ F_1 = \lambda(F_1)^{-1} F_1^2,\end{aligned}$$

so F_1 is the unique positive root of the operator $(G^*)^{-1} \circ G$ with $G = \lambda(F_1)^{1/2} F$ and, by Proposition 12, $(F_1^*)^{-1} = F_1$, that is $\lambda(F_1) = 1$ and the factorization is unique. In a similar way, we can obtain the uniqueness in the case $-F = F_2 \circ F_1$.

COROLLARY 14 (Essential uniqueness). *Let* V *be a topologically simple* H^**-triple system over* $\mathbb{K}$ *(*$\mathbb{K} = \mathbb{R}$ *or* $\mathbb{C}$*) and* V' *another* H^**-triple system with the same underlying* $\mathbb{K}$*-triple system of* V*. Then either* V *or* V^b *is isogenic to* V'.

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