

ANNALI DELLA
SCUOLA NORMALE SUPERIORE DI PISA
Classe di Scienze

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Annali della Scuola Normale Superiore di Pisa, Classe di Scienze 4^e série, tome 19,
n° 4 (1992), p. 507-514

http://www.numdam.org/item?id=ASNSP_1992_4_19_4_507_0

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Isomorphisms of H^* -Triple Systems ¹

A. CASTELLÓN SERRANO - J.A. CUENCA MIRA

Let A be a module over the commutative unit ring ϕ . We say that A is a ϕ -triple system if it is endowed with a trilinear map $\langle \cdot | \cdot \rangle$ of $A \times A \times A$ to A . If $\phi = \mathbb{R}$ or \mathbb{C} the map $*$: $A \rightarrow A$ that to each x assigns x^* is said to be a multiplicative involution if it satisfies $\langle xyz \rangle^* = \langle x^*y^*z^* \rangle$ for any $x, y, z \in A$ and it is involutive linear if $\phi = \mathbb{R}$ or involutive antilinear if $\phi = \mathbb{C}$. The ϕ -triple system A is said an H^* -triple system if its underlying ϕ -module ($\phi = \mathbb{R}$ or \mathbb{C}) is a Hilbert ϕ -space of inner product $(\cdot | \cdot)$ endowed with a multiplicative involution $x \mapsto x^*$ satisfying

$$(\langle xyz \rangle | t) = (x | \langle tz^*y^* \rangle) = (y | \langle z^*tx^* \rangle) = (z | \langle y^*x^*t \rangle)$$

for any $x, y, z \in A$. A ϕ -linear map F between the triple systems V and V' is said to be a morphism if $F\langle xyz \rangle = \langle F(x)F(y)F(z) \rangle$ for any $x, y, z \in V$. The concepts of isomorphism, automorphism between triple systems and $*$ -morphism, $*$ -isomorphism and $*$ -automorphism between H^* -triple systems are defined in an obvious way. A $*$ -isomorphism F between the H^* -triple systems V and V' is said to be an isogeny if there exists a real positive number λ , called the constant of the isogeny, such that $(F(x) | F(y)) = \lambda(x | y)$ for any $x, y \in V$. If F is a continuous morphism between the H^* -triple systems V and V' , we denote by $F^\square : V' \rightarrow V$ the adjoint operator of F and by $F^* : V \rightarrow V'$ the morphism given by $F^* : x \mapsto [F(x^*)]^*$. A subspace I of a triple system V is said to be an ideal of V if it satisfies $\langle IVV \rangle + \langle VIV \rangle + \langle VVI \rangle \subseteq I$. An H^* -triple system V is topologically simple if the triple product is non-zero and contains no proper closed ideals. In an H^* -triple system V we define the annihilator $\text{Ann}(V)$ of V to be the set $\{x \in V : \langle xVV \rangle = 0\}$. We observe that if V is an H^* -triple system, then $\text{Ann}(V) = \{x \in V : \langle VxV \rangle = 0\} = \{x \in V : \langle VVx \rangle = 0\}$, and the involution $*$ is isometric if $\text{Ann}(V) = 0$ ([3], [4] and [5]). $\text{Ann}(V)$ is a closed self-adjoint ideal of V . The centroid $Z(V)$ of an H^* -triple system V is the set of

¹ This work was partially supported by the "Plan Andaluz de Investigación y Desarrollo Tecnológico".

Pervenuto alla Redazione il 26 Luglio 1991.

linear maps $F : V \rightarrow V$ such that $F\langle xyz \rangle = \langle F(x)yz \rangle = \langle xF(y)z \rangle = \langle xyF(z) \rangle$ for any $x, y, z \in V$. In [6] we proved that if V is a topologically simple H^* -triple system then $(Z(V), \square)$ is $(\mathbb{C} \text{ Id}, -)$ in the complex case and either $(\mathbb{C} \text{ Id}, -)$ or $(\mathbb{R} \text{ Id}, \text{Id})$ in the real case.

PROPOSITION 1. *Let V and V' be two H^* -triple systems with continuous involution and $F : V \rightarrow V'$ a continuous morphism with dense range. Then $F^* \circ F^\square$ lies in $Z(V')$.*

PROOF. For any $x, y, z \in V, t \in V'$, we have

$$(F\langle xyz \rangle | t) = (\langle xyz \rangle | F^\square(t)) = (x | (F^\square(t)z^*y^*))$$

and on the other hand

$$\begin{aligned} (F\langle xyz \rangle | t) &= (\langle F(x)F(y)F(z) \rangle | t) = (F(x) | (tF(z)^*F(y)^*)) \\ &= (x | F^\square(tF(z)^*F(y)^*)). \end{aligned}$$

Hence $F^\square(tF(z)^*F(y)^*) = \langle F^\square(t)z^*y^* \rangle$. Substituting y for y^* and z for z^* , we can write $F^\square(tF^*(z)F^*(y)) = \langle F^\square(t)zy \rangle$, and by applying F^* to both members, we obtain

$$F^*F^\square(tF^*(z)F^*(y)) = \langle F^*F^\square(t)F^*(z)F^*(y) \rangle.$$

It follows, from continuity of F and the fact that F^* is a morphism, that F^* has dense range. Taking into account the above equality we have

$$F^*F^\square(tuv) = \langle F^*F^\square(t)uv \rangle,$$

for any $u, v \in V'$. Analogously we can prove that

$$F^*F^\square(tuv) = \langle tF^*F^\square(u)v \rangle = \langle tuF^*F^\square(v) \rangle,$$

for any $u, v \in V'$ and therefore $F^*F^\square \in Z(V')$.

PROPOSITION 2. (a) *Let V be an H^* -triple system with zero annihilator of norm $\| \cdot \|$. If V is endowed with another norm $\| \cdot \|_1$ such that $(V, \| \cdot \|_1)$ is a complete normed triple system, then $\| \cdot \|$ and $\| \cdot \|_1$ are equivalent.*

(b) *Let V and V' be two H^* -triple systems with zero annihilator and $F : V \rightarrow V'$ an (algebraic) isomorphism. Then F is continuous.*

PROOF. (a) Let V_1 be the triple system V with the norm $\| \cdot \|_1$. For any $x, y \in V$, we have $L(x, y) \in BL(V) \cap BL(V_1)$. So

$$L(x, y)^\square L(x, y) \in BL(V) \cap BL(V_1).$$

From the Banach inverse map theorem, we obtain

$$r(BL(V), L(x, y) \square L(x, y)) = r(BL(V_1), L(x, y) \square L(x, y)).$$

So

$$r(BL(V_1), L(x, y) \square L(x, y)) \leq \|L(x, y) \square L(x, y)\|_1,$$

and therefore

$$(3) \quad \|L(x, y)\|^2 \leq \|L(x, y) \square L(x, y)\|_1 \|L(x, y)\|_1 \leq \|x^*\|_1 \|y^*\|_1 \|x\|_1 \|y\|_1.$$

As in [7, (1-2-36)], it can be shown that $*$ is continuous for the topology induced by the norm $\|\cdot\|_1$, hence there exists a positive real k such that

$$(4) \quad \|x^*\|_1 \leq k\|x\|_1,$$

for any $x \in V_1$. Let $\{z_n\}$ be a sequence of elements of V with $\lim_{n \rightarrow \infty} z_n = 0$ in V_1 and $\lim_{n \rightarrow \infty} z_n = z$ in V . It follows from (3) and (4) that

$$\|L(x, y)\| \leq k\|x\|_1 \|y\|_1,$$

and therefore

$$\|L(z_n, y)\| \leq k\|z_n\|_1 \|y\|_1.$$

The limit of the sequence $\{L(z_n, y)\}$ with the norm $\|\cdot\|$ is therefore zero. Since relative to the norm $\|\cdot\|$, we have $\lim_{n \rightarrow \infty} L(z_n, y) = L(z, y)$, it follows that $L(z, y) = 0$ and $z = 0$. The closed map theorem implies that $x \mapsto x$ is a continuous map of V_1 to V . The Banach inverse map theorem finishes the proof.

(b) We define a new norm $\|\cdot\|_1$ on V by means $\|x\|_1 = \|F(x)\|$. Then $(V, \|\cdot\|_1)$ is a complete normed triple system. Part (a) proves that $\|\cdot\|$ and $\|\cdot\|_1$ are equivalent and this implies the continuity of F .

COROLLARY 5. *Let V and V' be topologically simple H^* -triple systems and $G : V \rightarrow V'$ a $*$ -isomorphism. Then G is an isogeny.*

PROOF. By Proposition 3, G is continuous. It follows from Proposition 1 and [6, Teorema 14] that $GG^\square = \lambda \text{Id}$ ($\lambda \in \mathbb{R}$). For any $x, y \in V'$, we have

$$(G^\square(x)|G^\square(y)) = (x|GG^\square(y)) = (x|\lambda y) = \lambda(x|y).$$

Hence λ is a positive real number and G is an isogeny of constant λ .

DEFINITION 6. Let V be a triple system over \mathbb{K} and $D : V \rightarrow V$ a linear map. We say that D is a derivation if it satisfies

$$D\langle xyz \rangle = \langle D(x)yz \rangle + \langle xD(y)z \rangle + \langle xyD(z) \rangle,$$

for any $x, y, z \in V$.

LEMMA 7. *Let V be a complex H^* -triple system with non-zero triple product, F a continuous automorphism of V and D a continuous derivation of V . Then*

- (a) *there exist $\lambda, \mu, \nu \in \text{Sp}(F)$, such that $\lambda\mu\nu \in \text{Sp}(F)$;*
- (b) *there exist $\lambda, \mu, \nu \in \text{Sp}(D)$, such that $\lambda + \mu + \nu \in \text{Sp}(D)$;*
- (c) *$\text{Sp}(F)$ cannot be contained in a halfline of origin 0 different from \mathbb{R}^+ and \mathbb{R}^- ;*
- (d) *$\text{Sp}(D)$ cannot be contained in a line other than for the lines containing the origin.*

PROOF. Let F be the Banach space of the continuous trilinear maps of $V \times V \times V$ into V . For any $T \in BL(V)$ we define the maps $\overset{a}{T}, \overset{b}{T}, \overset{c}{T}, \overset{d}{T} \in BL(F)$ by

$$\overset{a}{T}(f)(x, y, z) = T(f(x, y, z))$$

$$\overset{b}{T}(f)(x, y, z) = f(T(x), y, z)$$

$$\overset{c}{T}(f)(x, y, z) = f(x, T(y), z)$$

$$\overset{d}{T}(f)(x, y, z) = f(x, y, T(z)).$$

If $f_0(x, y, z) = \langle xyz \rangle$ then T is an automorphism iff $\overset{a}{T}(f_0) = \overset{b}{T} \overset{c}{T} \overset{d}{T}(f_0)$ and T is a derivation iff $\overset{a}{T}(f_0) = (\overset{b}{T} + \overset{c}{T} + \overset{d}{T})(f_0)$. The map $T \mapsto \overset{a}{T}$ is a continuous morphism which preserves the unity and the maps $T \mapsto \overset{x}{T}$, $x \in \{b, c, d\}$, are continuous skewmorphisms which preserve the unity. Hence

$$(8) \quad \text{Sp}(\overset{x}{T}) \subset \text{Sp}(T), \quad x \in \{a, b, c, d\}.$$

If F is an automorphism then $\overset{a}{F}(f_0) = \overset{b}{F} \overset{c}{F} \overset{d}{F}(f_0)$, that is, $(\overset{a}{F} - \overset{b}{F} \overset{c}{F} \overset{d}{F})(f_0) = 0$, and therefore $0 \in \text{Sp}(\overset{a}{F} - \overset{b}{F} \overset{c}{F} \overset{d}{F})$. It follows from the fact that $\{\overset{x}{F}\}_{x \in \{a, b, c, d\}}$ is a commutative set, that $0 \in \text{Sp}(\overset{a}{F}) - \text{Sp}(\overset{b}{F})\text{Sp}(\overset{c}{F})\text{Sp}(\overset{d}{F})$ (see [11, p. 280]). So there exist $\rho \in \text{Sp}(\overset{a}{F})$, $\lambda \in \text{Sp}(\overset{b}{F})$, $\mu \in \text{Sp}(\overset{c}{F})$, $\nu \in \text{Sp}(\overset{d}{F})$, such that $0 = \rho - \lambda\mu\nu$. Part (a) now follows from (8). In a similar way part (b) can be obtained. Parts (c) and (d) are consequence of (a) and (b).

Let V be a Hilbert space and $F \in BL(V)$. We recall that F is a positive operator if F is self-adjoint and $(F(x)|x) \geq 0$ for any $x \in V$. In the complex case the self-adjointness follows from the last condition.

LEMMA 9. *Let V and V' be two topologically simple complex H^* -triple systems and $F : V \rightarrow V'$ an isomorphism. Then either $(F^*)^{-1} \circ F$ or $-(F^*)^{-1} \circ F$ is a positive operator.*

PROOF. By [6, Teorema 14] and Proposition 1 we have $F^* \circ F^\square = \lambda \text{Id}$ ($\lambda \in \mathbb{C}$). Firstly we prove that $\lambda \in \mathbb{R} - \{0\}$. The fact that $\lambda \neq 0$ is obtained from the invertibility of F . So

$$(F^*)^{-1} \circ F = \frac{1}{\lambda} F^\square \circ F.$$

Since $F^\square \circ F$ is a positive operator

$$\text{Sp} \left(\frac{1}{\lambda} F^\square \circ F \right) \subset \frac{1}{\lambda} \mathbb{R}^+,$$

and, by Lemma 7, $\lambda \in \mathbb{R} - \{0\}$. Then we have either $(F^*)^{-1} \circ F$ or $-(F^*)^{-1} \circ F$ is a positive operator. This finishes the proof.

From the following lemmata we shall prove that the unique positive root of $(F^*)^{-1} \circ F$ or $-(F^*)^{-1} \circ F$ is also a morphism. Next we generalize a well know result (see [10, Lemme 8 p. 313]).

LEMMA 10. *Let E be a complex Banach space and \hat{F} the Banach space of the continuous multilinear maps of E^n into E . Let $D \in BL(E)$ and $F = e^D$. We define $D' \in BL(F)$ and $F' \in BL(\hat{F})$ by*

$$\begin{aligned} (D'f)(\zeta_1, \zeta_2, \dots, \zeta_n) &= D(f(\zeta_1, \zeta_2, \dots, \zeta_n)) - f(D\zeta_1, \zeta_2, \dots, \zeta_n) \\ &\quad - f(\zeta_1, D\zeta_2, \dots, \zeta_n) - \dots - f(\zeta_1, \zeta_2, \dots, D\zeta_n) \\ (F'f)(\zeta_1, \zeta_2, \dots, \zeta_n) &= F(f(F^{-1}\zeta_1, F^{-1}\zeta_2, \dots, F^{-1}\zeta_n)) \end{aligned}$$

with $\zeta_1, \zeta_2, \dots, \zeta_n \in E$, $f \in \hat{F}$. Then

- (a) $D'f = 0$ implies $F'f = f$.
- (b) If $F'f = f$, and $\text{Sp}D \subset \left\{ z \in \mathbb{C} : |\text{Im}(z)| < \frac{2\pi}{(n+1)} \right\}$, then $D'f = 0$.

PROOF. We define maps $x_1, x_1, \dots, x_n \in BL(\hat{F})$ by

$$\begin{aligned} (x_0f)(\zeta_1, \dots, \zeta_n) &= D(f(\zeta_1, \dots, \zeta_n)), \\ (x_i f)(\zeta_1, \dots, \zeta_n) &= -f(\zeta_1, \dots, D\zeta_i, \dots, \zeta_n), \quad i \in \{1, \dots, n\}, \end{aligned}$$

taking into account that $D' = \sum_{i=0}^n x_i$ and that the x_i pairwise commute, we can conclude the proof as in [10, Lemme 8 p. 314].

LEMMA 11. *Let V be a complex complete normed triple system and F a continuous automorphism of V such that*

$$\text{Sp}(F) \subset \left\{ z \in \mathbb{C} : |\arg(z)| < \frac{\pi}{2} \right\}.$$

Then there exists a unique continuous derivation $D : V \rightarrow V$ such that $e^D = F$ and

$$\text{Sp}(D) \subset \left\{ z \in \mathbb{C} : |\text{Im}(z)| < \frac{\pi}{2} \right\}.$$

PROOF. Let f be the triple product of V and \log the principal determination of the logarithm. Let $D = \log(F)$. Because F is an automorphism, it follows that $F'f = f$, with F' as in Lemma 10. The condition on F implies that

$$\text{Sp}(D) = \log(\text{Sp}(F)) \subset \left\{ z \in \mathbb{C} : |\text{Im}(z)| < \frac{\pi}{2} \right\},$$

and by Lemma 10 (b) we have that $D'f = 0$, that is D is a derivation.

PROPOSITION 12. *Let V be a topologically simple H^* -triple system and F a positive automorphism of V . Then there exists a unique positive automorphism G of V such that $G^2 = F$. Moreover if $(F^*)^{-1} = F$, then $(G^*)^{-1} = G$.*

PROOF. Let V be a complex topologically simple H^* -triple system. Since $\text{Sp}(F) \subseteq \mathbb{R}^+$, by Lemma 11 there exists a unique continuous derivation $D : V \rightarrow V$ such that $e^D = F$. Obviously $\frac{1}{2}D$ is a derivation of V , and by Lemma 10 (a) and the spectral mapping theorem, $G = e^{(1/2)D}$ is a positive automorphism of V such that $G^2 = F$. If V is a real H^* -triple system with $(Z(V), \square) = (\mathbb{R} \text{ Id}, \text{ Id})$, the unique positive automorphism \hat{G} of $\mathcal{C}(V)$, with $\hat{G}^2 = \mathcal{C}(F)$ can be obtained by arguing over the automorphism $\mathcal{C}(F)$ of the complexified $\mathcal{C}(V)$ of V given by $\mathcal{C}(F) : (a+bi) \mapsto F(a)+F(b)i$. A direct calculation proves that $\tau\hat{G}\tau$ is another positive root of $\mathcal{C}(F)$, where τ is the (real) involutive \mathbb{C} -antilinear automorphism of V given by $\tau : (a+bi) \mapsto (a-bi)$. So $\hat{G}(V) \subseteq V$ and $G = \hat{G}|_V$ is the unique positive automorphism of V such that $G^2 = F$. Finally, if V is a real H^* -triple system with $(Z(V), \square) = (\mathbb{C} \text{ Id}, -)$, that is, V is the realization of a complex H^* -triple system, then as in [1, Lemma 1.4.3] it can be proved that F is either \mathbb{C} -linear or \mathbb{C} -antilinear. But the (real) positivity of F implies that F must be a \mathbb{C} -linear automorphism of V . Hence this case follows from the complex one.

Suppose now that $(F^*)^{-1} = F$. By Proposition 1, we obtain $(G^*)^{-1} = \mu G$, for some $\mu \in Z(V)$ ($Z(V) = \mathbb{R}$ or \mathbb{C}), since G and G^* are positive operators $\mu > 0$. On the other hand, we have

$$\mu^2 F = (\mu G) \circ (\mu G) = (G^*)^{-1} \circ (G^*)^{-1} = (F^*)^{-1} = F$$

so $\mu = 1$ and the proposition is proved.

MAIN THEOREM 13. *Let V and V' be two topologically simple H^* -triple systems and $F : V \rightarrow V'$ an isomorphism. Then either $F : V \rightarrow V'$,*

or $-F : V^b \rightarrow V'$ splits in a unique way

$$\sigma F = F_2 \circ F_1, \quad \sigma \in \{1, -1\},$$

where F_1 is a positive automorphism, F_2 is a $*$ -isomorphism and V^b is the twin of V , that is, the H^* -triple system with the same Hilbert space and triple product as V and involution $x \mapsto -x^*$.

In particular if V and V' are isomorphic, then either V or V^b is $*$ -isomorphic to V' .

PROOF. Let $H = (F^*)^{-1} \circ F$. By Lemma 9 and the proposition above, we obtain that either H or $-H$ has a unique positive root F_1 . First we suppose that H is a positive operator. A direct calculation prove that $(H^*)^{-1} = H$, and by Proposition 12 we have $(F_1^*)^{-1} = F_1$. Then $F_2 = F \circ F_1^{-1}$ is a $*$ -isomorphism. Indeed from

$$F_1^{-1} \circ (F^*)^{-1} \circ F_2 = F_1^{-1} \circ (F^*)^{-1} \circ F \circ F_1^{-1} = F_1^{-1} \circ F_1^2 \circ F_1^{-1} = \text{Id},$$

we obtain

$$F_1^{-1} \circ (F^*)^{-1} = F_2^{-1},$$

or equivalently

$$F_2^* = F \circ F_1^*.$$

It follows from $(F_1^*)^{-1} = F_1$ that $F_2^* = F_2$, and F_2 is a $*$ -isomorphism. If H is a negative operator we argue over $-\text{Id} \circ H$ in a similar way, taking into account that $-\text{Id} : V \rightarrow V^b$ shows that $(-\text{Id})^* = \text{Id}$. Finally we prove the uniqueness of the factorization. Arguing as in the proof of Lemma 9, we have that $G^\square = \lambda(G)(G^*)^{-1}$ for every automorphism G , where $\lambda(G)$ is a non-zero real number. Moreover $\lambda(G) > 0$ if G is positive. Suppose that $F = F_2 \circ F_1$ with F_1 a positive automorphism and F_2 a $*$ -isomorphism. Then

$$\begin{aligned} (F^*)^{-1} \circ F &= (F_2^* \circ F_1^*)^{-1} \circ F_2 \circ F_1 = (F_1^*)^{-1} \circ (F_2^*)^{-1} \circ F_2 \circ F_1 \\ &= (F_1^*)^{-1} \circ F_1 = \lambda(F_1)^{-1} F_1^2, \end{aligned}$$

so F_1 is the unique positive root of the operator $(G^*)^{-1} \circ G$ with $G = \lambda(F_1)^{1/2} F$ and, by Proposition 12, $(F_1^*)^{-1} = F_1$, that is $\lambda(F_1) = 1$ and the factorization is unique. In a similar way, we can obtain the uniqueness in the case $-F = F_2 \circ F_1$.

COROLLARY 14 (Essential uniqueness). *Let V be a topologically simple H^* -triple system over \mathbb{K} ($\mathbb{K} = \mathbb{R}$ or \mathbb{C}) and V' another H^* -triple system with the same underlying \mathbb{K} -triple system of V . Then either V or V^b is isogenic to V' .*

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