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A SPECIAL CLASS OF TWO DIMENSIONAL EXPONENTIAL-BESSEL SERIES
OF FOX 'S H - FUNCTION

Abstract: In this paper, we present a special class of two-dimensional Exponential-Bessel series of Fox 's H-function [2], and present one two-dimensional series of this class.

The following formulae are required in the proof:

The integral [1, p.46, (1)]:

$$(1.1) \quad \int_0^{\pi} e^{-2iux} (\sin x)^{w_1-1} H_{p,q}^{m,n} \left[z (\sin x)^{-2h} \left| \begin{matrix} (a_p, e_p) \\ (b_q, f_q) \end{matrix} \right. \right] dx$$

$$= \frac{e^{-iu}}{2^{w_1-1}} H_{p+2,q+2}^{m+1,n} \left[z 2^{2h} \left| \begin{matrix} (a_p, e_p), (\frac{w_1+1}{2} \pm u, h) \\ (w_1, 2h), (b_q, f_q) \end{matrix} \right. \right],$$

where

$$h > 0, \sum_{j=1}^p e_j - \sum_{j=1}^q f_j \equiv A \leq 0, \sum_{j=1}^n e_j - \sum_{j=n+1}^p e_j + \sum_{j=1}^m f_j - \sum_{j=m+1}^q f_j \equiv B > 0,$$

$$|\arg z| < \frac{1}{2} B \pi, \operatorname{Re} w_1 - 2h \max_{1 \leq j \leq n} [\operatorname{Re}(a_j - 1)/e_j] > 0.$$

The integral [4, p. 94, (2.2)]:

$$(1.2) \quad \int_0^{\infty} y^{w_2-1} \operatorname{siny} J_{\nu}(y) H_{p,q}^{m,n} \left[zy^{2k} \left| \begin{matrix} (a_p, e_p) \\ (b_q, f_q) \end{matrix} \right. \right] dy$$

$$= 2^{w_2-1} \sqrt{\pi} H_{p+4, q+1}^{m+1, n+1} \left[\begin{matrix} \left(\frac{1-w_2-\nu}{2}, k \right), (a_p, e_p), \left(1 + \frac{\nu-w_2}{2}, k \right), \\ \left(1 - \frac{\nu+w_2}{2}, k \right), \left(\frac{1+\nu-w_2}{2}, k \right) \\ \left(\frac{1}{2} - w_2, 2k \right), (b_q, f_q) \end{matrix} \right],$$

where $k > 0$, $A \leq 0$, $B > 0$, $|\arg z| < \frac{1}{2} B \pi$, $\operatorname{Re}(w_2 + \nu) + 2k \min_{1 \leq j \leq m} [\operatorname{Re} b_j / f_j] > 0$.

The orthogonality property of the Bessel functions [3, p. 291, (6)]:

$$(1.3) \quad \int_0^{\infty} x^{-1} J_{a+2n+1}(x) J_{a+2m+1}(x) dx$$

$$= \begin{cases} 0, & \text{if } m \neq n; \\ (4n+2a+2)^{-1}, & \text{if } m = n, \operatorname{Re} a + m + n > -1. \end{cases}$$

2. TWO -DIMENSIONAL EXPONENTIAL-BESSEL SERIES. The two-dimensional Exponential-Bessel series to be established is

$$(2.1) \quad (\operatorname{sinx})^{w_1-1} y^{w_2} \operatorname{siny} H_{p,q}^{m,n} \left[z(\operatorname{sinx})^{-2h} y^{2k} \left| \begin{matrix} (a_p, e_p) \\ (b_q, f_q) \end{matrix} \right. \right]$$

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$$= 2^{1-w_1+w_2} \sum_{r=-\infty}^{\infty} \sum_{s=0}^{\infty} (a+2s+1) e^{2ir(x-\pi/2)} J_{a+2s+1}(y)$$

$$\times H_{p+6, q+2}^{m+2, n+1} \left[z^{2(h+k)} \left[\begin{array}{l} \left(-\frac{w_2+a+2s}{2}, k \right), (a_p, e_p), \left(\frac{w_1+1}{2} \pm r, h \right), \\ \left(1 + \frac{a+2s+1-w_2}{2}, k \right), \\ \left(1 - \frac{a+2s+1+w_2}{2}, k \right), \left(1 + \frac{a+2s-w_2}{2}, k \right) \\ (w_1, 2h), (1/2-w_2, k), (b_q, f_q) \end{array} \right. \right]$$

valid under the conditions of (1.1), (1.2) and (1.3).

Proof. Let

$$(2.2) \quad f(x, y) = (\sin x)^{w_1-1} y^{w_2} \sin y H_{p, q}^{m, n} \left[z(\sin x)^{-2h} y^{2k} \left[\begin{array}{l} (a_p, e_p) \\ (b_q, f_q) \end{array} \right. \right]$$

$$= \sum_{r=-\infty}^{\infty} \sum_{s=0}^{\infty} C_{r, s} e^{2irx} J_{a+2s+1}(y).$$

Equation (2.2) is valid, since $f(x, y)$ is continuous and bounded variation in the region $0 < x < \Pi$, $0 < y < \infty$.

Multiplying both sides of (2.2) by $y^{-1} J_{a+2v+1}(y)$ and integrating with respect to y from 0 to ∞ , then using (1.2) and (1.3). Now multiplying both sides of the resulting expression by e^{-2iux} and integrating with respect to x from 0 to Π , then using (1.1) and the orthogonality property of exponential functions, we obtain the value of $C_{r, s}$. Substituting this value of $C_{r, s}$ in (2.2), the expansion (2.1) is obtained.

Note : On applying the same procedure as above, we can establish three other forms of two-dimensional expansions of this class with the help of alternative forms of (1.1) and (1.2).

Since on specializing the parameters Fox's H-function yields almost all special functions appearing in applied mathematics and physical sciences. Therefore, the result presented in this paper is of a general character and hence may encompass several cases of interest.

R E F E R E N C E S

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