Erratum

Erratum to “Well-posedness and scattering for the KP-II equation in a critical space”

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1. Erratum

We will point out and correct an error which has occurred in Section 2 of [Hadac, Martin, Herr, Sebastian and Koch, Herbert, Well-posedness and scattering for the KP-II equation in a critical space, Ann. Inst. H. Poincaré Anal. Non Linéaire 26 (3) (2009) 917–941] and which came to our attention after publication of this work. It has no implications for the main results of the paper, but some adjustments are necessary in the section on $U^p$ and $V^p$ spaces.

In Definition 2.3 of the space $V^p$ we included the normalizing condition $\lim_{t \to \infty} v(t) = 0$. This, however, leads the problem that Theorem 2.8 on duality is incorrect as stated: Indeed, for fixed nonzero $\phi \in L^2$ consider the functional

$$T(u) = \langle \lim_{t \to \infty} u(t), \phi \rangle$$
on $U^p$. This cannot be represented by $V^p$ functions via the bilinear form $B$. The following modifications can be performed in order to resolve this issue:

(i) Define $Z_0$ as follows: $Z_0$ is defined as the set of finite partitions $-\infty < t_0 < t_1 < \cdots < t_K \leq \infty$.

(ii) Remove the notational convention for $u(-\infty)$ and $u(\infty)$ from Proposition 2.2, item (iv) (this change is purely notational).

(iii) We define $V^p$ as the normed space of all functions $v: \mathbb{R} \to L^2$ such that $\lim_{t \to \pm\infty} v(t)$ exist and for which the norm

$$\|v\|_{V^p} := \sup_{\{t_k\}_{k=0}^K \in \mathcal{Z}} \left( \sum_{k=1}^{K} \left\|v(t_k) - v(t_{k-1})\right\|_{L^2}^p \right)^{\frac{1}{p}}$$

is finite, where we use the convention that $v(-\infty) = \lim_{t \to -\infty} v(t)$ and $v(\infty) = 0$ (here, the difference is that $v(\infty) = 0$ does not necessarily coincide with the limit at $\infty$). This convention will also be used in the sequel. Likewise, let $V^p_-$ denote the closed subspace of all $v \in V^p$ with $\lim_{t \to -\infty} v(t) = 0$ (note that the space $V^p_-$ is unchanged).

(iv) Proposition 2.7 on the bilinear form remains unchanged. However, notice that our convention is $v(t_K) = 0$ since $t_K = \infty$ for all partitions $\{t_k\}_{k=0}^K \in \mathcal{Z}$.


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(v) In the proof of Theorem 2.8 we have \( \tilde{v}(t) = v(t) \) and the error in the calculation at the end of the proof is corrected.

(vi) The conclusion of Proposition 2.10 remains valid with the modified definition of \( V^p \) by a similar proof. Alternatively, it can be seen as follows: We know that it is correct for \( v - \lim_{t \to \infty} v(t) \) as proved in Section 2 and we obtain

\[
B\left(u, v - \lim_{t \to \infty} v(t)\right) = - \int_{-\infty}^{\infty} \left(u'(s), v(s) - \lim_{t \to \infty} v(t)\right) ds
\]

which is equivalent to

\[
B(u, v) + \lim_{t \to \infty} \langle u(t), v(t) \rangle = - \int_{-\infty}^{\infty} \left(u'(s), v(s) - \lim_{t \to \infty} v(t)\right) ds
\]

which shows the claim.